Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.2-Cosine/83-4.2.10-c+d-x-^m-a+b-cos-^n

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [189]. This is test number [83].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (189)	0.00 (0)
Mathematica	100.00 (189)	0.00 (0)
Maxima	74.07 (140)	25.93 (49)
Fricas	72.49 (137)	27.51 (52)
Maple	71.43 (135)	28.57 (54)
Giac	59.26 (112)	40.74 (77)
Mupad	39.15 (74)	60.85 (115)
Sympy	29.10 (55)	70.90 (134)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

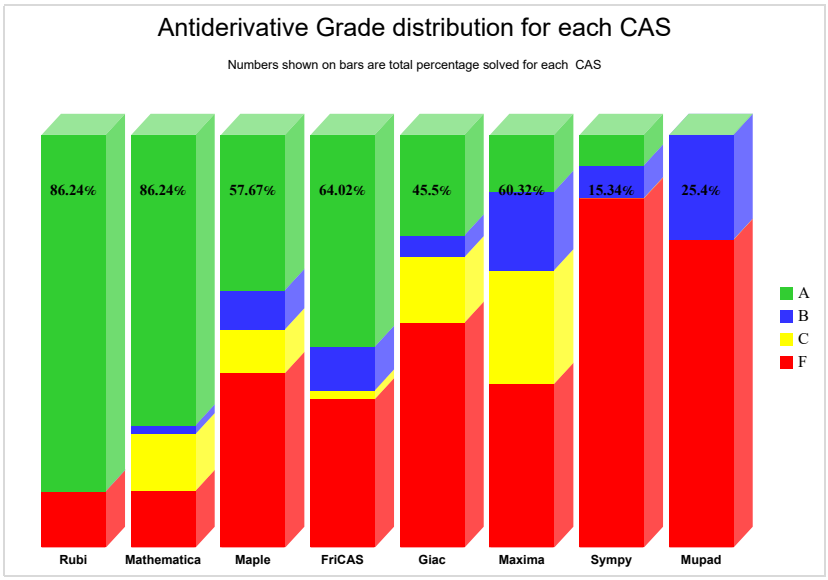
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

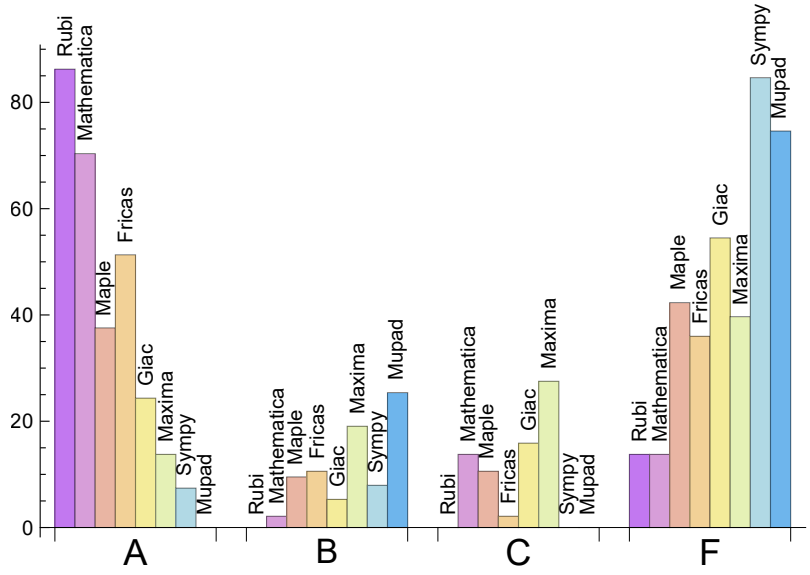
System	% A grade	% B grade	% C grade	% F grade
Rubi	86.243	0.000	0.000	13.757
Mathematica	70.370	2.116	13.757	13.757
Fricas	51.323	10.582	2.116	35.979
Maple	37.566	9.524	10.582	42.328
Giac	24.339	5.291	15.873	54.497
Maxima	13.757	19.048	27.513	39.683
Sympy	7.407	7.937	0.000	84.656
Mupad	0.000	25.397	0.000	74.603

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of

error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	52	17.31	0.00	82.69
Maxima	49	91.84	0.00	8.16
Maple	54	100.00	0.00	0.00
Giac	77	100.00	0.00	0.00
Mupad	115	0.00	100.00	0.00
Sympy	134	97.01	2.99	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.20
Fricas	0.25
Giac	0.41
Maxima	0.65
Maple	0.90
Mathematica	1.17
Sympy	6.46
Mupad	11.37

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	77.66	1.25	34.50	1.10
Mathematica	105.86	0.98	76.00	0.93
Rubi	110.17	1.00	89.00	1.00
Sympy	138.16	1.82	60.00	1.55
Maple	165.08	1.62	108.00	1.12
Fricas	201.34	1.60	114.00	1.18
Maxima	326.14	5.15	163.50	1.49
Giac	1602.83	11.15	80.50	1.14

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

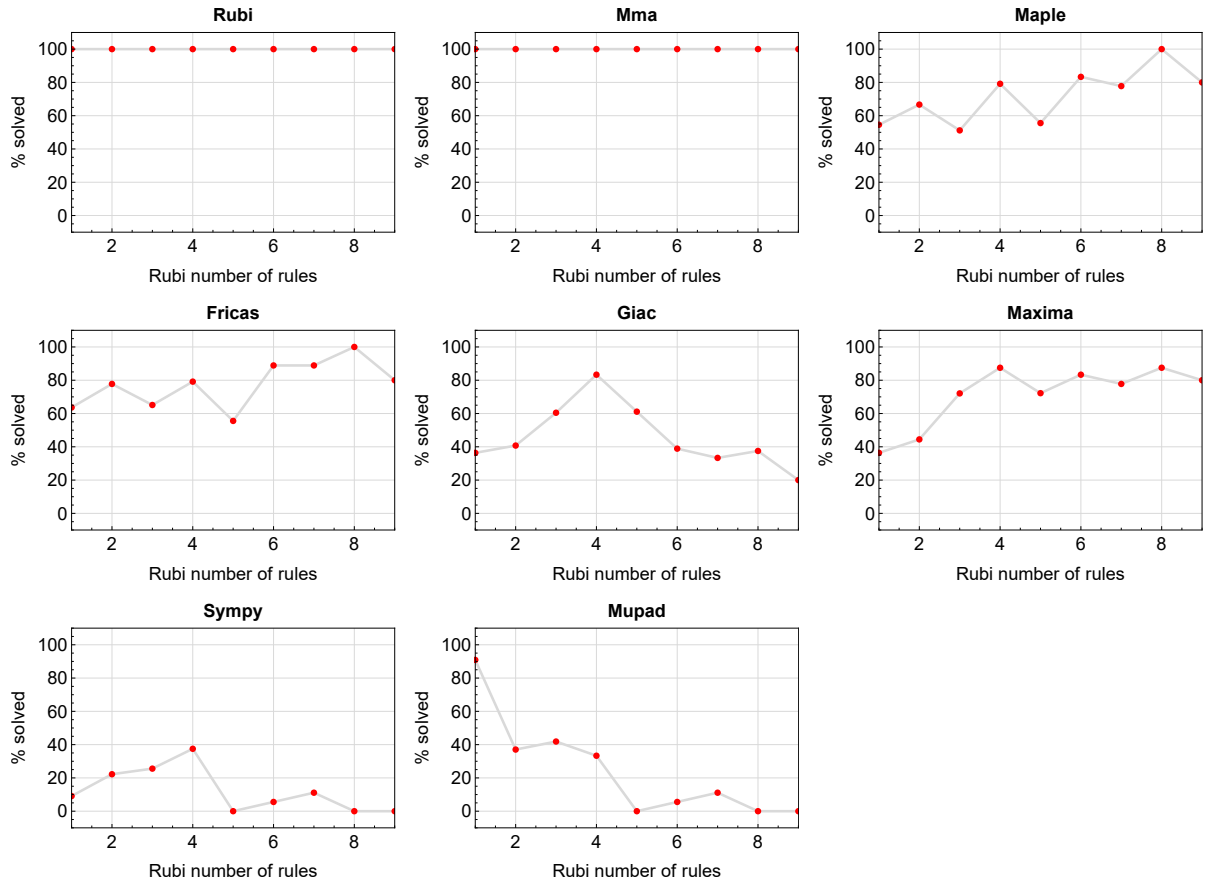


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

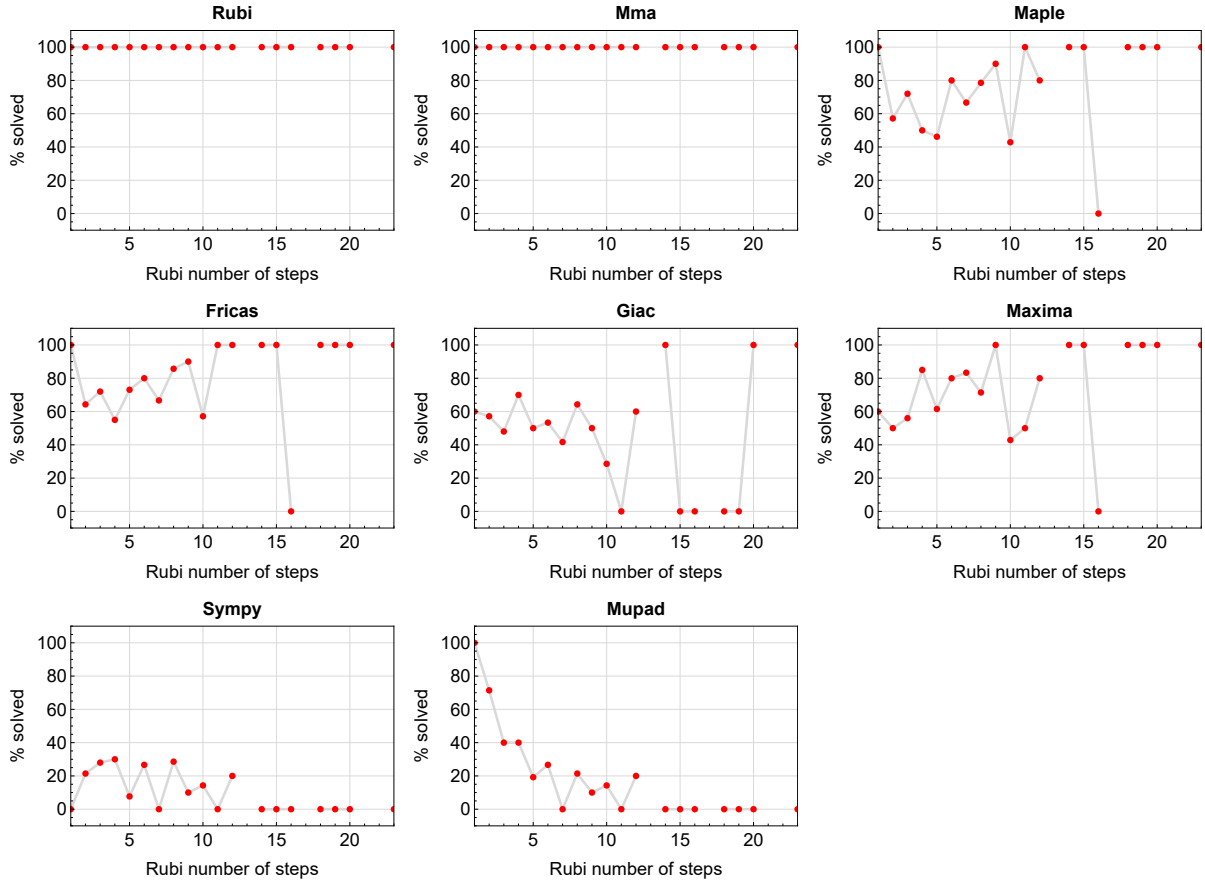


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

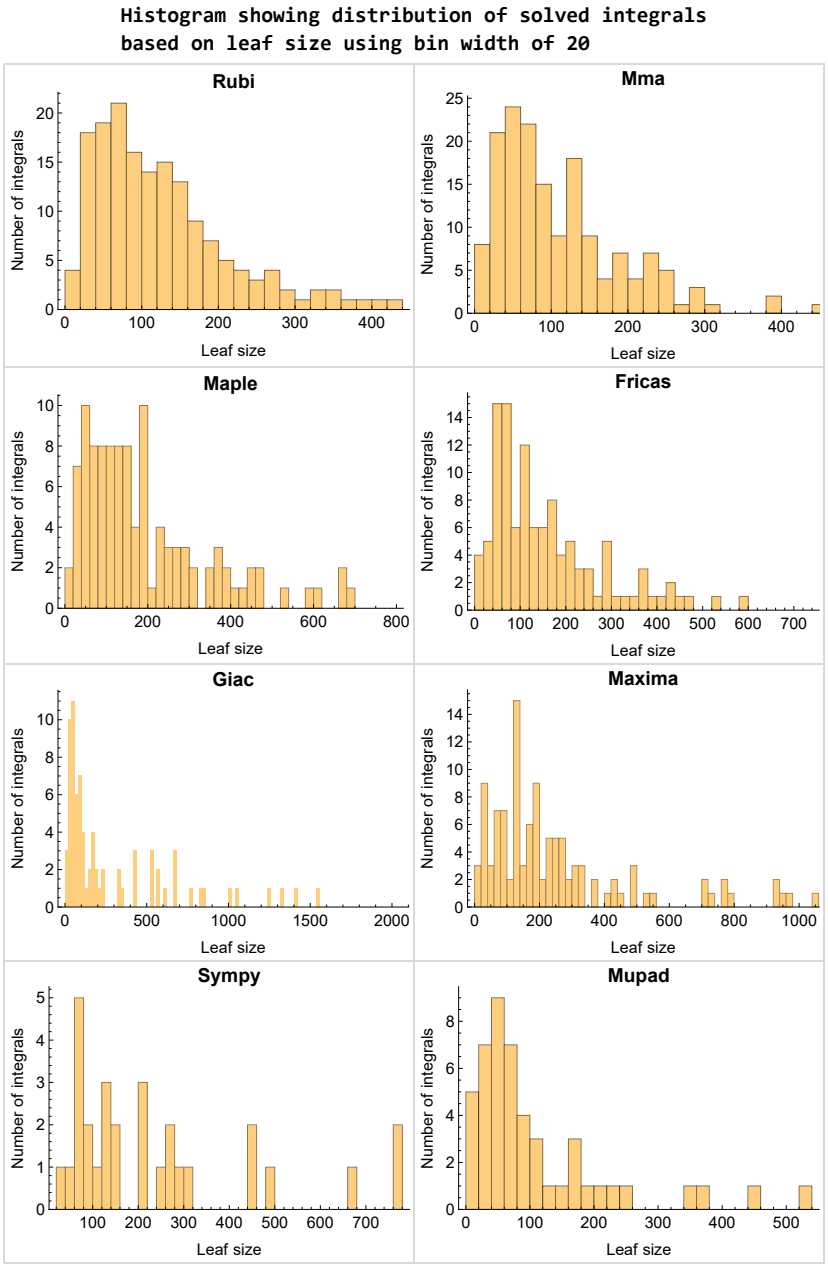


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

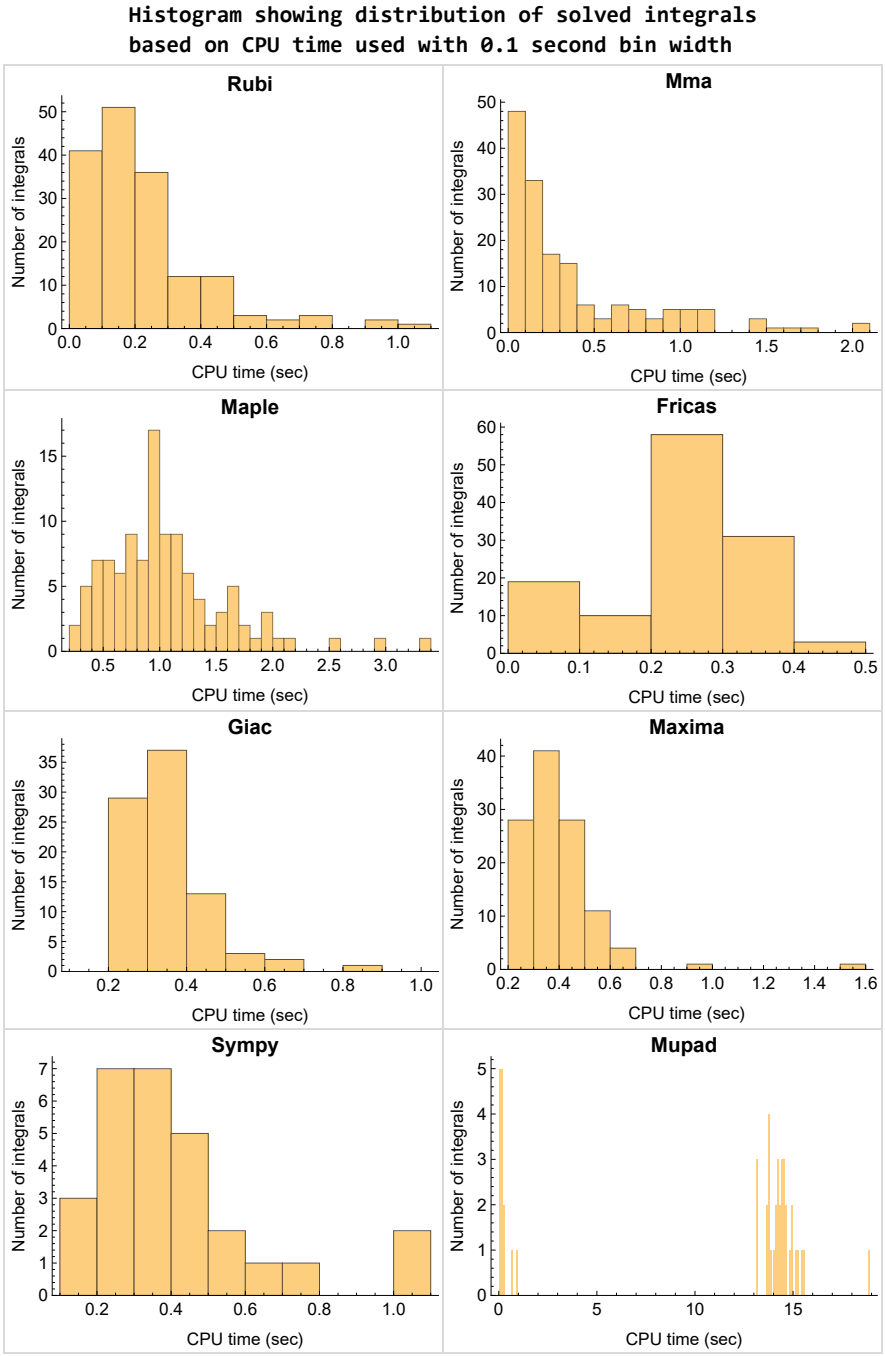


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

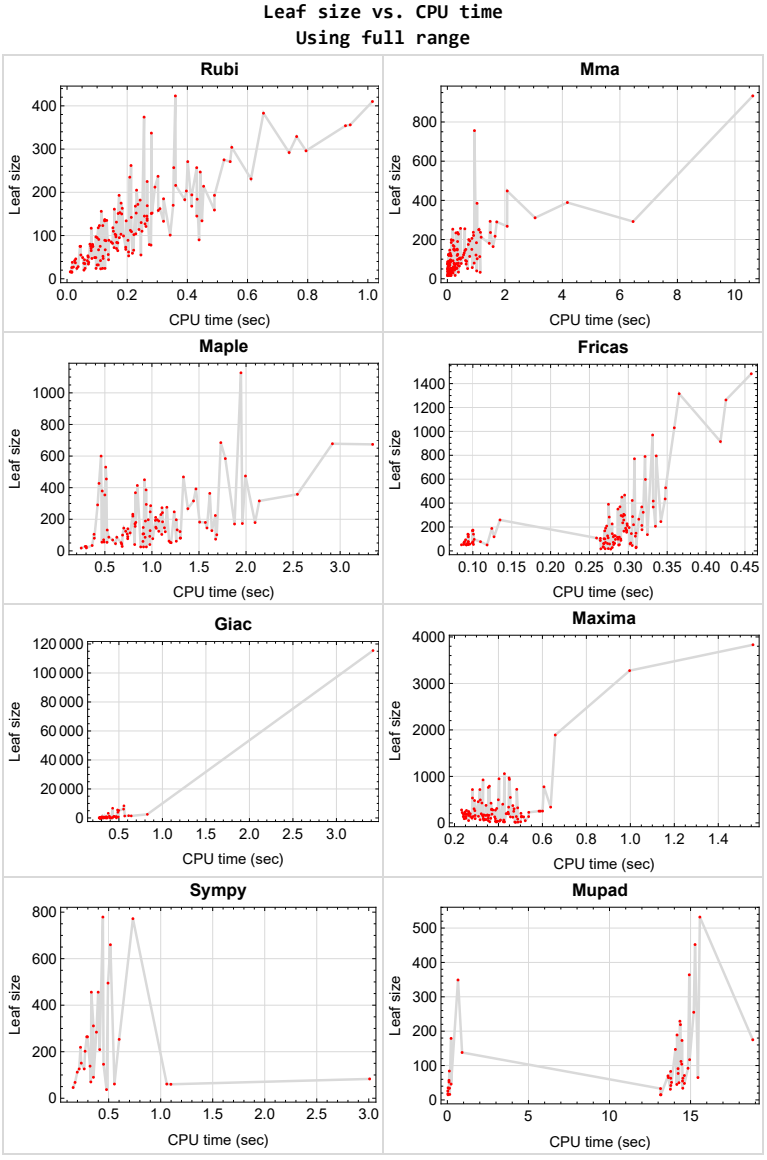


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{32, 36, 40, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 131, 132, 136, 137, 141, 142, 174, 179, 183, 184, 188}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {189}

Maple {173}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

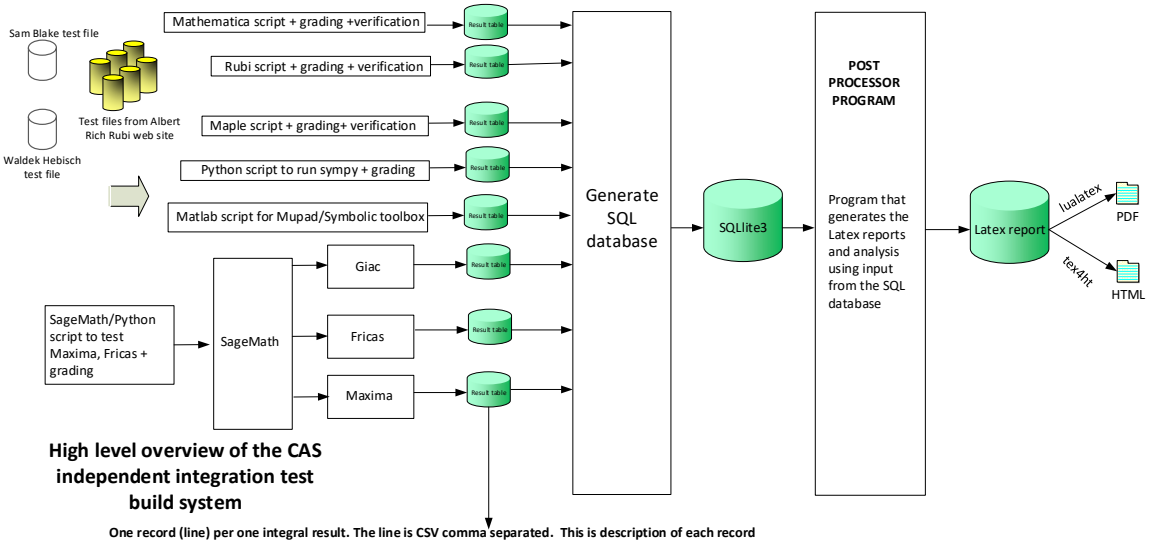
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
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2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 24
Giac 24
Mupad 24
Sympy 25

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 189 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 138, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 180, 181, 182, 185, 186 }

B grade { 39, 139, 187, 189 }

C grade { 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 135, 140, 146, 153, 160, 178 }

B grade { 29, 30, 33, 34, 37, 38, 39, 76, 79, 87, 128, 129, 133, 134, 138, 139, 187, 189 }

C grade { 13, 14, 84, 104, 105, 106, 107, 108, 109, 110, 143, 144, 145, 150, 151, 152, 157, 158, 159, 173 }

F normal fail { 67, 68, 69, 70, 71, 72, 73, 74, 81, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 111, 112, 113, 114, 115, 116, 117, 147, 148, 149, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 180, 181, 182, 185, 186 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 91, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 135, 140, 146, 153, 160, 173, 178 }

B grade { 8, 29, 30, 31, 33, 34, 37, 38, 39, 55, 128, 129, 133, 134, 138, 139, 185, 186, 187, 189 }

C grade { 76, 79, 84, 87 }

F normal fail { 170, 171, 172, 175, 176, 177, 180, 181, 182 }

F(-1) timedout fail { }

F(-2) exception fail { 75, 77, 78, 80, 81, 82, 83, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 97, 143, 144, 145, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 184 }

Maxima

A grade { 4, 12, 19, 23, 24, 25, 67, 68, 69, 70, 71, 72, 73, 74, 125, 144, 145, 146, 150, 151, 152, 153, 160, 164, 165, 166 }

B grade { 1, 2, 3, 9, 10, 11, 16, 17, 18, 29, 30, 33, 34, 35, 37, 38, 118, 119, 120, 123, 124, 128, 129, 130, 133, 134, 135, 138, 139, 140, 143, 157, 158, 159, 173, 178 }

C grade { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 26, 27, 28, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 121, 122, 126, 127, 147, 148, 149, 154, 155, 156, 167, 168, 169 }

F normal fail { 31, 39, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 161, 162, 163, 170, 171, 172, 175, 176, 177, 180, 181, 182 }

F(-1) timedout fail { }

F(-2) exception fail { 185, 186, 187, 189 }

Giac

A grade { 1, 2, 3, 4, 9, 10, 11, 12, 16, 17, 18, 19, 23, 24, 25, 118, 119, 120, 123, 124, 125, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 178 }

B grade { 6, 14, 21, 35, 122, 127, 130, 135, 140, 173 }

C grade { 5, 7, 8, 13, 15, 20, 22, 26, 27, 28, 41, 42, 43, 44, 48, 49, 50, 51, 56, 57, 58, 59, 63, 64, 65, 121, 126, 147, 148, 149 }

F normal fail { 29, 30, 31, 33, 34, 37, 38, 39, 45, 46, 47, 52, 53, 54, 55, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 128, 129, 133, 134, 138, 139, 170, 171, 172, 175, 176, 177, 180, 181, 182, 185, 186, 187, 189 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 9, 10, 11, 12, 16, 17, 18, 19, 23, 24, 25, 35, 64, 65, 76, 79, 84, 87, 89, 90, 91, 92, 118, 119, 120, 123, 124, 125, 130, 135, 140, 143, 144, 145, 146, 150, 151, 152, 153, 157, 158, 159, 160, 173 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 26, 27, 28, 29, 30, 31, 33, 34, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 71, 72, 73, 74, 81, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 126, 127, 128, 129, 133, 134, 138, 139, 147, 148, 149, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 189 }

F(-2) exception fail { }

Sympy

A grade { 4, 19, 23, 24, 25, 26, 63, 64, 65, 66, 120, 125, 130, 135 }

B grade { 1, 2, 3, 9, 10, 11, 12, 16, 17, 18, 118, 119, 123, 124, 140 }

C grade { }

F normal fail { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 126, 127, 128, 129, 133, 134, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187 }

F(-1) timedout fail { 55, 56, 92, 189 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	76	143	481	169	311	170	219
N.S.	1	1.00	0.84	1.57	5.29	1.86	3.42	1.87	2.41
time (sec)	N/A	0.105	0.314	1.046	0.291	0.298	0.353	0.293	14.378

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	107	278	109	202	110	147
N.S.	1	1.00	0.87	1.53	3.97	1.56	2.89	1.57	2.10
time (sec)	N/A	0.080	0.210	0.905	0.232	0.274	0.273	0.295	14.064

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	60	136	62	112	64	84
N.S.	1	1.00	0.90	1.22	2.78	1.27	2.29	1.31	1.71
time (sec)	N/A	0.052	0.172	0.905	0.330	0.273	0.198	0.313	0.122

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	28	50	28	46	30	34
N.S.	1	1.00	0.96	1.04	1.85	1.04	1.70	1.11	1.26
time (sec)	N/A	0.021	0.113	0.684	0.264	0.279	0.158	0.324	0.093

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	77	142	62	0	577	0
N.S.	1	1.00	0.96	1.48	2.73	1.19	0.00	11.10	0.00
time (sec)	N/A	0.120	0.113	0.743	0.309	0.268	0.000	0.342	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	114	164	96	0	523	0
N.S.	1	1.00	0.89	1.56	2.25	1.32	0.00	7.16	0.00
time (sec)	N/A	0.195	0.417	0.769	0.327	0.283	0.000	0.314	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	89	148	199	165	0	5518	0
N.S.	1	1.00	0.86	1.42	1.91	1.59	0.00	53.06	0.00
time (sec)	N/A	0.229	0.668	0.908	0.435	0.294	0.000	0.484	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	144	184	249	235	0	8378	0
N.S.	1	1.00	1.13	1.45	1.96	1.85	0.00	65.97	0.00
time (sec)	N/A	0.259	0.611	1.113	0.469	0.293	0.000	0.558	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	132	145	717	287	660	222	349
N.S.	1	1.00	0.82	0.90	4.45	1.78	4.10	1.38	2.17
time (sec)	N/A	0.157	0.585	1.585	0.282	0.305	0.517	0.332	0.656

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	106	121	428	190	456	153	229
N.S.	1	1.00	0.79	0.90	3.19	1.42	3.40	1.14	1.71
time (sec)	N/A	0.125	0.447	1.142	0.325	0.271	0.399	0.301	14.343

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	77	81	222	113	264	94	179
N.S.	1	1.00	0.81	0.85	2.34	1.19	2.78	0.99	1.88
time (sec)	N/A	0.100	0.326	1.304	0.235	0.286	0.290	0.307	0.229

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	46	90	53	126	48	57
N.S.	1	1.00	0.91	0.84	1.64	0.96	2.29	0.87	1.04
time (sec)	N/A	0.046	0.269	0.613	0.244	0.281	0.217	0.292	0.103

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	107	163	71	0	610	0
N.S.	1	1.00	0.83	1.37	2.09	0.91	0.00	7.82	0.00
time (sec)	N/A	0.279	0.264	0.738	0.357	0.281	0.000	0.352	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	75	155	171	100	0	534	0
N.S.	1	1.00	0.90	1.87	2.06	1.20	0.00	6.43	0.00
time (sec)	N/A	0.169	0.654	0.822	0.350	0.270	0.000	0.356	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	102	193	204	176	0	5136	0
N.S.	1	1.00	0.91	1.72	1.82	1.57	0.00	45.86	0.00
time (sec)	N/A	0.211	1.037	1.037	0.443	0.297	0.000	0.499	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	385	173	925	350	772	351	532
N.S.	1	1.00	1.71	0.77	4.11	1.56	3.43	1.56	2.36
time (sec)	N/A	0.266	1.032	1.961	0.330	0.287	0.733	0.324	15.574

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	121	224	535	227	495	231	364
N.S.	1	1.00	0.69	1.28	3.06	1.30	2.83	1.32	2.08
time (sec)	N/A	0.180	0.996	1.674	0.282	0.298	0.494	0.311	14.923

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	93	128	267	128	284	137	173
N.S.	1	1.00	0.76	1.04	2.17	1.04	2.31	1.11	1.41
time (sec)	N/A	0.119	0.634	1.637	0.255	0.282	0.382	0.309	14.472

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	61	103	60	126	69	77
N.S.	1	1.00	0.69	0.81	1.37	0.80	1.68	0.92	1.03
time (sec)	N/A	0.045	0.227	1.259	0.279	0.268	0.265	0.299	14.235

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	103	171	278	123	0	6075	0
N.S.	1	1.00	0.85	1.41	2.30	1.02	0.00	50.21	0.00
time (sec)	N/A	0.262	0.409	0.820	0.374	0.309	0.000	0.554	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	200	247	304	177	0	1000	0
N.S.	1	1.00	1.38	1.70	2.10	1.22	0.00	6.90	0.00
time (sec)	N/A	0.257	0.744	0.982	0.471	0.318	0.000	0.440	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	221	316	339	289	0	115446	0
N.S.	1	1.00	1.20	1.72	1.84	1.57	0.00	627.42	0.00
time (sec)	N/A	0.433	0.913	1.442	0.637	0.293	0.000	3.420	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	100	102	303	115	253	108	138
N.S.	1	1.00	0.58	0.59	1.76	0.67	1.47	0.63	0.80
time (sec)	N/A	0.235	0.457	1.689	0.289	0.288	0.601	0.354	0.908

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	92	81	188	88	209	84	104
N.S.	1	1.00	0.69	0.60	1.40	0.66	1.56	0.63	0.78
time (sec)	N/A	0.197	0.197	1.300	0.248	0.290	0.414	0.398	14.456

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	53	62	98	63	138	64	63
N.S.	1	1.00	0.66	0.78	1.22	0.79	1.72	0.80	0.79
time (sec)	N/A	0.076	0.135	1.182	0.252	0.282	0.321	0.412	14.527

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	52	91	49	60	428	0
N.S.	1	1.00	0.88	0.88	1.54	0.83	1.02	7.25	0.00
time (sec)	N/A	0.217	0.158	0.673	0.331	0.288	1.098	0.433	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	79	90	717	66	0	3220	0
N.S.	1	1.00	1.20	1.36	10.86	1.00	0.00	48.79	0.00
time (sec)	N/A	0.222	0.230	0.744	0.316	0.303	0.000	0.377	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	119	124	790	100	0	3920	0
N.S.	1	1.00	1.32	1.38	8.78	1.11	0.00	43.56	0.00
time (sec)	N/A	0.439	0.342	0.721	0.359	0.290	0.000	0.489	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	196	685	722	970	0	0	0
N.S.	1	1.00	0.96	3.34	3.52	4.73	0.00	0.00	0.00
time (sec)	N/A	0.231	0.198	1.732	0.483	0.331	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	130	392	402	598	0	0	0
N.S.	1	1.00	0.95	2.86	2.93	4.36	0.00	0.00	0.00
time (sec)	N/A	0.125	0.101	1.468	0.420	0.322	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	87	128	0	306	0	0	0
N.S.	1	1.00	1.16	1.71	0.00	4.08	0.00	0.00	0.00
time (sec)	N/A	0.043	0.010	0.981	0.000	0.294	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.29
time (sec)	N/A	0.025	4.359	0.528	0.577	0.289	0.379	0.499	13.675

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	109	316	1059	790	0	0	0
N.S.	1	1.00	0.96	2.77	9.29	6.93	0.00	0.00	0.00
time (sec)	N/A	0.239	0.529	2.141	0.427	0.322	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	75	170	324	450	0	0	0
N.S.	1	1.00	0.91	2.07	3.95	5.49	0.00	0.00	0.00
time (sec)	N/A	0.151	0.264	1.878	0.487	0.292	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	36	52	159	45	0	1404	55
N.S.	1	1.00	1.29	1.86	5.68	1.61	0.00	50.14	1.96
time (sec)	N/A	0.037	0.017	1.204	0.352	0.305	0.000	0.642	14.505

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	290	18	14	18	18
N.S.	1	1.00	1.12	1.00	18.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.048	6.061	0.534	0.547	0.293	0.409	0.381	13.698

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	311	1127	3831	1315	0	0	0
N.S.	1	1.00	0.92	3.34	11.37	3.90	0.00	0.00	0.00
time (sec)	N/A	0.280	3.050	1.946	1.558	0.365	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	184	584	1891	795	0	0	0
N.S.	1	1.00	0.95	3.03	9.80	4.12	0.00	0.00	0.00
time (sec)	N/A	0.173	1.011	1.780	0.658	0.336	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	389	267	0	435	0	0	0
N.S.	1	1.00	3.32	2.28	0.00	3.72	0.00	0.00	0.00
time (sec)	N/A	0.080	4.174	1.382	0.000	0.347	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	290	18	14	18	18
N.S.	1	1.00	1.12	1.00	18.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.043	1.726	0.023	0.579	0.322	0.403	0.309	0.003

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	124	232	263	190	0	1246	0
N.S.	1	1.00	0.64	1.20	1.36	0.98	0.00	6.42	0.00
time (sec)	N/A	0.415	0.063	0.797	0.293	0.297	0.000	0.390	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	122	189	242	156	0	779	0
N.S.	1	1.00	0.72	1.12	1.43	0.92	0.00	4.61	0.00
time (sec)	N/A	0.267	0.094	0.931	0.264	0.298	0.000	0.357	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	122	144	196	126	0	426	0
N.S.	1	1.00	0.86	1.01	1.38	0.89	0.00	3.00	0.00
time (sec)	N/A	0.214	0.044	0.701	0.240	0.273	0.000	0.340	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	124	100	159	108	0	168	0
N.S.	1	1.00	1.05	0.85	1.35	0.92	0.00	1.42	0.00
time (sec)	N/A	0.153	0.061	0.688	0.294	0.259	0.000	0.322	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	147	140	129	144	0	0	0
N.S.	1	1.00	1.06	1.01	0.93	1.04	0.00	0.00	0.00
time (sec)	N/A	0.266	0.344	0.744	0.506	0.279	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	190	180	129	208	0	0	0
N.S.	1	1.00	1.13	1.07	0.77	1.24	0.00	0.00	0.00
time (sec)	N/A	0.415	0.355	0.829	0.515	0.272	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	228	220	129	296	0	0	0
N.S.	1	1.00	1.18	1.14	0.67	1.53	0.00	0.00	0.00
time (sec)	N/A	0.490	0.386	0.798	0.455	0.291	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	150	242	295	258	0	1331	0
N.S.	1	1.00	0.65	1.05	1.28	1.12	0.00	5.76	0.00
time (sec)	N/A	0.612	0.851	1.112	0.369	0.294	0.000	0.567	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	150	197	274	195	0	821	0
N.S.	1	1.00	0.74	0.97	1.35	0.96	0.00	4.04	0.00
time (sec)	N/A	0.397	0.643	0.961	0.386	0.301	0.000	0.472	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	150	150	229	148	0	436	0
N.S.	1	1.00	0.95	0.95	1.45	0.94	0.00	2.76	0.00
time (sec)	N/A	0.303	0.248	1.041	0.444	0.310	0.000	0.395	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	145	108	187	114	0	167	0
N.S.	1	1.00	1.12	0.83	1.44	0.88	0.00	1.28	0.00
time (sec)	N/A	0.246	0.250	0.913	0.371	0.286	0.000	0.328	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	175	146	135	136	0	0	0
N.S.	1	1.00	1.30	1.08	1.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.266	0.729	1.133	0.537	0.325	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	181	189	136	206	0	0	0
N.S.	1	1.00	1.06	1.11	0.80	1.21	0.00	0.00	0.00
time (sec)	N/A	0.353	1.458	1.073	0.481	0.335	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	237	230	136	323	0	0	0
N.S.	1	1.00	1.10	1.06	0.63	1.50	0.00	0.00	0.00
time (sec)	N/A	0.361	1.150	1.096	0.506	0.319	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	237	273	136	417	0	0	0
N.S.	1	1.00	0.96	1.11	0.55	1.69	0.00	0.00	0.00
time (sec)	N/A	0.443	0.904	1.107	0.504	0.332	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	236	474	547	368	0	2480	0
N.S.	1	1.00	0.58	1.16	1.33	0.90	0.00	6.05	0.00
time (sec)	N/A	1.015	1.499	1.994	0.454	0.317	0.000	0.827	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	251	386	497	299	0	1549	0
N.S.	1	1.00	0.71	1.09	1.40	0.84	0.00	4.38	0.00
time (sec)	N/A	0.926	1.100	0.938	0.399	0.295	0.000	0.612	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	234	294	424	245	0	848	0
N.S.	1	1.00	0.77	0.97	1.39	0.81	0.00	2.79	0.00
time (sec)	N/A	0.548	0.298	0.940	0.366	0.342	0.000	0.467	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	236	212	377	213	0	332	0
N.S.	1	1.00	0.92	0.82	1.47	0.83	0.00	1.29	0.00
time (sec)	N/A	0.431	0.356	1.040	0.344	0.317	0.000	0.330	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	293	286	253	265	0	0	0
N.S.	1	1.00	1.08	1.06	0.93	0.98	0.00	0.00	0.00
time (sec)	N/A	0.542	1.488	0.987	0.591	0.313	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	268	368	253	367	0	0	0
N.S.	1	1.00	0.92	1.26	0.87	1.26	0.00	0.00	0.00
time (sec)	N/A	0.738	2.080	0.825	0.585	0.332	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	448	450	254	528	0	0	0
N.S.	1	1.00	1.26	1.26	0.71	1.48	0.00	0.00	0.00
time (sec)	N/A	0.941	2.082	0.921	0.601	0.348	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	55	34	74	35	83	69	0
N.S.	1	1.00	1.12	0.69	1.51	0.71	1.69	1.41	0.00
time (sec)	N/A	0.096	0.016	0.365	0.337	0.268	3.008	0.295	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	48	27	67	26	61	53	26
N.S.	1	1.00	1.33	0.75	1.86	0.72	1.69	1.47	0.72
time (sec)	N/A	0.056	0.009	0.302	0.316	0.310	0.555	0.286	0.031

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	51	19	60	18	37	35	18
N.S.	1	1.00	2.12	0.79	2.50	0.75	1.54	1.46	0.75
time (sec)	N/A	0.033	0.009	0.304	0.333	0.265	0.479	0.300	0.037

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	63	28	21	31	61	0	0
N.S.	1	1.00	1.80	0.80	0.60	0.89	1.74	0.00	0.00
time (sec)	N/A	0.059	0.051	0.292	0.428	0.268	1.059	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	122	0	235	174	0	0	0
N.S.	1	1.00	0.67	0.00	1.28	0.95	0.00	0.00	0.00
time (sec)	N/A	0.391	0.114	0.000	0.248	0.100	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	122	0	186	140	0	0	0
N.S.	1	1.00	0.80	0.00	1.22	0.92	0.00	0.00	0.00
time (sec)	N/A	0.283	0.049	0.000	0.262	0.101	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	122	0	186	140	0	0	0
N.S.	1	1.00	0.80	0.00	1.22	0.92	0.00	0.00	0.00
time (sec)	N/A	0.230	0.053	0.000	0.253	0.094	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	124	0	137	118	0	0	0
N.S.	1	1.00	0.92	0.00	1.01	0.87	0.00	0.00	0.00
time (sec)	N/A	0.126	0.063	0.000	0.253	0.127	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	124	0	138	118	0	0	0
N.S.	1	1.00	0.92	0.00	1.02	0.87	0.00	0.00	0.00
time (sec)	N/A	0.132	0.066	0.000	0.242	0.094	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	121	0	138	169	0	0	0
N.S.	1	1.00	0.80	0.00	0.91	1.12	0.00	0.00	0.00
time (sec)	N/A	0.170	0.055	0.000	0.273	0.100	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	121	0	138	169	0	0	0
N.S.	1	1.00	0.79	0.00	0.90	1.10	0.00	0.00	0.00
time (sec)	N/A	0.176	0.056	0.000	0.285	0.100	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	182	125	0	137	259	0	0	0
N.S.	1	1.00	0.69	0.00	0.75	1.42	0.00	0.00	0.00
time (sec)	N/A	0.243	0.053	0.000	0.240	0.135	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.023	26.255	0.119	0.701	0.000	6.357	0.329	13.311

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	133	0	57	0	0	15
N.S.	1	1.00	1.00	8.31	0.00	3.56	0.00	0.00	0.94
time (sec)	N/A	0.012	0.009	1.268	0.000	0.096	0.000	0.000	0.042

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	12	14	14
N.S.	1	1.00	1.14	0.86	1.00	0.00	0.86	1.00	1.00
time (sec)	N/A	0.033	0.808	0.151	0.574	0.000	2.994	0.344	13.504

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.048	2.291	0.131	0.696	0.000	87.663	0.368	13.726

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	179	0	68	0	0	35
N.S.	1	1.00	0.86	4.26	0.00	1.62	0.00	0.00	0.83
time (sec)	N/A	0.025	0.029	2.096	0.000	0.099	0.000	0.000	0.047

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	12	14	14
N.S.	1	1.00	1.14	0.86	1.00	0.00	0.86	1.00	1.00
time (sec)	N/A	0.037	7.418	0.146	0.599	0.000	37.920	0.328	13.556

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	1.020	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	10	0	10	10	10
N.S.	1	1.00	1.20	0.80	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.095	6.110	0.195	0.414	0.000	92.969	0.323	13.057

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.022	0.278	0.178	0.644	0.000	3.071	0.345	13.326

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	0	51	0	0	15
N.S.	1	1.00	1.00	1.12	0.00	3.19	0.00	0.00	0.94
time (sec)	N/A	0.011	0.009	0.249	0.000	0.085	0.000	0.000	13.172

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.038	0.453	0.118	0.577	0.000	10.382	0.326	13.056

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.044	2.672	0.130	0.662	0.000	14.099	0.357	13.610

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	182	0	93	0	0	42
N.S.	1	1.00	1.00	4.79	0.00	2.45	0.00	0.00	1.11
time (sec)	N/A	0.018	0.051	1.507	0.000	0.091	0.000	0.000	13.781

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.035	10.729	0.130	0.630	0.000	36.925	0.352	13.676

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	33	0	0	0	0	0	51
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	1.34
time (sec)	N/A	0.072	1.141	0.000	0.000	0.000	0.000	0.000	13.825

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	15
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.75
time (sec)	N/A	0.056	0.132	0.000	0.000	0.000	0.000	0.000	13.152

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	15	0	0	16
N.S.	1	1.00	0.71	0.00	0.00	0.62	0.00	0.00	0.67
time (sec)	N/A	0.060	0.111	0.000	0.000	0.277	0.000	0.000	0.147

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	33	0	0	0	0	0	31
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.66
time (sec)	N/A	0.083	0.184	0.000	0.000	0.000	0.000	0.000	13.756

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	29	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.196	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	0.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.270	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	51	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.069	1.111	0.345	0.672	0.260	8.741	1.195	14.057

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	253	0	0	188	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.522	0.188	0.000	0.000	0.124	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	150	0	0	136	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.309	0.215	0.000	0.000	0.094	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	122	0	0	96	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.164	0.052	0.000	0.000	0.102	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.29
time (sec)	N/A	0.024	9.537	0.372	0.479	0.286	2.123	0.434	14.366

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.055	0.858	0.326	0.532	0.259	5.245	0.365	13.861

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	455	0	54	0	0	0
N.S.	1	1.00	1.00	6.07	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.110	0.022	0.516	0.000	0.091	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	354	0	54	0	0	0
N.S.	1	1.00	1.00	4.48	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.084	0.020	0.500	0.000	0.088	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	291	0	54	0	0	0
N.S.	1	1.00	1.00	3.88	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.086	0.018	0.423	0.000	0.101	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	379	0	50	0	0	0
N.S.	1	1.00	1.00	4.80	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.085	0.017	0.472	0.000	0.118	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	427	0	50	0	0	0
N.S.	1	1.00	0.95	6.57	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.080	0.029	0.438	0.000	0.092	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	530	0	54	0	0	0
N.S.	1	1.00	1.00	7.07	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.079	0.019	0.509	0.000	0.094	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	600	0	54	0	0	0
N.S.	1	1.00	1.00	8.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.086	0.018	0.460	0.000	0.094	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	92	0	0	77	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.188	0.115	0.000	0.000	0.094	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	96	0	0	77	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.179	0.114	0.000	0.000	0.110	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	90	0	0	77	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.163	0.105	0.000	0.000	0.100	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	90	0	0	69	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.160	0.125	0.000	0.000	0.089	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	64	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.147	0.065	0.000	0.000	0.093	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	91	0	0	77	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.162	0.118	0.000	0.000	0.094	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	95	0	0	77	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.174	0.095	0.000	0.000	0.089	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	122	104	456	168	264	154	189
N.S.	1	1.00	1.37	1.17	5.12	1.89	2.97	1.73	2.12
time (sec)	N/A	0.139	0.564	1.105	0.308	0.270	0.296	0.282	14.166

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	80	77	235	102	151	92	112
N.S.	1	1.00	1.19	1.15	3.51	1.52	2.25	1.37	1.67
time (sec)	N/A	0.105	0.383	1.009	0.265	0.269	0.237	0.290	14.410

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	52	40	91	51	68	44	52
N.S.	1	1.00	1.18	0.91	2.07	1.16	1.55	1.00	1.18
time (sec)	N/A	0.058	0.704	0.817	0.294	0.288	0.177	0.278	0.104

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	86	172	71	0	673	0
N.S.	1	1.00	0.85	1.32	2.65	1.09	0.00	10.35	0.00
time (sec)	N/A	0.169	0.216	0.928	0.333	0.271	0.000	0.301	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	124	196	105	0	535	0
N.S.	1	1.00	0.88	1.39	2.20	1.18	0.00	6.01	0.00
time (sec)	N/A	0.203	0.482	1.073	0.322	0.264	0.000	0.306	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	217	180	949	369	779	335	452
N.S.	1	1.00	0.92	0.76	4.00	1.56	3.29	1.41	1.91
time (sec)	N/A	0.303	1.659	1.568	0.403	0.289	0.444	0.310	15.277

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	193	122	494	212	456	203	255
N.S.	1	1.00	1.15	0.73	2.94	1.26	2.71	1.21	1.52
time (sec)	N/A	0.227	0.781	1.299	0.317	0.299	0.333	0.280	15.196

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	80	92	197	98	219	103	117
N.S.	1	1.00	0.68	0.78	1.67	0.83	1.86	0.87	0.99
time (sec)	N/A	0.104	0.932	0.977	0.272	0.287	0.229	0.297	14.941

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	114	197	339	145	0	6693	0
N.S.	1	1.00	0.79	1.36	2.34	1.00	0.00	46.16	0.00
time (sec)	N/A	0.431	0.777	1.057	0.446	0.304	0.000	0.428	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	206	276	372	219	0	1049	0
N.S.	1	1.00	1.30	1.74	2.34	1.38	0.00	6.60	0.00
time (sec)	N/A	0.490	0.854	1.159	0.465	0.307	0.000	0.448	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	151	364	935	422	0	0	0
N.S.	1	1.00	1.13	2.72	6.98	3.15	0.00	0.00	0.00
time (sec)	N/A	0.449	0.342	1.615	0.450	0.305	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	125	197	284	226	0	0	0
N.S.	1	1.00	1.24	1.95	2.81	2.24	0.00	0.00	0.00
time (sec)	N/A	0.343	0.359	1.259	0.368	0.279	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	70	40	160	58	70	194	65
N.S.	1	1.00	1.43	0.82	3.27	1.18	1.43	3.96	1.33
time (sec)	N/A	0.092	0.088	0.978	0.343	0.276	0.328	0.333	15.456

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	285	27	27	22	22
N.S.	1	1.00	1.10	1.00	14.25	1.35	1.35	1.10	1.10
time (sec)	N/A	0.066	3.932	0.416	0.659	0.280	1.046	0.291	14.949

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	442	51	58	22	22
N.S.	1	1.00	1.10	1.00	22.10	2.55	2.90	1.10	1.10
time (sec)	N/A	0.060	2.508	0.382	0.974	0.260	1.882	0.506	14.914

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	250	678	3275	771	0	0	0
N.S.	1	1.00	0.92	2.50	12.08	2.85	0.00	0.00	0.00
time (sec)	N/A	0.401	1.094	2.918	0.997	0.308	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	212	358	776	390	0	0	0
N.S.	1	1.00	1.00	1.69	3.66	1.84	0.00	0.00	0.00
time (sec)	N/A	0.292	1.175	2.546	0.607	0.274	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	113	74	763	118	146	661	175
N.S.	1	1.00	0.92	0.60	6.20	0.96	1.19	5.37	1.42
time (sec)	N/A	0.108	1.128	1.677	0.354	0.281	0.451	0.494	18.838

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	2913	57	54	22	22
N.S.	1	1.00	1.10	1.00	145.65	2.85	2.70	1.10	1.10
time (sec)	N/A	0.067	11.283	0.699	10.732	0.280	1.772	0.306	14.722

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	3521	99	105	22	22
N.S.	1	1.00	1.10	1.00	176.05	4.95	5.25	1.10	1.10
time (sec)	N/A	0.059	12.478	0.709	19.786	0.283	4.511	0.878	14.159

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	164	468	967	467	0	0	0
N.S.	1	1.00	1.23	3.52	7.27	3.51	0.00	0.00	0.00
time (sec)	N/A	0.321	1.590	1.335	0.448	0.295	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	292	247	310	283	0	0	0
N.S.	1	1.00	2.86	2.42	3.04	2.77	0.00	0.00	0.00
time (sec)	N/A	0.222	6.458	1.237	0.392	0.275	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	57	54	160	59	90	189	65
N.S.	1	1.00	1.14	1.08	3.20	1.18	1.80	3.78	1.30
time (sec)	N/A	0.071	0.866	1.194	0.264	0.302	0.353	0.330	13.633

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	285	28	29	25	23
N.S.	1	1.00	1.10	1.00	13.57	1.33	1.38	1.19	1.10
time (sec)	N/A	0.069	3.628	0.366	0.686	0.293	1.581	0.269	13.474

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	442	52	60	25	23
N.S.	1	1.00	1.10	1.00	21.05	2.48	2.86	1.19	1.10
time (sec)	N/A	0.065	2.516	0.375	0.954	0.266	3.184	0.480	13.597

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	53	132	206	0	0	98	83
N.S.	1	1.00	0.48	1.20	1.87	0.00	0.00	0.89	0.75
time (sec)	N/A	0.154	0.334	0.525	0.502	0.000	0.000	0.296	13.774

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	44	105	122	0	0	77	63
N.S.	1	1.00	0.50	1.19	1.39	0.00	0.00	0.88	0.72
time (sec)	N/A	0.129	0.247	0.385	0.465	0.000	0.000	0.286	13.779

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	34	80	61	0	0	57	46
N.S.	1	1.00	0.64	1.51	1.15	0.00	0.00	1.08	0.87
time (sec)	N/A	0.069	0.385	0.388	0.424	0.000	0.000	0.286	0.230

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	20	32	0	30	33
N.S.	1	1.00	1.12	1.65	0.77	1.23	0.00	1.15	1.27
time (sec)	N/A	0.016	0.079	0.679	0.390	0.310	0.000	0.273	13.148

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	55	0	61	0	0	166	0
N.S.	1	1.00	0.65	0.00	0.73	0.00	0.00	1.98	0.00
time (sec)	N/A	0.165	0.315	0.000	0.409	0.000	0.000	0.308	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	75	0	193	0	0	560	0
N.S.	1	1.00	0.68	0.00	1.75	0.00	0.00	5.09	0.00
time (sec)	N/A	0.249	0.246	0.000	0.446	0.000	0.000	0.327	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	98	0	227	0	0	662	0
N.S.	1	1.00	0.65	0.00	1.50	0.00	0.00	4.38	0.00
time (sec)	N/A	0.280	0.352	0.000	0.538	0.000	0.000	0.314	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	33	87	48	0	0	55	91
N.S.	1	1.00	0.49	1.28	0.71	0.00	0.00	0.81	1.34
time (sec)	N/A	0.195	0.134	0.542	0.354	0.000	0.000	0.285	14.227

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	29	70	36	0	0	43	70
N.S.	1	1.00	0.55	1.32	0.68	0.00	0.00	0.81	1.32
time (sec)	N/A	0.205	0.101	0.487	0.396	0.000	0.000	0.279	13.609

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	22	55	24	0	0	31	50
N.S.	1	1.00	0.69	1.72	0.75	0.00	0.00	0.97	1.56
time (sec)	N/A	0.104	0.074	0.469	0.360	0.000	0.000	0.299	14.284

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	18	25	12	18	0	17	34
N.S.	1	1.00	1.20	1.67	0.80	1.20	0.00	1.13	2.27
time (sec)	N/A	0.016	0.045	0.881	0.478	0.270	0.000	0.282	0.131

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	17	0	0	16	0
N.S.	1	1.00	1.00	0.00	0.74	0.00	0.00	0.70	0.00
time (sec)	N/A	0.112	0.009	0.000	0.489	0.000	0.000	0.296	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	33	0	23	0	0	34	0
N.S.	1	1.00	0.79	0.00	0.55	0.00	0.00	0.81	0.00
time (sec)	N/A	0.111	0.117	0.000	0.501	0.000	0.000	0.286	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	44	0	19	0	0	48	0
N.S.	1	1.00	0.66	0.00	0.28	0.00	0.00	0.72	0.00
time (sec)	N/A	0.127	0.131	0.000	0.475	0.000	0.000	0.280	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	34	86	129	0	0	55	92
N.S.	1	1.00	0.47	1.19	1.79	0.00	0.00	0.76	1.28
time (sec)	N/A	0.157	0.161	0.627	0.383	0.000	0.000	0.282	14.844

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	30	69	100	0	0	51	71
N.S.	1	1.00	0.54	1.23	1.79	0.00	0.00	0.91	1.27
time (sec)	N/A	0.137	0.126	0.583	0.353	0.000	0.000	0.289	14.619

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	54	72	0	0	31	48
N.S.	1	1.00	0.68	1.59	2.12	0.00	0.00	0.91	1.41
time (sec)	N/A	0.071	0.092	0.523	0.336	0.000	0.000	0.279	14.621

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	19	25	23	19	0	26	34
N.S.	1	1.00	1.19	1.56	1.44	1.19	0.00	1.62	2.12
time (sec)	N/A	0.016	0.059	0.941	0.423	0.274	0.000	0.289	14.520

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	0	16	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.126	0.086	0.000	0.000	0.000	0.000	0.290	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	34	0	0	0	0	34	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.138	0.033	0.000	0.000	0.000	0.000	0.471	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	45	0	0	0	0	48	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.164	0.149	0.000	0.000	0.000	0.000	0.362	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	185	67	0	98	0	0	113	0
N.S.	1	1.00	0.36	0.00	0.53	0.00	0.00	0.61	0.00
time (sec)	N/A	0.321	0.383	0.000	0.365	0.000	0.000	0.346	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	54	0	72	0	0	85	0
N.S.	1	1.00	0.37	0.00	0.50	0.00	0.00	0.59	0.00
time (sec)	N/A	0.265	0.296	0.000	0.344	0.000	0.000	0.363	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	45	0	48	0	0	59	0
N.S.	1	1.00	0.51	0.00	0.54	0.00	0.00	0.66	0.00
time (sec)	N/A	0.126	0.117	0.000	0.522	0.000	0.000	0.359	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	36	0	29	0	0	32	0
N.S.	1	1.00	0.65	0.00	0.53	0.00	0.00	0.58	0.00
time (sec)	N/A	0.246	0.024	0.000	0.393	0.000	0.000	0.285	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	53	0	37	0	0	62	0
N.S.	1	1.00	0.67	0.00	0.47	0.00	0.00	0.78	0.00
time (sec)	N/A	0.274	0.139	0.000	0.405	0.000	0.000	0.341	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	66	0	33	0	0	92	0
N.S.	1	1.00	0.61	0.00	0.30	0.00	0.00	0.84	0.00
time (sec)	N/A	0.214	0.065	0.000	0.386	0.000	0.000	0.366	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	374	199	0	0	0	0	0	0
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.137	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	146	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	89	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	40	56	90	126	0	93	45
N.S.	1	1.00	0.87	1.22	1.96	2.74	0.00	2.02	0.98
time (sec)	N/A	0.029	0.018	0.491	0.386	0.283	0.000	0.349	14.153

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	30	17	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.67	0.94	1.00	1.00
time (sec)	N/A	0.091	1.091	0.172	0.578	0.255	1.864	0.563	14.357

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	235	170	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	0.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	117	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	83	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	25	81	87	0	20	0
N.S.	1	1.00	0.97	0.68	2.19	2.35	0.00	0.54	0.00
time (sec)	N/A	0.028	0.073	0.911	0.488	0.264	0.000	0.416	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	13	15	25	15	15	15
N.S.	1	1.00	1.13	0.87	1.00	1.67	1.00	1.00	1.00
time (sec)	N/A	0.110	4.668	0.313	0.519	0.253	1.832	0.783	13.977

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	423	423	257	0	0	0	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	0.458	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	257	257	185	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	0.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	165	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.232	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	36	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	2.57	1.00	1.00	1.00
time (sec)	N/A	0.096	11.208	0.400	0.531	0.252	9.327	1.071	14.491

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	0	15	18	18
N.S.	1	1.00	1.11	0.89	1.00	0.00	0.83	1.00	1.00
time (sec)	N/A	0.077	2.246	0.137	0.557	0.000	1.657	0.633	14.846

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	383	383	290	0	0	1030	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	2.69	0.00	0.00	0.00
time (sec)	N/A	0.653	1.718	0.000	0.000	0.359	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	329	329	255	0	0	1263	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	3.84	0.00	0.00	0.00
time (sec)	N/A	0.763	0.607	0.000	0.000	0.426	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	756	414	0	915	0	0	0
N.S.	1	1.00	3.53	1.93	0.00	4.28	0.00	0.00	0.00
time (sec)	N/A	0.454	0.942	0.845	0.000	0.419	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	13	10	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.08	0.83	1.17	1.17
time (sec)	N/A	0.082	1.453	0.201	0.358	0.313	2.084	0.277	14.017

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	933	674	0	1482	0	0	0
N.S.	1	1.00	3.15	2.28	0.00	5.01	0.00	0.00	0.00
time (sec)	N/A	0.794	10.621	3.345	0.000	0.458	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [180] had the largest ratio of [.642900000000000027]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.00	14	0.143
2	A	4	2	1.00	14	0.143
3	A	3	2	1.00	14	0.143
4	A	2	2	1.00	12	0.167
5	A	3	3	1.00	14	0.214
6	A	4	4	1.00	14	0.286
7	A	5	4	1.00	14	0.286
8	A	6	4	1.00	14	0.286
9	A	6	4	1.00	16	0.250
10	A	4	3	1.00	16	0.188
11	A	4	4	1.00	16	0.250
12	A	2	1	1.00	14	0.071
13	A	5	4	1.00	16	0.250
14	A	5	5	1.00	16	0.312
15	A	7	6	1.00	16	0.375
16	A	12	4	1.00	16	0.250
17	A	8	4	1.00	16	0.250
18	A	6	4	1.00	16	0.250
19	A	3	3	1.00	14	0.214
20	A	8	4	1.00	16	0.250
21	A	8	4	1.00	16	0.250
22	A	12	5	1.00	16	0.312
23	A	8	3	1.00	12	0.250
24	A	8	4	1.00	12	0.333

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	10	0.200
26	A	8	4	1.00	12	0.333
27	A	8	4	1.00	12	0.333
28	A	14	5	1.00	12	0.417
29	A	9	5	1.00	14	0.357
30	A	7	4	1.00	14	0.286
31	A	5	3	1.00	12	0.250
32	N/A	0	0	1.00	14	0.000
33	A	6	6	1.00	16	0.375
34	A	5	5	1.00	16	0.312
35	A	2	2	1.00	14	0.143
36	N/A	0	0	1.00	16	0.000
37	A	15	8	1.00	16	0.500
38	A	9	6	1.00	16	0.375
39	A	6	4	1.00	14	0.286
40	N/A	0	0	1.00	16	0.000
41	A	8	6	1.00	16	0.375
42	A	7	6	1.00	16	0.375
43	A	6	6	1.00	16	0.375
44	A	5	5	1.00	16	0.312
45	A	6	6	1.00	16	0.375
46	A	7	6	1.00	16	0.375
47	A	8	6	1.00	16	0.375
48	A	10	9	1.00	18	0.500
49	A	9	8	1.00	18	0.444
50	A	8	7	1.00	18	0.389
51	A	7	6	1.00	18	0.333
52	A	7	7	1.00	18	0.389
53	A	9	8	1.00	18	0.444
54	A	9	9	1.00	18	0.500
55	A	11	8	1.00	18	0.444
56	A	23	8	1.00	18	0.444
57	A	20	8	1.00	18	0.444
58	A	14	7	1.00	18	0.389
59	A	12	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	12	6	1.00	18	0.333
61	A	18	7	1.00	18	0.389
62	A	19	8	1.00	18	0.444
63	A	4	3	1.00	8	0.375
64	A	3	3	1.00	8	0.375
65	A	2	2	1.00	8	0.250
66	A	3	3	1.00	8	0.375
67	A	5	3	1.00	16	0.188
68	A	4	3	1.00	16	0.188
69	A	4	3	1.00	16	0.188
70	A	3	2	1.00	16	0.125
71	A	3	2	1.00	16	0.125
72	A	4	3	1.00	16	0.188
73	A	4	3	1.00	16	0.188
74	A	5	3	1.00	16	0.188
75	N/A	0	0	1.00	12	0.000
76	A	1	1	1.00	10	0.100
77	N/A	0	0	1.00	14	0.000
78	N/A	0	0	1.00	12	0.000
79	A	2	2	1.00	10	0.200
80	N/A	0	0	1.00	14	0.000
81	A	2	1	1.00	28	0.036
82	N/A	0	0	1.00	10	0.000
83	N/A	0	0	1.00	12	0.000
84	A	1	1	1.00	10	0.100
85	N/A	0	0	1.00	14	0.000
86	N/A	0	0	1.00	12	0.000
87	A	2	2	1.00	10	0.200
88	N/A	0	0	1.00	14	0.000
89	A	2	1	1.00	25	0.040
90	A	2	1	1.00	17	0.059
91	A	2	1	1.00	20	0.050
92	A	3	1	1.00	20	0.050
93	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	4	2	1.00	20	0.100
95	A	4	2	1.00	20	0.100
96	A	5	2	1.00	20	0.100
97	A	7	5	1.00	24	0.208
98	N/A	0	0	1.00	18	0.000
99	A	8	3	1.00	16	0.188
100	A	5	3	1.00	16	0.188
101	A	3	2	1.00	14	0.143
102	N/A	0	0	1.00	14	0.000
103	N/A	0	0	1.00	16	0.000
104	A	3	2	1.00	12	0.167
105	A	3	2	1.00	12	0.167
106	A	3	2	1.00	12	0.167
107	A	3	2	1.00	10	0.200
108	A	3	2	1.00	12	0.167
109	A	3	2	1.00	12	0.167
110	A	3	2	1.00	12	0.167
111	A	5	3	1.00	14	0.214
112	A	5	3	1.00	14	0.214
113	A	5	3	1.00	14	0.214
114	A	5	3	1.00	12	0.250
115	A	5	3	1.00	14	0.214
116	A	5	3	1.00	14	0.214
117	A	5	3	1.00	14	0.214
118	A	6	3	1.00	18	0.167
119	A	5	3	1.00	18	0.167
120	A	4	3	1.00	16	0.188
121	A	5	4	1.00	18	0.222
122	A	6	5	1.00	18	0.278
123	A	10	6	1.00	20	0.300
124	A	9	7	1.00	20	0.350
125	A	6	4	1.00	18	0.222
126	A	9	5	1.00	20	0.250
127	A	9	5	1.00	20	0.250
128	A	7	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	6	6	1.00	20	0.300
130	A	3	3	1.00	18	0.167
131	N/A	0	0	1.00	20	0.000
132	N/A	0	0	1.00	20	0.000
133	A	10	9	1.00	20	0.450
134	A	9	9	1.00	20	0.450
135	A	4	4	1.00	18	0.222
136	N/A	0	0	1.00	20	0.000
137	N/A	0	0	1.00	20	0.000
138	A	7	7	1.00	21	0.333
139	A	6	6	1.00	21	0.286
140	A	3	3	1.00	19	0.158
141	N/A	0	0	1.00	21	0.000
142	N/A	0	0	1.00	21	0.000
143	A	5	3	1.00	18	0.167
144	A	4	3	1.00	18	0.167
145	A	3	3	1.00	16	0.188
146	A	1	1	1.00	14	0.071
147	A	4	4	1.00	18	0.222
148	A	5	5	1.00	18	0.278
149	A	6	5	1.00	18	0.278
150	A	5	3	1.00	14	0.214
151	A	4	3	1.00	14	0.214
152	A	3	3	1.00	12	0.250
153	A	1	1	1.00	10	0.100
154	A	2	2	1.00	14	0.143
155	A	3	3	1.00	14	0.214
156	A	4	3	1.00	14	0.214
157	A	5	3	1.00	15	0.200
158	A	4	3	1.00	15	0.200
159	A	3	3	1.00	13	0.231
160	A	1	1	1.00	11	0.091
161	A	2	2	1.00	15	0.133
162	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	A	4	3	1.00	15	0.200
164	A	9	5	1.00	14	0.357
165	A	7	5	1.00	14	0.357
166	A	4	4	1.00	12	0.333
167	A	5	3	1.00	14	0.214
168	A	5	3	1.00	14	0.214
169	A	7	4	1.00	14	0.286
170	A	10	6	1.00	18	0.333
171	A	8	5	1.00	18	0.278
172	A	6	4	1.00	16	0.250
173	A	2	2	1.00	14	0.143
174	N/A	0	0	1.00	18	0.000
175	A	10	6	1.00	15	0.400
176	A	8	5	1.00	15	0.333
177	A	6	4	1.00	13	0.308
178	A	2	2	1.00	11	0.182
179	N/A	0	0	1.00	15	0.000
180	A	16	9	1.00	14	0.643
181	A	10	7	1.00	14	0.500
182	A	7	5	1.00	12	0.417
183	N/A	0	0	1.00	14	0.000
184	N/A	0	0	1.00	18	0.000
185	A	12	7	1.00	12	0.583
186	A	10	6	1.00	16	0.375
187	A	8	5	1.00	14	0.357
188	N/A	0	0	1.00	12	0.000
189	A	11	8	1.00	18	0.444

CHAPTER 3

LISTING OF INTEGRALS

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Optimal result

Integrand size = 14, antiderivative size = 91

$$\int (c+dx)^4 \cos(a+bx)\,dx = -\frac{24d^3(c+dx)\cos(a+bx)}{b^4} + \frac{4d(c+dx)^3\cos(a+bx)}{b^2} + \frac{24d^4\sin(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} + \frac{(c+dx)^4\sin(a+bx)}{b}$$

```
[Out] -24*d^3*(d*x+c)*cos(b*x+a)/b^4+4*d*(d*x+c)^3*cos(b*x+a)/b^2+24*d^4*sin(b*x+
a)/b^5-12*d^2*(d*x+c)^2*sin(b*x+a)/b^3+(d*x+c)^4*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2717}

$$\int (c+dx)^4 \cos(a+bx)\,dx = \frac{24d^4\sin(a+bx)}{b^5} - \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} + \frac{4d(c+dx)^3\cos(a+bx)}{b^2} + \frac{(c+dx)^4\sin(a+bx)}{b}$$

```
[In] Int[(c + d*x)^4*Cos[a + b*x],x]
```

```
[Out] (-24*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 +
 (24*d^4*Sin[a + b*x])/b^5 - (12*d^2*(c + d*x)^2*Sin[a + b*x])/b^3 + ((c +
d*x)^4*Sin[a + b*x])/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c+dx)^4\sin(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\sin(a+bx)\,dx}{b} \\
&= \frac{4d(c+dx)^3\cos(a+bx)}{b^2} + \frac{(c+dx)^4\sin(a+bx)}{b} - \frac{(12d^2)\int (c+dx)^2\cos(a+bx)\,dx}{b^2} \\
&= \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} \\
&\quad + \frac{(c+dx)^4\sin(a+bx)}{b} + \frac{(24d^3)\int (c+dx)\sin(a+bx)\,dx}{b^3} \\
&= -\frac{24d^3(c+dx)\cos(a+bx)}{b^4} + \frac{4d(c+dx)^3\cos(a+bx)}{b^2} \\
&\quad - \frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} + \frac{(c+dx)^4\sin(a+bx)}{b} + \frac{(24d^4)\int \cos(a+bx)\,dx}{b^4} \\
&= -\frac{24d^3(c+dx)\cos(a+bx)}{b^4} + \frac{4d(c+dx)^3\cos(a+bx)}{b^2} \\
&\quad + \frac{24d^4\sin(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} + \frac{(c+dx)^4\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int (c+dx)^4\cos(a+bx)\,dx \\
&= \frac{4bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx) + \left(24d^4 - 12b^2d^2(c+dx)^2 + b^4(c+dx)^4\right)\sin(a+bx)}{b^5}
\end{aligned}$$

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x],x]
```

```
[Out] (4*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (24*d^4 - 12*b^2
*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Sin[a + b*x])/b^5
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.57

method	result
parallelrisch	$\frac{-12b\,d^2\left(\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2-2d^2\right)x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\left(2(dx+c)^4b^4-24d^2(dx+c)^2b^2+48d^4\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+8b\left(\left(x^2d^2+\right.\right.}{b^5\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$
risch	$\frac{4d\left(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c\right)\cos(bx+a)}{b^4}+\frac{\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12\right.}{b^5}$
norman	$\frac{\left(-8b^2c^3d+48c\,d^3\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^4}+\frac{4d^4x^3}{b^2}+\frac{2\left(b^4c^4-12b^2c^2d^2+24d^4\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^5}+\frac{12c\,d^3x^2}{b^2}+\frac{12d^2\left(b^2c^2-2d^2\right)x}{b^4}-\frac{4d^4x^3\left(\tan\right.}{}$
parts	$\frac{\sin(bx+a)d^4x^4}{b}+\frac{4\sin(bx+a)c\,d^3x^3}{b}+\frac{6\sin(bx+a)c^2d^2x^2}{b}+\frac{4\sin(bx+a)c^3dx}{b}+\frac{\sin(bx+a)c^4}{b}-\frac{4d\left(\frac{a^3d^3\cos(bx}{b^3}\right.}{}$
meijerg	$\frac{16d^4\cos(a)\sqrt{\pi}\left(-\frac{x\left(b^2\right)^{\frac{5}{2}}\left(-\frac{5x^2b^2}{2}+15\right)\cos(bx)}{10\sqrt{\pi}\,b^4}+\frac{\left(b^2\right)^{\frac{5}{2}}\left(\frac{5}{8}x^4b^4-\frac{15}{2}x^2b^2+15\right)\sin(bx)}{10\sqrt{\pi}\,b^5}\right)}{b^4\sqrt{b^2}}-\frac{16d^4\sin(a)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}x^4\right.}{}$
derivativedivides	$\frac{a^4d^4\sin(bx+a)}{b^4}-\frac{4a^3c\,d^3\sin(bx+a)}{b^3}-\frac{4a^3d^4(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^4}+\frac{6a^2c^2d^2\sin(bx+a)}{b^2}+\frac{12a^2c\,d^3(\cos(bx+a)+(bx+a)\sin(}{b^3}$
default	$\frac{a^4d^4\sin(bx+a)}{b^4}-\frac{4a^3c\,d^3\sin(bx+a)}{b^3}-\frac{4a^3d^4(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^4}+\frac{6a^2c^2d^2\sin(bx+a)}{b^2}+\frac{12a^2c\,d^3(\cos(bx+a)+(bx+a)\sin(}{b^3}$

```
[In] int((d*x+c)^4*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (-12*b*d^2*((1/3*x^2*d^2+c*d*x+c^2)*b^2-2*d^2)*x*tan(1/2*b*x+1/2*a)^2+(2*(d
*x+c)^4*b^4-24*d^2*(d*x+c)^2*b^2+48*d^4)*tan(1/2*b*x+1/2*a)+8*b*((d^2*x^2+c
*d*x+c^2)*b^2-6*d^2)*d*(1/2*d*x+c))/b^5/(1+tan(1/2*b*x+1/2*a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.86

$$\int (c+dx)^4\cos(a+bx)\,dx$$
$$=\frac{4\left(b^3d^4x^3+3\,b^3cd^3x^2+b^3c^3d-6\,bcd^3+3\left(b^3c^2d^2-2\,bd^4\right)x\right)\cos\left(bx+a\right)+\left(b^4d^4x^4+4\,b^4cd^3x^3+b^4c^4-1\right.}{b^5}$$

```
[In] integrate((d*x+c)^4*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2
- 2*b*d^4)*x)*cos(b*x + a) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*
b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b
^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(92) = 184$.

Time = 0.35 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.42

$$\int (c+dx)^4 \cos(a+bx)\,dx$$

$$= \begin{cases} \frac{c^4 \sin(a+bx)}{b} + \frac{4c^3 dx \sin(a+bx)}{b} + \frac{6c^2 d^2 x^2 \sin(a+bx)}{b} + \frac{4cd^3 x^3 \sin(a+bx)}{b} + \frac{d^4 x^4 \sin(a+bx)}{b} + \frac{4c^3 d \cos(a+bx)}{b^2} + \frac{12c^2 d^2 x \cos(a+b} \\ \left(c^4 x + 2c^3 dx^2 + 2c^2 d^2 x^3 + cd^3 x^4 + \frac{d^4 x^5}{5}\right) \cos(a) \end{cases}$$

```
[In] integrate((d*x+c)**4*cos(b*x+a),x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)/b + 4*c**3*d*x*sin(a + b*x)/b + 6*c**2*d**2*x*
*2*sin(a + b*x)/b + 4*c*d**3*x**3*sin(a + b*x)/b + d**4*x**4*sin(a + b*x)/b
 + 4*c**3*d*cos(a + b*x)/b**2 + 12*c**2*d**2*x*cos(a + b*x)/b**2 + 12*c*d**
3*x**2*cos(a + b*x)/b**2 + 4*d**4*x**3*cos(a + b*x)/b**2 - 12*c**2*d**2*sin
(a + b*x)/b**3 - 24*c*d**3*x*sin(a + b*x)/b**3 - 12*d**4*x**2*sin(a + b*x)/
b**3 - 24*c*d**3*cos(a + b*x)/b**4 - 24*d**4*x*cos(a + b*x)/b**4 + 24*d**4*
sin(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 +
 c*d**3*x**4 + d**4*x**5/5)*cos(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(91) = 182$.

Time = 0.29 (sec) , antiderivative size = 481, normalized size of antiderivative = 5.29

$$\int (c+dx)^4 \cos(a+bx)\,dx$$

$$= \frac{c^4 \sin(bx+a) - \frac{4ac^3 d \sin(bx+a)}{b} + \frac{6a^2 c^2 d^2 \sin(bx+a)}{b^2} - \frac{4a^3 cd^3 \sin(bx+a)}{b^3} + \frac{a^4 d^4 \sin(bx+a)}{b^4} + \frac{4((bx+a)\sin(bx+a)+\cos(bx+a))c^3}{b}}{}$$

```
[In] integrate((d*x+c)^4*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c^4*sin(b*x + a) - 4*a*c^3*d*sin(b*x + a)/b + 6*a^2*c^2*d^2*sin(b*x + a)/b
^2 - 4*a^3*c*d^3*sin(b*x + a)/b^3 + a^4*d^4*sin(b*x + a)/b^4 + 4*((b*x + a)
*sin(b*x + a) + cos(b*x + a))*c^3*d/b - 12*((b*x + a)*sin(b*x + a) + cos(b*
x + a))*a*c^2*d^2/b^2 + 12*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a^2*c*d^
3/b^3 - 4*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a^3*d^4/b^4 + 6*(2*(b*x +
 a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 12*(2*(b*x
 + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 + 6*(2*(b*
x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 + 4*(3*((
b*x + a)^2 - 2)*cos(b*x + a) + ((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*
d^3/b^3 - 4*(3*((b*x + a)^2 - 2)*cos(b*x + a) + ((b*x + a)^3 - 6*b*x - 6*a)
*sin(b*x + a))*a*d^4/b^4 + (4*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) + ((
b*x + a)^4 - 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.87

$$\int (c+dx)^4 \cos(a+bx)\,dx$$
$$= \frac{4\left(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3\right)\cos\left(bx+a\right)}{b^5}$$
$$+\frac{\left(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4\right)\sin\left(bx+\right.}{b^5}$$

```
[In] integrate((d*x+c)^4*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x
- 6*b*c*d^3)*cos(b*x + a)/b^5 + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*
d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^
2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.41

$$\int (c+dx)^4 \cos(a+bx)\,dx = \frac{\sin\left(a+b\,x\right)\,\left(b^4\,c^4-12\,b^2\,c^2\,d^2+24\,d^4\right)}{b^5}$$
$$-\frac{4\,\cos\left(a+b\,x\right)\,\left(6\,c\,d^3-b^2\,c^3\,d\right)}{b^4}+\frac{4\,d^4\,x^3\,\cos\left(a+b\,x\right)}{b^2}$$
$$-\frac{12\,x\,\cos\left(a+b\,x\right)\,\left(2\,d^4-b^2\,c^2\,d^2\right)}{b^4}$$
$$+\frac{d^4\,x^4\,\sin\left(a+b\,x\right)}{b}-\frac{4\,x\,\sin\left(a+b\,x\right)\,\left(6\,c\,d^3-b^2\,c^3\,d\right)}{b^3}$$
$$-\frac{6\,x^2\,\sin\left(a+b\,x\right)\,\left(2\,d^4-b^2\,c^2\,d^2\right)}{b^3}$$
$$+\frac{12\,c\,d^3\,x^2\,\cos\left(a+b\,x\right)}{b^2}+\frac{4\,c\,d^3\,x^3\,\sin\left(a+b\,x\right)}{b}$$

```
[In] int(cos(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (sin(a + b*x)*(24*d^4 + b^4*c^4 - 12*b^2*c^2*d^2))/b^5 - (4*cos(a + b*x)*(6
*c*d^3 - b^2*c^3*d))/b^4 + (4*d^4*x^3*cos(a + b*x))/b^2 - (12*x*cos(a + b*x
)*(2*d^4 - b^2*c^2*d^2))/b^4 + (d^4*x^4*sin(a + b*x))/b - (4*x*sin(a + b*x)
*(6*c*d^3 - b^2*c^3*d))/b^3 - (6*x^2*sin(a + b*x)*(2*d^4 - b^2*c^2*d^2))/b^
3 + (12*c*d^3*x^2*cos(a + b*x))/b^2 + (4*c*d^3*x^3*sin(a + b*x))/b
```

3.2 $\int (c+dx)^3 \cos(a+bx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 70

$$\int (c+dx)^3 \cos(a+bx)\,dx = -\frac{6d^3\cos(a+bx)}{b^4} + \frac{3d(c+dx)^2\cos(a+bx)}{b^2} - \frac{6d^2(c+dx)\sin(a+bx)}{b^3} + \frac{(c+dx)^3\sin(a+bx)}{b}$$

```
[Out] -6*d^3*cos(b*x+a)/b^4+3*d*(d*x+c)^2*cos(b*x+a)/b^2-6*d^2*(d*x+c)*sin(b*x+a)
/b^3+(d*x+c)^3*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2718}

$$\int (c+dx)^3 \cos(a+bx)\,dx = -\frac{6d^3\cos(a+bx)}{b^4} - \frac{6d^2(c+dx)\sin(a+bx)}{b^3} + \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{(c+dx)^3\sin(a+bx)}{b}$$

```
[In] Int[(c + d*x)^3*Cos[a + b*x],x]
```

```
[Out] (-6*d^3*Cos[a + b*x])/b^4 + (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 - (6*d^2*(c
+ d*x)*Sin[a + b*x])/b^3 + ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c+dx)^3\sin(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\sin(a+bx)\,dx}{b} \\
&= \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{(c+dx)^3\sin(a+bx)}{b} - \frac{(6d^2)\int (c+dx)\cos(a+bx)\,dx}{b^2} \\
&= \frac{3d(c+dx)^2\cos(a+bx)}{b^2} - \frac{6d^2(c+dx)\sin(a+bx)}{b^3} \\
&\quad + \frac{(c+dx)^3\sin(a+bx)}{b} + \frac{(6d^3)\int \sin(a+bx)\,dx}{b^3} \\
&= -\frac{6d^3\cos(a+bx)}{b^4} + \frac{3d(c+dx)^2\cos(a+bx)}{b^2} \\
&\quad - \frac{6d^2(c+dx)\sin(a+bx)}{b^3} + \frac{(c+dx)^3\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\begin{aligned}
&\int (c+dx)^3\cos(a+bx)\,dx \\
&= \frac{3d(-2d^2+b^2(c+dx)^2)\cos(a+bx) + b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\sin(a+bx)}{b^4}
\end{aligned}$$

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x],x]
```

```
[Out] (3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + b*(c + d*x)*(-6*d^2 + b^2*(c
 + d*x)^2)*Sin[a + b*x])/b^4
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.53

method	result
risch	$\frac{3d\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\cos(bx+a)}{b^4}+\frac{\left(b^2d^3x^3+3b^2cd^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c\right)\sin(bx+a)}{b^3}$
parallelrisch	$\frac{-6b^2d^2\left(\frac{dx}{2}+c\right)x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2b(dx+c)\left((dx+c)^2b^2-6d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+6d\left(\left(\frac{1}{2}x^2d^2+cdx+c^2\right)b^2-2d^2\right)}{b^4\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$
norman	$\frac{\frac{\left(-6b^2c^2d+12d^3\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^4}+\frac{3d^3x^2}{b^2}+\frac{6cd^2x}{b^2}+\frac{2c\left(b^2c^2-6d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{2d^3x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{3d^3x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{6}{}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
parts	$\frac{\sin(bx+a)d^3x^3}{b}+\frac{3\sin(bx+a)cd^2x^2}{b}+\frac{3\sin(bx+a)c^2dx}{b}+\frac{\sin(bx+a)c^3}{b}-\frac{3d\left(-\frac{a^2d^2\cos(bx+a)}{b^2}+\frac{2acd\cos(bx+a)}{b}-\frac{2c}{}\right.}{}$
derivativedivides	$\frac{-\frac{a^3d^3\sin(bx+a)}{b^3}+\frac{3a^2cd^2\sin(bx+a)}{b^2}+\frac{3a^2d^3(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^3}-\frac{3ac^2d\sin(bx+a)}{b}-\frac{6acd^2(\cos(bx+a)+(bx+a)\sin(bx+}{b^2}$
default	$\frac{-\frac{a^3d^3\sin(bx+a)}{b^3}+\frac{3a^2cd^2\sin(bx+a)}{b^2}+\frac{3a^2d^3(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^3}-\frac{3ac^2d\sin(bx+a)}{b}-\frac{6acd^2(\cos(bx+a)+(bx+a)\sin(bx+}{b^2}$
meijerg	$\frac{8d^3\cos(a)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2b^2}{2}+3\right)\cos(bx)}{4\sqrt{\pi}}-\frac{xb\left(-\frac{x^2b^2}{2}+3\right)\sin(bx)}{4\sqrt{\pi}}\right)}{b^4}-\frac{8d^3\sin(a)\sqrt{\pi}\left(\frac{xb\left(-\frac{5x^2b^2}{2}+15\right)\cos(bx)}{20\sqrt{\pi}}-\frac{\left(-\frac{15x}{}\right.}{b^4}$

```
[In] int((d*x+c)^3*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 3*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^4*cos(b*x+a)+1/b^3*(b^2*d^3*x
^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^2)*sin(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

$$\int (c+dx)^3\cos(a+bx)\,dx$$
$$=\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)+\left(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3-6\,bcd^2+3\left(b^3c^2d-2\,bd^3\right)x\right.}{b^4}$$

```
[In] integrate((d*x+c)^3*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + (b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*
sin(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(70) = 140$.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.89

$$\int (c+dx)^3 \cos(a+bx)\,dx$$

$$= \begin{cases} \frac{c^3 \sin(a+bx)}{b} + \frac{3c^2 dx \sin(a+bx)}{b} + \frac{3cd^2x^2 \sin(a+bx)}{b} + \frac{d^3x^3 \sin(a+bx)}{b} + \frac{3c^2 d\cos(a+bx)}{b^2} + \frac{6cd^2x\cos(a+bx)}{b^2} + \frac{3d^3x^2\cos(a+bx)}{b^2} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cos(a) \end{cases}$$

```
[In] integrate((d*x+c)**3*cos(b*x+a),x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)/b + 3*c**2*d*x*sin(a + b*x)/b + 3*c*d**2*x**2*
sin(a + b*x)/b + d**3*x**3*sin(a + b*x)/b + 3*c**2*d*cos(a + b*x)/b**2 + 6*
c*d**2*x*cos(a + b*x)/b**2 + 3*d**3*x**2*cos(a + b*x)/b**2 - 6*c*d**2*sin(a
 + b*x)/b**3 - 6*d**3*x*sin(a + b*x)/b**3 - 6*d**3*cos(a + b*x)/b**4, Ne(b,
 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(70) = 140$.

Time = 0.23 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.97

$$\int (c+dx)^3 \cos(a+bx)\,dx$$

$$= \frac{c^3 \sin(bx+a) - \frac{3\,ac^2 d\sin(bx+a)}{b} + \frac{3\,a^2cd^2\sin(bx+a)}{b^2} - \frac{a^3d^3\sin(bx+a)}{b^3} + \frac{3\,((bx+a)\sin(bx+a)+\cos(bx+a))c^2d}{b} - \frac{6\,((bx+a)\sin(bx}{}$$

```
[In] integrate((d*x+c)^3*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c^3*sin(b*x + a) - 3*a*c^2*d*sin(b*x + a)/b + 3*a^2*c*d^2*sin(b*x + a)/b^2
 - a^3*d^3*sin(b*x + a)/b^3 + 3*((b*x + a)*sin(b*x + a) + cos(b*x + a))*c^2
*d/b - 6*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a*c*d^2/b^2 + 3*((b*x + a)
*sin(b*x + a) + cos(b*x + a))*a^2*d^3/b^3 + 3*(2*(b*x + a)*cos(b*x + a) + (
(b*x + a)^2 - 2)*sin(b*x + a))*c*d^2/b^2 - 3*(2*(b*x + a)*cos(b*x + a) + ((
b*x + a)^2 - 2)*sin(b*x + a))*a*d^3/b^3 + (3*((b*x + a)^2 - 2)*cos(b*x + a)
 + ((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

$$\begin{aligned}&\int (c+dx)^3\cos(a+bx)\,dx\\&=\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)}{b^4}\\&\quad+\frac{\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-6\,bd^3x-6\,bcd^2\right)\sin\left(bx+a\right)}{b^4}\end{aligned}$$

```
[In] integrate((d*x+c)^3*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)/b^4 + (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^
2)*sin(b*x + a)/b^4
```

Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.10

$$\begin{aligned}\int (c+dx)^3\cos(a+bx)\,dx=&\frac{3\,d^3\,x^2\,\cos\left(a+b\,x\right)}{b^2}-\frac{\sin\left(a+b\,x\right)\,\left(6\,c\,d^2-b^2\,c^3\right)}{b^3}\\&-\frac{3\,\cos\left(a+b\,x\right)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^4}+\frac{d^3\,x^3\,\sin\left(a+b\,x\right)}{b}\\&-\frac{3\,x\,\sin\left(a+b\,x\right)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^3}\\&+\frac{6\,c\,d^2\,x\,\cos\left(a+b\,x\right)}{b^2}+\frac{3\,c\,d^2\,x^2\,\sin\left(a+b\,x\right)}{b}\end{aligned}$$

```
[In] int(cos(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (3*d^3*x^2*cos(a + b*x))/b^2 - (sin(a + b*x)*(6*c*d^2 - b^2*c^3))/b^3 - (3*
cos(a + b*x)*(2*d^3 - b^2*c^2*d))/b^4 + (d^3*x^3*sin(a + b*x))/b - (3*x*sin
(a + b*x)*(2*d^3 - b^2*c^2*d))/b^3 + (6*c*d^2*x*cos(a + b*x))/b^2 + (3*c*d^
2*x^2*sin(a + b*x))/b
```

3.3 $\int (c+dx)^2 \cos(a+bx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (c+dx)^2 \cos(a+bx)\,dx = \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{2d^2\sin(a+bx)}{b^3} + \frac{(c+dx)^2\sin(a+bx)}{b}$$

```
[Out] 2*d*(d*x+c)*cos(b*x+a)/b^2-2*d^2*sin(b*x+a)/b^3+(d*x+c)^2*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2717}

$$\int (c+dx)^2 \cos(a+bx)\,dx = -\frac{2d^2\sin(a+bx)}{b^3} + \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{(c+dx)^2\sin(a+bx)}{b}$$

```
[In] Int[(c + d*x)^2*Cos[a + b*x],x]
```

```
[Out] (2*d*(c + d*x)*Cos[a + b*x])/b^2 - (2*d^2*Sin[a + b*x])/b^3 + ((c + d*x)^2*
Sin[a + b*x])/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(c+dx)^2\sin(a+bx)}{b} - \frac{(2d)\int (c+dx)\sin(a+bx)\,dx}{b} \\ &= \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{(c+dx)^2\sin(a+bx)}{b} - \frac{(2d^2)\int \cos(a+bx)\,dx}{b^2} \\ &= \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{2d^2\sin(a+bx)}{b^3} + \frac{(c+dx)^2\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int (c+dx)^2\cos(a+bx)\,dx = \frac{2bd(c+dx)\cos(a+bx) + (-2d^2 + b^2(c+dx)^2)\sin(a+bx)}{b^3}$$

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x],x]
```

```
[Out] (2*b*d*(c + d*x)*Cos[a + b*x] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a + b*x])/b^
3
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

method	result
risch	$\frac{2d(dx+c)\cos(bx+a)}{b^2}+\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{b^3}$
parallelrisch	$\frac{-2d^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)xb+\left(2(dx+c)^2b^2-4d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+4bd\left(\frac{dx}{2}+c\right)}{b^3\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$
parts	$\frac{\sin(bx+a)x^2d^2}{b}+\frac{2\sin(bx+a)cdx}{b}+\frac{\sin(bx+a)c^2}{b}-\frac{2d\left(\frac{da\cos(bx+a)}{b}-c\cos(bx+a)+\frac{d(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}\right)}{b^2}$
norman	$\frac{\frac{4cd}{b^2}+\frac{2d^2x}{b^2}+\frac{2\left(b^2c^2-2d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{2d^2x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2d^2x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{4cdx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
derivativedivides	$\frac{\frac{a^2d^2\sin(bx+a)}{b^2}-\frac{2acd\sin(bx+a)}{b}-\frac{2a\,d^2(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^2}+c^2\sin(bx+a)+\frac{2cd(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}+\frac{d^2(}{}}{b}$
default	$\frac{\frac{a^2d^2\sin(bx+a)}{b^2}-\frac{2acd\sin(bx+a)}{b}-\frac{2a\,d^2(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^2}+c^2\sin(bx+a)+\frac{2cd(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}+\frac{d^2(}{}}{b}$
meijerg	$\frac{4d^2\cos(a)\sqrt{\pi}\left(\frac{x\left(b^2\right)^{\frac{3}{2}}\cos(bx)}{2\sqrt{\pi}\,b^2}-\frac{\left(b^2\right)^{\frac{3}{2}}\left(-\frac{3x^2b^2}{2}+3\right)\sin(bx)}{6\sqrt{\pi}\,b^3}\right)}{b^2\sqrt{b^2}}-\frac{4d^2\sin(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2b^2}{2}+1\right)\cos(bx)}{2\sqrt{\pi}}+\frac{xb\sin(bx}{2\sqrt{\pi}}\right.}{b^3}$

```
[In] int((d*x+c)^2*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(d*x+c)*cos(b*x+a)/b^2+(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^3*sin(
b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int (c+dx)^2\cos(a+bx)\,dx = \frac{2\,(bd^2x+bcd)\cos(bx+a)+(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin(bx+a)}{b^3}$$

```
[In] integrate((d*x+c)^2*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*(b*d^2*x + b*c*d)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 -
2*d^2)*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (c+dx)^2 \cos(a+bx)\,dx$$

$$= \begin{cases} \frac{c^2 \sin(a+bx)}{b} + \frac{2cdx \sin(a+bx)}{b} + \frac{d^2x^2 \sin(a+bx)}{b} + \frac{2cd \cos(a+bx)}{b^2} + \frac{2d^2x \cos(a+bx)}{b^2} - \frac{2d^2 \sin(a+bx)}{b^3} & \text{for } b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cos(a) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x+c)**2*cos(b*x+a),x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)/b + 2*c*d*x*sin(a + b*x)/b + d**2*x**2*sin(a +
 b*x)/b + 2*c*d*cos(a + b*x)/b**2 + 2*d**2*x*cos(a + b*x)/b**2 - 2*d**2*sin
(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cos(a), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(49) = 98$.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.78

$$\int (c+dx)^2 \cos(a+bx)\,dx$$

$$= \frac{c^2 \sin(bx+a) - \frac{2\,acd \sin(bx+a)}{b} + \frac{a^2d^2 \sin(bx+a)}{b^2} + \frac{2\left((bx+a)\sin(bx+a)+\cos(bx+a)\right)cd}{b} - \frac{2\left((bx+a)\sin(bx+a)+\cos(bx+a)\right)ad^2}{b^2} + \frac{}{}}{b}$$

```
[In] integrate((d*x+c)^2*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c^2*sin(b*x + a) - 2*a*c*d*sin(b*x + a)/b + a^2*d^2*sin(b*x + a)/b^2 + 2*(
(b*x + a)*sin(b*x + a) + cos(b*x + a))*c*d/b - 2*((b*x + a)*sin(b*x + a) +
cos(b*x + a))*a*d^2/b^2 + (2*(b*x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin
(b*x + a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int (c+dx)^2 \cos(a+bx)\,dx = \frac{2\,(bd^2x+bcd)\cos(bx+a)}{b^3} + \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin(bx+a)}{b^3}$$

```
[In] integrate((d*x+c)^2*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 - 2*d^2)*sin(b*x + a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.71

$$\int (c+dx)^2 \cos(a+bx)\,dx = \frac{d^2\,x^2\,\sin(a+b\,x)}{b} - \frac{\sin(a+b\,x)\,(2\,d^2-b^2\,c^2)}{b^3} + \frac{2\,c\,d\,\cos(a+b\,x)}{b^2} + \frac{2\,d^2\,x\,\cos(a+b\,x)}{b^2} + \frac{2\,c\,d\,x\,\sin(a+b\,x)}{b}$$

```
[In] int(cos(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (d^2*x^2*sin(a + b*x))/b - (sin(a + b*x)*(2*d^2 - b^2*c^2))/b^3 + (2*c*d*co
s(a + b*x))/b^2 + (2*d^2*x*cos(a + b*x))/b^2 + (2*c*d*x*sin(a + b*x))/b
```

3.4 $\int (c+dx)\cos(a+bx)\,dx$

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Giac [A] (verification not implemented) . . . 95
Mupad [B] (verification not implemented) . . . 95

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int (c+dx)\cos(a+bx)\,dx = \frac{d\cos(a+bx)}{b^2} + \frac{(c+dx)\sin(a+bx)}{b}$$

```
[Out] d*cos(b*x+a)/b^2+(d*x+c)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3377, 2718}

$$\int (c+dx)\cos(a+bx)\,dx = \frac{d\cos(a+bx)}{b^2} + \frac{(c+dx)\sin(a+bx)}{b}$$

```
[In] Int[(c + d*x)*Cos[a + b*x],x]
```

```
[Out] (d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \sin(a+bx)\,dx}{b} \\ &= \frac{d\cos(a+bx)}{b^2} + \frac{(c+dx)\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (c+dx)\cos(a+bx)\,dx = \frac{d\cos(a+bx) + b(c+dx)\sin(a+bx)}{b^2}$$

```
[In] Integrate[(c + d*x)*Cos[a + b*x],x]
[Out] (d*Cos[a + b*x] + b*(c + d*x)*Sin[a + b*x])/b^2
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result
risch	$\frac{d\cos(bx+a)}{b^2} + \frac{(dx+c)\sin(bx+a)}{b}$
parallelrisch	$\frac{(dx+c)b\sin(bx+a)+d(\cos(bx+a)-1)}{b^2}$
parts	$\frac{\sin(bx+a)dx}{b} + \frac{\sin(bx+a)c}{b} + \frac{d\cos(bx+a)}{b^2}$
derivativedivides	$\frac{-\frac{da\sin(bx+a)}{b}+c\sin(bx+a)+\frac{d(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}}{b}$
default	$\frac{-\frac{da\sin(bx+a)}{b}+c\sin(bx+a)+\frac{d(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}}{b}$
norman	$\frac{\frac{2d}{b^2}+\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{2dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
meijerg	$\frac{2d\cos(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(bx)}{2\sqrt{\pi}}+\frac{xb\sin(bx)}{2\sqrt{\pi}}\right)}{b^2} - \frac{2d\sin(a)\sqrt{\pi}\left(-\frac{xb\cos(bx)}{2\sqrt{\pi}}+\frac{\sin(bx)}{2\sqrt{\pi}}\right)}{b^2} + \frac{c\cos(a)\sin(bx)}{b} - \frac{c\sin(a)\sqrt{\pi}\left($

```
[In] int((d*x+c)*cos(b*x+a),x,method=_RETURNVERBOSE)
[Out] d*cos(b*x+a)/b^2+(d*x+c)*sin(b*x+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos(a+bx)\,dx = \frac{d\cos(bx+a)+(bdx+bc)\sin(bx+a)}{b^2}$$

```
[In] integrate((d*x+c)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a) + (b*d*x + b*c)*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int (c+dx)\cos(a+bx)\,dx = \begin{cases} \frac{c\sin(a+bx)}{b} + \frac{dx\sin(a+bx)}{b} + \frac{d\cos(a+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\cos(a) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x+c)*cos(b*x+a),x)
```

```
[Out] Piecewise((c*sin(a + b*x)/b + d*x*sin(a + b*x)/b + d*cos(a + b*x)/b**2, Ne(
b, 0)), ((c*x + d*x**2/2)*cos(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int (c+dx)\cos(a+bx)\,dx = \frac{c\sin(bx+a) - \frac{ad\sin(bx+a)}{b} + \frac{((bx+a)\sin(bx+a)+\cos(bx+a))d}{b}}{b}$$

```
[In] integrate((d*x+c)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (c*sin(b*x + a) - a*d*sin(b*x + a)/b + ((b*x + a)*sin(b*x + a) + cos(b*x +
a))*d/b)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int (c+dx)\cos(a+bx)\,dx = \frac{d\cos(bx+a)}{b^2} + \frac{(bdx+bc)\sin(bx+a)}{b^2}$$

```
[In] integrate((d*x+c)*cos(b*x+a),x, algorithm="giac")
[Out] d*cos(b*x + a)/b^2 + (b*d*x + b*c)*sin(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int (c+dx)\cos(a+bx)\,dx = \frac{c\sin(a+b\,x) + d\,x\,\sin(a+b\,x)}{b} + \frac{d\,\cos(a+b\,x)}{b^2}$$

```
[In] int(cos(a + b*x)*(c + d*x),x)
[Out] (c*sin(a + b*x) + d*x*sin(a + b*x))/b + (d*cos(a + b*x))/b^2
```

3.5 $\int \frac{\cos(a+bx)}{c+dx}\,dx$

Optimal result . . . 96
Rubi [A] (verified) . . . 96
Mathematica [A] (verified) . . . 97
Maple [A] (verified) . . . 97
Fricas [A] (verification not implemented) . . . 98
Sympy [F] . . . 98
Maxima [C] (verification not implemented) . . . 98
Giac [C] (verification not implemented) . . . 99
Mupad [F(-1)] . . . 99

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = \frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] Ci(b*c/d+b*x)*cos(a-b*c/d)/d-Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3384, 3380, 3383}

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = \frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[In] Int[Cos[a + b*x]/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d - (Sin[a - (b*c)/d]*SinInte
gral[(b*c)/d + b*x])/d
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx - \sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx \\ &= \frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = \frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right) - \sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x] - Sin[a - (b*c)/d]*SinIntegral
[(b*c)/d + b*x])/d
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

method	result	size
derivativedivides	$-\frac{\operatorname{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}+\frac{\operatorname{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}$	77
default	$-\frac{\operatorname{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}+\frac{\operatorname{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}$	77
risch	$-\frac{e^{-\frac{i(ad-bc)}{d}}\operatorname{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{2d}-\frac{e^{\frac{i(ad-bc)}{d}}\operatorname{Ei}_1\left(-ixb-ia-\frac{-iad+ibc}{d}\right)}{2d}$	96

```
[In] int(cos(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a
*d+b*c)/d)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = \frac{\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right) - \sin\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)}{d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - sin(-(b*c - a*d)/d)*si
n_integral((b*d*x + b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+bx\right)}{c+dx}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.73

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = -\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) - b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) - i\,E_1\left(-\frac{i\,bc+}{2\,bd}\right.$$

```
[In] integrate(cos(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integra
l_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(I*exp_
integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(1, -(I*
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 577, normalized size of antiderivative = 11.10

$$\int \frac{\cos(a+bx)}{c+dx}\,dx$$
$$= \frac{\Re\left(\mathrm{Ci}\left(bx+\frac{bc}{d}\right)\right)\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{bc}{2\,d}\right)^2+\Re\left(\mathrm{Ci}\left(-bx-\frac{bc}{d}\right)\right)\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{bc}{2\,d}\right)^2-2\,\Im\left(\mathrm{Ci}\left(bx+\frac{bc}{d}\right)\right)\tan\left(\frac{1}{2}\right.}{}$$

```
[In] integrate(cos(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + r
eal_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*imag
_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*imag_part(
cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*sin_integral((b
*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(b*x +
 b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*x - b*c/
d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a
)*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 - re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 + 4*real_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 4*real_part(cos_integral(-b*x -
b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - real_part(cos_integral(b*x + b*c/d))*ta
n(1/2*b*c/d)^2 - real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 2
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*a) - 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a) + 2
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*imag_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(1/
2*b*c/d) + real_part(cos_integral(b*x + b*c/d)) + real_part(cos_integral(-b
*x - b*c/d)))/(d*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2 + d*tan(1/2
*b*c/d)^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)}{c+d\,x}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x), x)
```

3.6 $\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 100
Rubi [A] (verified) . . . 100
Mathematica [A] (verified) . . . 101
Maple [A] (verified) . . . 102
Fricas [A] (verification not implemented) . . . 102
Sympy [F] . . . 102
Maxima [C] (verification not implemented) . . . 103
Giac [B] (verification not implemented) . . . 103
Mupad [F(-1)] . . . 104

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = -\frac{\cos(a+bx)}{d(c+dx)} - \frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2} - \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}$$

```
[Out] -cos(b*x+a)/d/(d*x+c)-b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2-b*Ci(b*c/d+b*x)*sin(
a-b*c/d)/d^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3380, 3383}

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = -\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{\cos(a+bx)}{d(c+dx)}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(Cos[a + b*x]/(d*(c + d*x))) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)
/d])/d^2 - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cos(a+bx)}{d(c+dx)} - \frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d} \\ &= -\frac{\cos(a+bx)}{d(c+dx)} - \frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} - \frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} \\ &= -\frac{\cos(a+bx)}{d(c+dx)} - \frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2} - \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\begin{aligned}&\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx \\ &= -\frac{\frac{d\cos(a+bx)}{c+dx} + b\,\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right) + b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}\end{aligned}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(((d*Cos[a + b*x])/(c + d*x) + b*CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d]
 + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])/d^2)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

method	result	size
derivativedivides	$b\left(-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}}{d}\right)$	114
default	$b\left(-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}}{d}\right)$	114
risch	$\frac{ib\,\mathrm{e}^{-\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{2d^2}-\frac{ib\,\mathrm{e}^{\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(-ixb-ia-\frac{-iad+ibc}{d}\right)}{2d^2}-\frac{(-2bxd-2bc)\cos(bx+a)}{2d(dx+c)(-bxd-bc)}$	140

```
[In] int(cos(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-cos(b*x+a)/(-a*d+b*c+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b
*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx$$
$$= -\frac{(bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+d\cos\left(bx+a\right)}{d^3x+cd^2}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + (b*d*x
+ b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + d*cos(b*x + a))/
(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.25

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = -\frac{b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(-i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_2\left(\right.\right.}{2\,(bcd+(bx+a)d^2-ad^2)b}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2*(-I
*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(2,
 -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b*c*d + (b*x +
 a)*d^2 - a*d^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(73) = 146$.

Time = 0.31 (sec) , antiderivative size = 523, normalized size of antiderivative = 7.16

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = -\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\,\mathrm{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+b^3c\,\mathrm{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right.\right.}{d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(((d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)/d) +
b^3*c*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a
*d)/d)*sin(-(b*c - a*d)/d) - a*b^2*d*cos_integral(((d*x + c)*(b - b*c/(d*x
+ c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)/d) - (d*x + c)*(b -
b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c - a*d)/d)*sin_integral(-((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - b^3*c*cos(-(b*c
- a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*
c - a*d)/d) + a*b^2*d*cos(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c
/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + b^2*d*cos(-(d*x + c)*(b - b*c
/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^2,x)
[Out] int(cos(a + b*x)/(c + d*x)^2, x)
```

3.7 $\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3} + \frac{b\sin(a+bx)}{2d^2(c+dx)} + \frac{b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

```
[Out] -1/2*b^2*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^3-1/2*cos(b*x+a)/d/(d*x+c)^2+1/2*b^2*
Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+1/2*b*sin(b*x+a)/d^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3380, 3383}

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = -\frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3} + \frac{b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{2d^3} + \frac{b\sin(a+bx)}{2d^2(c+dx)} - \frac{\cos(a+bx)}{2d(c+dx)^2}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -1/2*Cos[a + b*x]/(d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)
/d + b*x])/(2*d^3) + (b*Sin[a + b*x])/(2*d^2*(c + d*x)) + (b^2*Sin[a - (b*c
)/d]*SinIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos(a+bx)}{2d(c+dx)^2} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{2d} \\
&= -\frac{\cos(a+bx)}{2d(c+dx)^2} + \frac{b\sin(a+bx)}{2d^2(c+dx)} - \frac{b^2\int \frac{\cos(a+bx)}{c+dx}\,dx}{2d^2} \\
&= -\frac{\cos(a+bx)}{2d(c+dx)^2} + \frac{b\sin(a+bx)}{2d^2(c+dx)} - \frac{\left(b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2} \\
&\quad + \frac{\left(b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2} \\
&= -\frac{\cos(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3} \\
&\quad + \frac{b\sin(a+bx)}{2d^2(c+dx)} + \frac{b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{-b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d(-d\cos(a+bx)+b(c+dx)\sin(a+bx))}{(c+dx)^2}+b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^3,x]
```

```
[Out] (-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + (d*(-(d*Cos[a + b*x]) +
 b*(c + d*x)*Sin[a + b*x]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[
b*(c/d + x)])/(2*d^3)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.42

method	result
derivativedivides	$b^2\left(-\frac{\cos(bx+a)}{2(-ad+bc+d(bx+a))^2d}-\frac{-\frac{\sin(bx+a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{\operatorname{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}+\frac{\operatorname{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos}{d}}{d}}{2d}\right.$
default	$b^2\left(-\frac{\cos(bx+a)}{2(-ad+bc+d(bx+a))^2d}-\frac{-\frac{\sin(bx+a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{\operatorname{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}+\frac{\operatorname{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos}{d}}{d}}{2d}\right.$
risch	$\frac{b^2e^{-\frac{i(ad-bc)}{d}}\operatorname{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{4d^3}+\frac{b^2e^{\frac{i(ad-bc)}{d}}\operatorname{Ei}_1\left(-ixb-ia-\frac{-iad+ibc}{d}\right)}{4d^3}+\frac{(-2b^2d^3x^2-4b^2cd^2x-2b^2c^2d)\cos}{4d^2(dx+c)^2(x^2d^2b^2+2b^2cdx+}$

```
[In] int(cos(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*cos(b*x+a)/(-a*d+b*c+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-a*d+b*c+d*
(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c
)/d)*cos((-a*d+b*c)/d)/d)/d)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = -\frac{d^2\cos(bx+a) + (b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right) - (b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\sin\left(-\frac{bc-}{}\right.}{2\,(d^5x^2 + 2\,cd^4x + c^2d^3)}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c -
a*d)/d)*cos_integral((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - (b*d^2*x + b*c*d)*si
n(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.91

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = -\frac{b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) + E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) + b^3\left(-i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) + i\,E_3\left(-\right.\right.}{2\left(b^2c^2d - 2\,abcd^2 + (bx+a)^2d^3 + a^2d^3 + 2\,(bcd^2 - ad^3)(bx+a)\right)b}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3*(-I
*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(3,
 -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^2*c^2*d - 2*
a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 5518, normalized size of antiderivative = 53.06

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(co
s_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2
*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*
tan(1/2*b*c/d) - 4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part
(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4
*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/
2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^
2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-b*x -
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d^2*x^2*rea
l_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b^2*d^2*
x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*
b*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2
*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*imag_part(cos_i
ntegral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 8*b^2*c
*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/
d) - b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*b*c/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)
^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 8*b^2*c*d*x*si
n_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b^
2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^
2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*
b*c/d)^2 + b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*d^2*x^2*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 4*b^2*d^2*x^2*sin_in
tegral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a) - 2*b^2*c*d*x*real_part(c
```

```
os_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*d*x*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*d^2*x^2*
imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b^2*
d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)
 + 4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d
) + 8*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2
*a)*tan(1/2*b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*b^2*d^2*x^2*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^2*c^2*imag_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*c
^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/
2*b*c/d) - 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*b*c/d) - 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1
/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/
d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_integral(
b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b^2*d^2*x^2*sin_integr
al((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*b^2*c^2*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*c^
2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d)^2 + 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)
*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/
2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*x)^2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2 - 4*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*
x)^2*tan(1/2*a) + 4*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2
*b*x)^2*tan(1/2*a) - 8*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)
^2*tan(1/2*a) - b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)
^2 - b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - b^2*c
^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*c
^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b^
2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)
 - 4*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
b*c/d) + 8*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b
*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1
/2*b*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*
tan(1/2*b*c/d) + 4*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a)*tan(1/2*b*c/d) + 4*b^2*c^2*real_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^2*c*d*x*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*b^2*c*d*x*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 8*b^2*c*d*x*sin_in
tegral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - b^2*d^2*x^2*real_part
```

```
(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 - b^2*d^2*x^2*real_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - b^2*c^2*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - b^2*c^2*real_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos
_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b^2*c*d*x*imag_part
(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 8*b^2*c*d*x*sin_
integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b*d^2*x*tan(1/2*b
*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + b^2*c^2*real_part(cos_integral(b*x + b*
c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*c^2*real_part(cos_integral(-b*x -
 b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^
2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1
/2*b*x)^2 + 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^
2 - 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*b^2*d
^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 4*b^2*d^2*x^2*sin
_integral((b*d*x + b*c)/d)*tan(1/2*a) - 2*b^2*c^2*imag_part(cos_integral(b*
x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c^2*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 4*b^2*c^2*sin_integral((b*d*x + b*
c)/d)*tan(1/2*b*x)^2*tan(1/2*a) - 2*b^2*c*d*x*real_part(cos_integral(b*x +
b*c/d))*tan(1/2*a)^2 - 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*ta
n(1/2*a)^2 + 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c
/d) - 2*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) +
4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d) + 2*b^2*c^2*imag
_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b^2*c^2*
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 4*b^2
*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 8*b^2*c*
d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^2*
c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 2*b
^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2
*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)
- 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b
^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 - 2*b^2*c*d*
x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 + 2*b^2*c^2*imag_p
art(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*c^2*imag
_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b^2*c^2*s
in_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b*c*d*tan(1/2*
b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*b*c*d*tan(1/2*b*x)*tan(1/2*a)^2*tan(
1/2*b*c/d)^2 + 2*d^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2
*x^2*real_part(cos_integral(b*x + b*c/d)) + b^2*d^2*x^2*real_part(cos_integ
ral(-b*x - b*c/d)) + b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b
*x)^2 + b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2 - 4*b^
2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 4*b^2*c*d*x*imag_
part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 8*b^2*c*d*x*sin_integral((b*d
*x + b*c)/d)*tan(1/2*a) + 4*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a) - b^2*c^2*rea
l_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 - b^2*c^2*real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 + 4*
```

```
b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 4*b^2*c*d*x
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) + 8*b^2*c*d*x*sin_int
egral((b*d*x + b*c)/d)*tan(1/2*b*c/d) + 4*b^2*c^2*real_part(cos_integral(b*
x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 4*b^2*c^2*real_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - b^2*c^2*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*c/d)^2 - b^2*c^2*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*c/d)^2 - 4*b*d^2*x*tan(1/2*b*x)*tan(1/2*b*c/d)^2 - 4*b*d^2*x*tan
(1/2*a)*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))
 + 2*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d)) - 2*b^2*c^2*imag_part(
cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*b^2*c^2*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a) - 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a
) + 4*b*c*d*tan(1/2*b*x)^2*tan(1/2*a) + 4*b*c*d*tan(1/2*b*x)*tan(1/2*a)^2 +
 2*d^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c^2*imag_part(cos_integral(b*x +
 b*c/d))*tan(1/2*b*c/d) - 2*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*c/d) + 4*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d) - 4*
b*c*d*tan(1/2*b*x)*tan(1/2*b*c/d)^2 - 2*d^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2
 - 4*b*c*d*tan(1/2*a)*tan(1/2*b*c/d)^2 - 8*d^2*tan(1/2*b*x)*tan(1/2*a)*tan(
1/2*b*c/d)^2 - 2*d^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*c^2*real_part(cos_
integral(b*x + b*c/d)) + b^2*c^2*real_part(cos_integral(-b*x - b*c/d)) - 4*
b*d^2*x*tan(1/2*b*x) - 4*b*d^2*x*tan(1/2*a) - 4*b*c*d*tan(1/2*b*x) - 2*d^2*
tan(1/2*b*x)^2 - 4*b*c*d*tan(1/2*a) - 8*d^2*tan(1/2*b*x)*tan(1/2*a) - 2*d^2
*tan(1/2*a)^2 + 2*d^2*tan(1/2*b*c/d)^2 + 2*d^2)/(d^5*x^2*tan(1/2*b*x)^2*tan
(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*
b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1/2*b*x)^2*tan
(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x
)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2 +
 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2 + d^5*x^2*tan(1/2*a)^2 + c^2*d^3*tan(1/2
*b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x)^2*ta
n(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*
b*x)^2 + 2*c*d^4*x*tan(1/2*a)^2 + 2*c*d^4*x*tan(1/2*b*c/d)^2 + d^5*x^2 + c^
2*d^3*tan(1/2*b*x)^2 + c^2*d^3*tan(1/2*a)^2 + c^2*d^3*tan(1/2*b*c/d)^2 + 2*
c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^3}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^3, x)
```

3.8 $\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$

Optimal result . . . 113
Rubi [A] (verified) . . . 113
Mathematica [A] (verified) . . . 115
Maple [A] (verified) . . . 115
Fricas [B] (verification not implemented) . . . 116
Sympy [F] . . . 116
Maxima [C] (verification not implemented) . . . 116
Giac [C] (verification not implemented) . . . 117
Mupad [F(-1)] . . . 122

Optimal result

Integrand size = 14, antiderivative size = 127

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b^3\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{6d^4} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} + \frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{6d^4}$$

```
[Out] -1/3*cos(b*x+a)/d/(d*x+c)^3+1/6*b^2*cos(b*x+a)/d^3/(d*x+c)+1/6*b^3*cos(a-b*
c/d)*Si(b*c/d+b*x)/d^4+1/6*b^3*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^4+1/6*b*sin(b*x
+a)/d^2/(d*x+c)^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3380, 3383}

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = \frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{6d^4} + \frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{6d^4} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} - \frac{\cos(a+bx)}{3d(c+dx)^3}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^4,x]
```

```
[Out] -1/3*Cos[a + b*x]/(d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(6*d^3*(c + d*x)) +
(b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(6*d^4) + (b*Sin[a + b*x]
)/(6*d^2*(c + d*x)^2) + (b^3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(
6*d^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cos(a+bx)}{3d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{3d} \\
&= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} - \frac{b^2\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{6d^2} \\
&= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} + \frac{b^3\int \frac{\sin(a+bx)}{c+dx}\,dx}{6d^3} \\
&= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} \\
&\quad + \frac{\left(b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{6d^3} + \frac{\left(b^3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{6d^3} \\
&= -\frac{\cos(a+bx)}{3d(c+dx)^3} + \frac{b^2\cos(a+bx)}{6d^3(c+dx)} + \frac{b^3\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{6d^4} \\
&\quad + \frac{b\sin(a+bx)}{6d^2(c+dx)^2} + \frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{6d^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{d\cos(bx)\left(\left(-2d^2+b^2(c+dx)^2\right)\cos(a)+bd(c+dx)\sin(a)\right)+d\left(bd(c+dx)\cos(a)-\left(-2d^2+b^2(c+dx)^2\right)\right.}{6d^4(c+dx)}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^4,x]
```

```
[Out] (d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) + d*
(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + b^3*(
c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*Si
nIntegral[b*(c/d + x)]))/(6*d^4*(c + d*x)^3)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.45

method	result
derivativedivides	$b^3\left(-\frac{\cos(bx+a)}{3(-ad+bc+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-ad+bc+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right.}{d}}{2d}}{3d}\right.$
default	$b^3\left(-\frac{\cos(bx+a)}{3(-ad+bc+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-ad+bc+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right.}{d}}{2d}}{3d}\right.$
risch	$-\frac{ib^3\mathrm{e}^{-\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{12d^4}+\frac{ib^3\mathrm{e}^{\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(-ixb-ia-\frac{-iad+ibc}{d}\right)}{12d^4}-\frac{(-2b^5d^5x^5-10b^5cd^4x^4-20b$

```
[In] int(cos(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3*cos(b*x+a)/(-a*d+b*c+d*(b*x+a))^3/d-1/3*(-1/2*sin(b*x+a)/(-a*d+b*
c+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+b*c+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+
b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/
d)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(117) = 234$.

Time $= 0.29$ (sec) , antiderivative size $= 235$, normalized size of antiderivative $= 1.85$

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right) + (b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\cos}{6\,(d^7x^3 + 3\,cd^6x^2 + 3}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral
((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + b^3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) +
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + (b*d^3*x +
 b*c*d^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time $= 0.47$ (sec) , antiderivative size $= 249$, normalized size of antiderivative $= 1.96$

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) + E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) + b^4\left(-i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) + i\,E_4\left(-\right.\right.}{2\,\left(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 + (bx+a)^3d^4 - a^3d^4 + 3\,(bcd^3 - ad^4)(bx+a)^2 + 3\,(b^2c^2d^2 - 2\,a\right.}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^4*(-I
*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(4,
 -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^3*c^3*d - 3*
a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*
d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 8378, normalized size of antiderivative = 65.97

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral
((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*
x^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/
2*b*c/d) + 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)
^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(b*x +
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 +
 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^3*c*d^2*x^2*sin_int
egral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^3*d
^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + b
^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 4*b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*a)*tan(1/2*b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^3*d^3*x^3*sin_integral((b*d*x
 + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 6*b^3*c*d^2*x^2*real_
part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
+ 6*b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d) - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(-b*x -
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*sin_integral((b*d*
x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos
_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^3*
c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*t
an(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(b*x +
b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c^2*d*x*imag_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^
2 + 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2
```

```
*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a) + 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*x)^2*tan(1/2*a) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c*d^2*x^2*sin_integral(
(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^3*d^3*x^3*real_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 12*b^3*c*d^
2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/
2*b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b
*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^3*c*d^2*x^2*sin_integral((b*d*x + b*
c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 2*b^3*d^3*x^3*real_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^3*d^3*x^3*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^3*c^2*d*x
*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) + 6*b^3*c^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*
tan(1/2*a)^2*tan(1/2*b*c/d) - 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x +
b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*sin_
integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*r
eal_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^3*d^3
*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*
b^3*c^2*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*
tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_par
t(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^3*c*d^2*x
^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^3*
x^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^3*c^3*imag_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^3*c^3
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*
b*c/d)^2 + 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan
(1/2*b*x)^2 - b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x
)^2 + 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 6*b^3*c*
d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 6*
b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
a) - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 + b^3*d^
3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 2*b^3*d^3*x^3*si
n_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 - 3*b^3*c^2*d*x*imag_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 3*b^3*c^2*d*x*imag_part(co
s_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c^2*d*x*sin_i
ntegral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c*d^2*x^2*real
_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 6*b^3*c*d^
2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) +
```

```
 4*b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/
d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2
*b*c/d) + 8*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*
c/d) + 12*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*t
an(1/2*a)*tan(1/2*b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^3*c^2*d*x*sin_integral
((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 6*b^3*c*d^2*x^
2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^3*
c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)
 + 2*b^3*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)
^2*tan(1/2*b*c/d) + 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2
*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - b^3*d^3*x^3*imag_part(cos_integral(b*
x + b*c/d))*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*
c/d))*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/
2*b*c/d)^2 - 3*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(1/2*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(-b*x - b*c/d))*
tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_
integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^3*c^3*real_part(c
os_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^
3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1
/2*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 - 3*b^3*c^2*d*x*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/d)*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2*t
an(1/2*b*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(
1/2*b*x)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*
b*x)^2 + 6*b^3*c*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 2*b
^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*b^3*d^3*x^3*
real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) + 6*b^3*c^2*d*x*real_part(
cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) + 6*b^3*c^2*d*x*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a) - 3*b^3*c*d^2*x^2
*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 + 3*b^3*c*d^2*x^2*imag_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - 6*b^3*c*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(1/2*a)^2 + 2*b^2*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
 - b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2
 + b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2
 - 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*
b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*b^3*d^3
*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) - 6*b^3*c^2*d*x*r
eal_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 6*b^3*c
^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)
 + 12*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*
b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*
```

```
tan(1/2*b*c/d) + 24*b^3*c*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*
tan(1/2*b*c/d) + 4*b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^3*c^3*imag_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^3*c^3*sin_integral((b*d
*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 6*b^3*c^2*d*x*real_
part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^3*c^2*d*x
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 3*b^3*
c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)^2 + 3*b^3*c*d
^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 6*b^3*c*d^2
*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - 2*b^2*d^3*x^2*tan(1/2
*b*x)^2*tan(1/2*b*c/d)^2 - b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan
(1/2*b*x)^2*tan(1/2*b*c/d)^2 + b^3*c^3*imag_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)
*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integr
al(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 8*b^2*d^3*x^2*tan(1/2*b*x)*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^3*x^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 +
 b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 - b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d
)^2 + 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + 2*b^2*c^2*d*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + b^3*d^3*x^3*i
mag_part(cos_integral(b*x + b*c/d)) - b^3*d^3*x^3*imag_part(cos_integral(-b
*x - b*c/d)) + 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d) + 3*b^3*c^2*d*x*
imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2 - 3*b^3*c^2*d*x*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2 + 6*b^3*c^2*d*x*sin_integral(
(b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(b*
x + b*c/d))*tan(1/2*a) + 6*b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b*c/
d))*tan(1/2*a) + 2*b^3*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a) + 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*
x)^2*tan(1/2*a) - 3*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/
2*a)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 -
 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*b^2*c*d^2*x*t
an(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(b*x + b
*c/d))*tan(1/2*b*c/d) - 6*b^3*c*d^2*x^2*real_part(cos_integral(-b*x - b*c/d
))*tan(1/2*b*c/d) - 2*b^3*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*
b*x)^2*tan(1/2*b*c/d) - 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan
(1/2*b*x)^2*tan(1/2*b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*
c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^3*c^2*d*x*sin_integral((b*d*x
+ b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d) + 2*b^3*c^3*real_part(cos_integral(b*x
+ b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^3*c^3*real_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 3*b^3*c^2*d*x*imag_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(
-b*x - b*c/d))*tan(1/2*b*c/d)^2 - 6*b^3*c^2*d*x*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*c/d)^2 - 4*b^2*c*d^2*x*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 2*b^3
```

```
*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b
^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 -
16*b^2*c*d^2*x*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b*d^3*x*tan(1/2
*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b^2*c*d^2*x*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 - 4*b*d^3*x*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^3*c*d^2
*x^2*imag_part(cos_integral(b*x + b*c/d)) - 3*b^3*c*d^2*x^2*imag_part(cos_i
ntegral(-b*x - b*c/d)) + 6*b^3*c*d^2*x^2*sin_integral((b*d*x + b*c)/d) - 2*
b^2*d^3*x^2*tan(1/2*b*x)^2 + b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*t
an(1/2*b*x)^2 - b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^
2 + 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2 + 6*b^3*c^2*d*x*
real_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 6*b^3*c^2*d*x*real_part(c
os_integral(-b*x - b*c/d))*tan(1/2*a) - 8*b^2*d^3*x^2*tan(1/2*b*x)*tan(1/2*
a) - 2*b^2*d^3*x^2*tan(1/2*a)^2 - b^3*c^3*imag_part(cos_integral(b*x + b*c/
d))*tan(1/2*a)^2 + b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)
^2 - 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 2*b^2*c^2*d*tan
(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*c/d) - 6*b^3*c^2*d*x*real_part(cos_integral(-b*x - b*c/d))*tan
(1/2*b*c/d) + 4*b^3*c^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan
(1/2*b*c/d) - 4*b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*ta
n(1/2*b*c/d) + 8*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b
*c/d) + 2*b^2*d^3*x^2*tan(1/2*b*c/d)^2 - b^3*c^3*imag_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*c/d)^2 + b^3*c^3*imag_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*c/d)^2 - 2*b^3*c^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d)^
2 - 2*b^2*c^2*d*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 8*b^2*c^2*d*tan(1/2*b*x)*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d)^2 - 2*b^2*c^2*d*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 4*b*c*d^2*tan(1/2*b*x
)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 4*d^3*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 + 3*b^3*c^2*d*x*imag_part(cos_integral(b*x + b*c/d)) - 3*b^3*c^2*
d*x*imag_part(cos_integral(-b*x - b*c/d)) + 6*b^3*c^2*d*x*sin_integral((b*d
*x + b*c)/d) - 4*b^2*c*d^2*x*tan(1/2*b*x)^2 + 2*b^3*c^3*real_part(cos_integ
ral(b*x + b*c/d))*tan(1/2*a) + 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/
d))*tan(1/2*a) - 16*b^2*c*d^2*x*tan(1/2*b*x)*tan(1/2*a) - 4*b*d^3*x*tan(1/2
*b*x)^2*tan(1/2*a) - 4*b^2*c*d^2*x*tan(1/2*a)^2 - 4*b*d^3*x*tan(1/2*b*x)*ta
n(1/2*a)^2 - 2*b^3*c^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d)
- 2*b^3*c^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) + 4*b^2*c*
d^2*x*tan(1/2*b*c/d)^2 + 4*b*d^3*x*tan(1/2*b*x)*tan(1/2*b*c/d)^2 + 4*b*d^3*
x*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*b^2*d^3*x^2 + b^3*c^3*imag_part(cos_integ
ral(b*x + b*c/d)) - b^3*c^3*imag_part(cos_integral(-b*x - b*c/d)) + 2*b^3*c
^3*sin_integral((b*d*x + b*c)/d) - 2*b^2*c^2*d*tan(1/2*b*x)^2 - 8*b^2*c^2*d
*tan(1/2*b*x)*tan(1/2*a) - 4*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 2*b^2*c^2*
d*tan(1/2*a)^2 - 4*b*c*d^2*tan(1/2*b*x)*tan(1/2*a)^2 - 4*d^3*tan(1/2*b*x)^2
*tan(1/2*a)^2 + 2*b^2*c^2*d*tan(1/2*b*c/d)^2 + 4*b*c*d^2*tan(1/2*b*x)*tan(1
/2*b*c/d)^2 + 4*d^3*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 4*b*c*d^2*tan(1/2*a)*
tan(1/2*b*c/d)^2 + 16*d^3*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*d^3*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*b^2*c*d^2*x + 4*b*d^3*x*tan(1/2*b*x) + 4*
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b*d^3*x*tan(1/2*a) + 2*b^2*c^2*d + 4*b*c*d^2*tan(1/2*b*x) + 4*d^3*tan(1/2*b
*x)^2 + 4*b*c*d^2*tan(1/2*a) + 16*d^3*tan(1/2*b*x)*tan(1/2*a) + 4*d^3*tan(1
/2*a)^2 - 4*d^3*tan(1/2*b*c/d)^2 - 4*d^3)/(d^7*x^3*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/
d)^2 + d^7*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^7*x^3*tan(1/2*b*x)^2*tan(1/2
*b*c/d)^2 + d^7*x^3*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*c^2*d^5*x*tan(1/2*b*x
)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 3*c*d^6*x^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2*tan(1/2*a)^2*t
an(1/2*b*c/d)^2 + c^3*d^4*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^
7*x^3*tan(1/2*b*x)^2 + d^7*x^3*tan(1/2*a)^2 + 3*c^2*d^5*x*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + d^7*x^3*tan(1/2*b*c/d)^2 + 3*c^2*d^5*x*tan(1/2*b*x)^2*tan(1/2*
b*c/d)^2 + 3*c^2*d^5*x*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2*tan(1/2*
b*x)^2 + 3*c*d^6*x^2*tan(1/2*a)^2 + c^3*d^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + 3
*c*d^6*x^2*tan(1/2*b*c/d)^2 + c^3*d^4*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + c^3
*d^4*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^7*x^3 + 3*c^2*d^5*x*tan(1/2*b*x)^2 +
 3*c^2*d^5*x*tan(1/2*a)^2 + 3*c^2*d^5*x*tan(1/2*b*c/d)^2 + 3*c*d^6*x^2 + c^
3*d^4*tan(1/2*b*x)^2 + c^3*d^4*tan(1/2*a)^2 + c^3*d^4*tan(1/2*b*c/d)^2 + 3*
c^2*d^5*x + c^3*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^4}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^4,x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^4, x)
```

3.9 $\int (c+dx)^4 \cos^2(a+bx)\,dx$

Optimal result . . . 123
Rubi [A] (verified) . . . 123
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Fricas [A] (verification not implemented) . . . 126
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Optimal result

Integrand size = 16, antiderivative size = 161

$$\begin{aligned}\int (c+dx)^4 \cos^2(a+bx)\,dx = {} & \frac{3d^4x}{4b^4} - \frac{d(c+dx)^3}{2b^2} + \frac{(c+dx)^5}{10d} - \frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4} \\ & + \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} + \frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5} \\ & - \frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b^3} \\ & + \frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

```
[Out] 3/4*d^4*x/b^4-1/2*d*(d*x+c)^3/b^2+1/10*(d*x+c)^5/d-3/2*d^3*(d*x+c)*cos(b*x+
a)^2/b^4+d*(d*x+c)^3*cos(b*x+a)^2/b^2+3/4*d^4*cos(b*x+a)*sin(b*x+a)/b^5-3/2
*d^2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3+1/2*(d*x+c)^4*cos(b*x+a)*sin(b*x+a
)/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$\begin{aligned}\int (c+dx)^4 \cos^2(a+bx)\,dx = {} & \frac{3d^4\sin(a+bx)\cos(a+bx)}{4b^5} - \frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4} \\ & - \frac{3d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b^3} \\ & + \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} + \frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b} \\ & + \frac{3d^4x}{4b^4} - \frac{d(c+dx)^3}{2b^2} + \frac{(c+dx)^5}{10d}\end{aligned}$$

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2,x]
```

```
[Out] (3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^3*(
c + d*x)*Cos[a + b*x]^2)/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*
d^4*Cos[a + b*x]*Sin[a + b*x])/(4*b^5) - (3*d^2*(c + d*x)^2*Cos[a + b*x]*Si
n[a + b*x])/(2*b^3) + ((c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} + \frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad + \frac{1}{2}\int (c+dx)^4\,dx - \frac{(3d^2)\int (c+dx)^2\cos^2(a+bx)\,dx}{b^2} \\
&= \frac{(c+dx)^5}{10d} - \frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4} + \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} \\
&\quad - \frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b^3} + \frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad - \frac{(3d^2)\int (c+dx)^2\,dx}{2b^2} + \frac{(3d^4)\int \cos^2(a+bx)\,dx}{2b^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{d(c+dx)^3}{2b^2} + \frac{(c+dx)^5}{10d} - \frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4} + \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} \\
&\quad + \frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5} - \frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b^3} \\
&\quad + \frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b} + \frac{(3d^4)\int 1\,dx}{4b^4} \\
&= \frac{3d^4x}{4b^4} - \frac{d(c+dx)^3}{2b^2} + \frac{(c+dx)^5}{10d} - \frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4} \\
&\quad + \frac{d(c+dx)^3\cos^2(a+bx)}{b^2} + \frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5} \\
&\quad - \frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b^3} + \frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$
\begin{aligned}
&\int (c+dx)^4\cos^2(a+bx)\,dx \\
&= \frac{8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)+20bd(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+1}{80b^5}
\end{aligned}
$$

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2,x]
```

```
[Out] (8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) + 20
*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 10*(3*d^4 -
6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(80*b^5)
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{\left(2(dx+c)^4b^4-6d^2(dx+c)^2b^2+3d^4\right)\sin(2bx+2a)+4b\left((dx+c)\left((dx+c)^2b^2-\frac{3d^2}{2}\right)d\cos(2bx+2a)+x\left(\frac{1}{5}d^4x^4+c\,d^3x^3+2c^2\right.\right.}{8b^5}$
risch	$\frac{d^4x^5}{10}+\frac{c\,d^3x^4}{2}+d^2c^2x^3+d\,c^3x^2+\frac{c^4x}{2}+\frac{c^5}{10d}+\frac{d\left(2b^2d^3x^3+6b^2c\,d^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3d^2c\right)\cos}{4b^4}$
norman	$-\frac{\left(2b^4c^4-6b^2c^2d^2+3d^4\right)\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^5}+\frac{d^4x^5\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{10}+\frac{d^4x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4c\,d^3x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{d^2\left(2b^2c^2+d^2\right)x^3}{2b^2}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((d*x+c)^4*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((2*(d*x+c)^4*b^4-6*d^2*(d*x+c)^2*b^2+3*d^4)*sin(2*b*x+2*a)+4*b*((d*x+c
)*((d*x+c)^2*b^2-3/2*d^2)*d*cos(2*b*x+2*a)+x*(1/5*d^4*x^4+c*d^3*x^3+2*c^2*d
^2*x^2+2*c^3*d*x+c^4)*b^4-b^2*c^3*d+3/2*c*d^3))/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.78

$$\int (c+dx)^4 \cos^2(a+bx)\,dx$$
$$= \frac{2\,b^5d^4x^5 + 10\,b^5cd^3x^4 + 10\,(2\,b^5c^2d^2 - b^3d^4)x^3 + 10\,(2\,b^5c^3d - 3\,b^3cd^3)x^2 + 10\,(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 2\,b^3c^3$$

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 10*(2*b^5*c^2*d^2 - b^3*d^4)*x^3 +
 10*(2*b^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 +
 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*
(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2
*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)
*sin(b*x + a) + 5*(2*b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.

Time = 0.52 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.10

$$\int (c+dx)^4 \cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4 x \sin^2(a+bx)}{2} + \frac{c^4 x \cos^2(a+bx)}{2} + c^3 dx^2 \sin^2(a+bx) + c^3 dx^2 \cos^2(a+bx) + c^2 d^2 x^3 \sin^2(a+bx) + c^2 d^2 x^3 \cos \\ \left(c^4 x + 2c^3 dx^2 + 2c^2 d^2 x^3 + cd^3 x^4 + \frac{d^4 x^5}{5}\right) \cos^2(a) \end{cases}$$

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sin(a + b*x)**2/2 + c**4*x*cos(a + b*x)**2/2 + c**3*d*x**
2*sin(a + b*x)**2 + c**3*d*x**2*cos(a + b*x)**2 + c**2*d**2*x**3*sin(a + b*
x)**2 + c**2*d**2*x**3*cos(a + b*x)**2 + c*d**3*x**4*sin(a + b*x)**2/2 + c*
d**3*x**4*cos(a + b*x)**2/2 + d**4*x**5*sin(a + b*x)**2/10 + d**4*x**5*cos(
a + b*x)**2/10 + c**4*sin(a + b*x)*cos(a + b*x)/(2*b) + 2*c**3*d*x*sin(a +
b*x)*cos(a + b*x)/b + 3*c**2*d**2*x**2*sin(a + b*x)*cos(a + b*x)/b + 2*c*d*
*3*x**3*sin(a + b*x)*cos(a + b*x)/b + d**4*x**4*sin(a + b*x)*cos(a + b*x)/(
```

```
2*b) + c**3*d*cos(a + b*x)**2/b**2 - 3*c**2*d**2*x*sin(a + b*x)**2/(2*b**2)
 + 3*c**2*d**2*x*cos(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*sin(a + b*x)**2/(
2*b**2) + 3*c*d**3*x**2*cos(a + b*x)**2/(2*b**2) - d**4*x**3*sin(a + b*x)**
2/(2*b**2) + d**4*x**3*cos(a + b*x)**2/(2*b**2) - 3*c**2*d**2*sin(a + b*x)*
cos(a + b*x)/(2*b**3) - 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)/b**3 - 3*d**4*
x**2*sin(a + b*x)*cos(a + b*x)/(2*b**3) - 3*c*d**3*cos(a + b*x)**2/(2*b**4)
 + 3*d**4*x*sin(a + b*x)**2/(4*b**4) - 3*d**4*x*cos(a + b*x)**2/(4*b**4) +
3*d**4*sin(a + b*x)*cos(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x
**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cos(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(147) = 294$.

Time $= 0.28$ (sec) , antiderivative size $= 717$, normalized size of antiderivative $= 4.45$

$$\int (c+dx)^4 \cos^2(a+bx)\,dx$$

$$= \frac{10\,(2\,bx+2\,a+\sin(2\,bx+2\,a))c^4 - \frac{40\,(2\,bx+2\,a+\sin(2\,bx+2\,a))ac^3d}{b} + \frac{60\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2c^2d^2}{b^2} - \frac{40\,(2\,bx+2\,a+\sin}{}$$

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/40*(10*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^4 - 40*(2*b*x + 2*a + sin(2*b*x
 + 2*a))*a*c^3*d/b + 60*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*c^2*d^2/b^2 -
40*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^3*c*d^3/b^3 + 10*(2*b*x + 2*a + sin(2
*b*x + 2*a))*a^4*d^4/b^4 + 20*(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a)
 + cos(2*b*x + 2*a))*c^3*d/b - 60*(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x +
2*a) + cos(2*b*x + 2*a))*a*c^2*d^2/b^2 + 60*(2*(b*x + a)^2 + 2*(b*x + a)*si
n(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^2*c*d^3/b^3 - 20*(2*(b*x + a)^2 + 2*(b
*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^3*d^4/b^4 + 10*(4*(b*x + a)^
3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*
c^2*d^2/b^2 - 20*(4*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x
+ a)^2 - 1)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 10*(4*(b*x + a)^3 + 6*(b*x + a)
*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a^2*d^4/b^4 + 1
0*(2*(b*x + a)^4 + 3*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^
3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*c*d^3/b^3 - 10*(2*(b*x + a)^4 + 3*(2*(b*
x + a)^2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x
+ 2*a))*a*d^4/b^4 + (4*(b*x + a)^5 + 10*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2
*b*x + 2*a) + 5*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*sin(2*b*x + 2*a))*d^4/b
^4)/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.38

$$\int (c+dx)^4 \cos^2(a+bx)\,dx = \frac{1}{10}\,d^4x^5 + \frac{1}{2}\,cd^3x^4 + c^2d^2x^3 + c^3dx^2 + \frac{1}{2}\,c^4x + \frac{(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 6\,b^3c^2d^2x + 2\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\cos(2\,bx + 2\,a)}{4\,b^5} + \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 8\,b^4c^3dx + 2\,b^4c^4 - 6\,b^2d^4x^2 - 12\,b^2cd^3x - 6\,b^2c^2d^2 + 3\,d^4)\sin(2\,bx$$

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x + 1/4*(2
*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*b^3*c^3*d - 3*b*d^4*x
- 3*b*c*d^3)*cos(2*b*x + 2*a)/b^5 + 1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 +
12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d
^3*x - 6*b^2*c^2*d^2 + 3*d^4)*sin(2*b*x + 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.17

$$\int (c+dx)^4 \cos^2(a+bx)\,dx = \frac{\frac{15\,d^4\,\sin(2\,a+2\,b\,x)}{2} + 10\,b^5\,c^4\,x + 5\,b^4\,c^4\,\sin(2\,a+2\,b\,x) + 2\,b^5\,d^4\,x^5 + 10\,b^3\,c^3\,d\,\cos(2\,a+2\,b\,x) + 20\,b^5\,c^3\,d\,x^2 -$$

```
[In] int(cos(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] ((15*d^4*sin(2*a + 2*b*x))/2 + 10*b^5*c^4*x + 5*b^4*c^4*sin(2*a + 2*b*x) +
2*b^5*d^4*x^5 + 10*b^3*c^3*d*cos(2*a + 2*b*x) + 20*b^5*c^3*d*x^2 + 10*b^5*c
*d^3*x^4 - 15*b^2*c^2*d^2*sin(2*a + 2*b*x) + 10*b^3*d^4*x^3*cos(2*a + 2*b*x
) + 20*b^5*c^2*d^2*x^3 - 15*b^2*d^4*x^2*sin(2*a + 2*b*x) + 5*b^4*d^4*x^4*si
n(2*a + 2*b*x) - 15*b*c*d^3*cos(2*a + 2*b*x) - 15*b*d^4*x*cos(2*a + 2*b*x)
+ 30*b^4*c^2*d^2*x^2*sin(2*a + 2*b*x) - 30*b^2*c*d^3*x*sin(2*a + 2*b*x) + 2
0*b^4*c^3*d*x*sin(2*a + 2*b*x) + 30*b^3*c^2*d^2*x*cos(2*a + 2*b*x) + 30*b^3
*c*d^3*x^2*cos(2*a + 2*b*x) + 20*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/(20*b^5)
```

3.10 $\int (c+dx)^3 \cos^2(a+bx)\,dx$

Optimal result . . . 129
Rubi [A] (verified) . . . 129
Mathematica [A] (verified) . . . 131
Maple [A] (verified) . . . 131
Fricas [A] (verification not implemented) . . . 132
Sympy [B] (verification not implemented) . . . 132
Maxima [B] (verification not implemented) . . . 133
Giac [A] (verification not implemented) . . . 133
Mupad [B] (verification not implemented) . . . 134

Optimal result

Integrand size = 16, antiderivative size = 134

$$\begin{aligned}\int (c+dx)^3 \cos^2(a+bx)\,dx = &-\frac{3cd^2x}{4b^2} - \frac{3d^3x^2}{8b^2} + \frac{(c+dx)^4}{8d} \\ &- \frac{3d^3\cos^2(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2} \\ &- \frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3} \\ &+ \frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

```
[Out] -3/4*c*d^2*x/b^2-3/8*d^3*x^2/b^2+1/8*(d*x+c)^4/d-3/8*d^3*cos(b*x+a)^2/b^4+3
/4*d*(d*x+c)^2*cos(b*x+a)^2/b^2-3/4*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^3+1
/2*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3392, 32, 3391}

$$\begin{aligned}\int (c+dx)^3 \cos^2(a+bx)\,dx = &-\frac{3d^3\cos^2(a+bx)}{8b^4} - \frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{4b^3} \\ &+ \frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2} \\ &+ \frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b} \\ &- \frac{3cd^2x}{4b^2} - \frac{3d^3x^2}{8b^2} + \frac{(c+dx)^4}{8d}\end{aligned}$$

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2,x]
```

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) - (3*d^3*Cos
[a + b*x]^2)/(8*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2} + \frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad + \frac{1}{2}\int (c+dx)^3\,dx - \frac{(3d^2)\int (c+dx)\cos^2(a+bx)\,dx}{2b^2} \\
&= \frac{(c+dx)^4}{8d} - \frac{3d^3\cos^2(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2} \\
&\quad - \frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3} \\
&\quad + \frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b} - \frac{(3d^2)\int (c+dx)\,dx}{4b^2} \\
&= -\frac{3cd^2x}{4b^2} - \frac{3d^3x^2}{8b^2} + \frac{(c+dx)^4}{8d} - \frac{3d^3\cos^2(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2} \\
&\quad - \frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3} + \frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int (c+dx)^3 \cos^2(a+bx)\,dx$$
$$= \frac{2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+3d(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))+2b(c+dx)\left(-3d^2+2b^2(c\right.}{16b^4}$$

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2,x]
```

```
[Out] (2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c
 + d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Si
n[2*(a + b*x)])/(16*b^4)
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{4b\sin(2bx+2a)(dx+c)\left((dx+c)^2b^2-\frac{3d^2}{2}\right)+6d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\cos(2bx+2a)+2\left(d^3x^4+4d^2cx^3+6dc^2x^2+4c^3x\right)b^4-6b^2$
risch	$\frac{d^3x^4}{8}+\frac{d^2cx^3}{2}+\frac{3dc^2x^2}{4}+\frac{c^3x}{2}+\frac{c^4}{8d}+\frac{3d\left(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2\right)\cos(2bx+2a)}{16b^4}+\frac{\left(2b^2d^3x^3+6b^2cd^2x^2\right.}{}$
norman	$\frac{d^2cx^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{d^3x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{d^3x^4}{8}+\frac{d^2cx^3}{2}+\frac{d^3x^4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{d^3x^4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8}+\frac{\left(-6b^2c^2d+3d^3\right)\left($
derivativedivides	$-\frac{a^3d^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{3a^2cd^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{3a^2d^3\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right.\right.}{b^3}$
default	$-\frac{a^3d^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{3a^2cd^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{3a^2d^3\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right.\right.}{b^3}$

```
[In] int((d*x+c)^3*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(4*b*sin(2*b*x+2*a)*(d*x+c)*((d*x+c)^2*b^2-3/2*d^2)+6*d*((d*x+c)^2*b^2
-1/2*d^2)*cos(2*b*x+2*a)+2*(d^3*x^4+4*c*d^2*x^3+6*c^2*d*x^2+4*c^3*x)*b^4-6*
b^2*c^2*d+3*d^3)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.42

$$\int (c+dx)^3 \cos^2(a+bx)\,dx$$
$$= \frac{b^4d^3x^4 + 4\,b^4cd^2x^3 + 3\,(2\,b^4c^2d - b^2d^3)x^2 + 3\,(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3)\cos(bx+a)^2 + 2\,(2\,b^3d^3x}{8\,b^4}$$

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 3*(2*b^4*c^2*d - b^2*d^3)*x^2 + 3*(2*b
^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*
x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3 - 3*b^2*c*d^2)*x)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(131) = 262.

Time = 0.40 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.40

$$\int (c+dx)^3 \cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3x\sin^2(a+bx)}{2} + \frac{c^3x\cos^2(a+bx)}{2} + \frac{3c^2dx^2\sin^2(a+bx)}{4} + \frac{3c^2dx^2\cos^2(a+bx)}{4} + \frac{cd^2x^3\sin^2(a+bx)}{2} + \frac{cd^2x^3\cos^2(a+bx)}{2} + \frac{d^3x^4\sin^2}{8} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cos^2(a) \end{cases}$$

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sin(a + b*x)**2/2 + c**3*x*cos(a + b*x)**2/2 + 3*c**2*d*x
**2*sin(a + b*x)**2/4 + 3*c**2*d*x**2*cos(a + b*x)**2/4 + c*d**2*x**3*sin(a
 + b*x)**2/2 + c*d**2*x**3*cos(a + b*x)**2/2 + d**3*x**4*sin(a + b*x)**2/8
+ d**3*x**4*cos(a + b*x)**2/8 + c**3*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c*
*2*d*x*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c*d**2*x**2*sin(a + b*x)*cos(a +
 b*x)/(2*b) + d**3*x**3*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c**2*d*cos(a +
b*x)**2/(4*b**2) - 3*c*d**2*x*sin(a + b*x)**2/(4*b**2) + 3*c*d**2*x*cos(a +
 b*x)**2/(4*b**2) - 3*d**3*x**2*sin(a + b*x)**2/(8*b**2) + 3*d**3*x**2*cos(
a + b*x)**2/(8*b**2) - 3*c*d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3) - 3*d**3
*x*sin(a + b*x)*cos(a + b*x)/(4*b**3) - 3*d**3*cos(a + b*x)**2/(8*b**4), Ne
(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a)**2,
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(120) = 240.

Time = 0.32 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.19

$$\int (c+dx)^3 \cos^2(a+bx)\,dx$$

$$= \frac{4\,(2\,bx+2\,a+\sin(2\,bx+2\,a))c^3 - \frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))ac^2d}{b} + \frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2cd^2}{b^2} - \frac{4\,(2\,bx+2\,a+\sin(2\,b}{b^3}$$

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^3 - 12*(2*b*x + 2*a + sin(2*b*x
+ 2*a))*a*c^2*d/b + 12*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*c*d^2/b^2 - 4*(
2*b*x + 2*a + sin(2*b*x + 2*a))*a^3*d^3/b^3 + 6*(2*(b*x + a)^2 + 2*(b*x + a
)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)^2 + 2*(b*x
 + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)^2 +
 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^2*d^3/b^3 + 2*(4*(b*x +
 a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*
a))*c*d^2/b^2 - 2*(4*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x
 + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(b*x + a)^4 + 3*(2*(b*x + a)^
2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))
*d^3/b^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.14

$$\int (c+dx)^3 \cos^2(a+bx)\,dx$$

$$= \frac{1}{8}\,d^3x^4 + \frac{1}{2}\,cd^2x^3 + \frac{3}{4}\,c^2dx^2 + \frac{1}{2}\,c^3x + \frac{3\,(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3)\cos(2\,bx+2\,a)}{16\,b^4} + \frac{(2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx + 2\,b^3c^3 - 3\,bd^3x - 3\,bcd^2)\sin(2\,bx+2\,a)}{8\,b^4}$$

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x + 3/16*(2*b^2*d^3*x
^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(2*b*x + 2*a)/b^4 + 1/8*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^
2)*sin(2*b*x + 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int (c+dx)^3 \cos^2(a+bx)\,dx$$

$$= \frac{4\,b^4\,c^3\,x - \frac{3\,d^3\,\cos(2\,a+2\,b\,x)}{2} + 2\,b^3\,c^3\,\sin(2\,a+2\,b\,x) + b^4\,d^3\,x^4 + 3\,b^2\,c^2\,d\,\cos(2\,a+2\,b\,x) + 6\,b^4\,c^2\,d\,x^2 + 4\,b^4$$

```
[In] int(cos(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] (4*b^4*c^3*x - (3*d^3*cos(2*a + 2*b*x))/2 + 2*b^3*c^3*sin(2*a + 2*b*x) + b^
4*d^3*x^4 + 3*b^2*c^2*d*cos(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*d^2*x^
3 + 3*b^2*d^3*x^2*cos(2*a + 2*b*x) + 2*b^3*d^3*x^3*sin(2*a + 2*b*x) - 3*b*c
*d^2*sin(2*a + 2*b*x) - 3*b*d^3*x*sin(2*a + 2*b*x) + 6*b^2*c*d^2*x*cos(2*a
+ 2*b*x) + 6*b^3*c^2*d*x*sin(2*a + 2*b*x) + 6*b^3*c*d^2*x^2*sin(2*a + 2*b*x
))/(8*b^4)
```

3.11 $\int (c+dx)^2 \cos^2(a+bx)\,dx$

Optimal result . . . 135
Rubi [A] (verified) . . . 135
Mathematica [A] (verified) . . . 137
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Giac [A] (verification not implemented) . . . 139
Mupad [B] (verification not implemented) . . . 140

Optimal result

Integrand size = 16, antiderivative size = 95

$$\int (c+dx)^2 \cos^2(a+bx)\,dx = -\frac{d^2 x}{4b^2} + \frac{(c+dx)^3}{6d} + \frac{d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2 \cos(a+bx)\sin(a+bx)}{4b^3} + \frac{(c+dx)^2 \cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] -1/4*d^2*x/b^2+1/6*(d*x+c)^3/d+1/2*d*(d*x+c)*cos(b*x+a)^2/b^2-1/4*d^2*cos(b
*x+a)*sin(b*x+a)/b^3+1/2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$\int (c+dx)^2 \cos^2(a+bx)\,dx = -\frac{d^2 \sin(a+bx)\cos(a+bx)}{4b^3} + \frac{d(c+dx)\cos^2(a+bx)}{2b^2} + \frac{(c+dx)^2 \sin(a+bx)\cos(a+bx)}{2b} - \frac{d^2 x}{4b^2} + \frac{(c+dx)^3}{6d}$$

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2,x]
```

```
[Out] -1/4*(d^2*x)/b^2 + (c + d*x)^3/(6*d) + (d*(c + d*x)*Cos[a + b*x]^2)/(2*b^2)
 - (d^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^3) + ((c + d*x)^2*Cos[a + b*x]*Sin[
a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(c+dx)\cos^2(a+bx)}{2b^2} + \frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad + \frac{1}{2}\int (c+dx)^2\,dx - \frac{d^2\int \cos^2(a+bx)\,dx}{2b^2} \\
&= \frac{(c+dx)^3}{6d} + \frac{d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3} \\
&\quad + \frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b} - \frac{d^2\int 1\,dx}{4b^2} \\
&= -\frac{d^2x}{4b^2} + \frac{(c+dx)^3}{6d} + \frac{d(c+dx)\cos^2(a+bx)}{2b^2} \\
&\quad - \frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3} + \frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int (c+dx)^2 \cos^2(a+bx)\,dx$$
$$= \frac{4b^3x(3c^2+3cdx+d^2x^2)+6bd(c+dx)\cos(2(a+bx))+3(-d^2+2b^2(c+dx)^2)\sin(2(a+bx))}{24b^3}$$

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2,x]
```

```
[Out] (4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 6*b*d*(c + d*x)*Cos[2*(a + b*x)] + 3
*(-d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(24*b^3)
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{\left(2(dx+c)^2b^2-d^2\right)\sin(2bx+2a)+4b\left(\frac{d(dx+c)\cos(2bx+2a)}{2}+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2-\frac{cd}{2}\right)}{8b^3}$
risch	$\frac{d^2x^3}{6}+\frac{cdx^2}{2}+\frac{c^2x}{2}+\frac{c^3}{6d}+\frac{d(dx+c)\cos(2bx+2a)}{4b^2}+\frac{\left(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2\right)\sin(2bx+2a)}{8b^3}$
derivativedivides	$\frac{a^2d^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(b$
default	$\frac{a^2d^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(b$
norman	$cdx^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{d^2x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{d^2x^3}{6}+\frac{cdx^2}{2}+\frac{d^2x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{d^2x^3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6}+\frac{\left(2b^2c^2-d^2\right)\tan\left(\frac{bx}{2}\right.}{2b^3}$

```
[In] int((d*x+c)^2*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((2*(d*x+c)^2*b^2-d^2)*sin(2*b*x+2*a)+4*b*(1/2*d*(d*x+c)*cos(2*b*x+2*a)
+x*(1/3*x^2*d^2+c*d*x+c^2)*b^2-1/2*c*d))/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int (c+dx)^2 \cos^2(a+bx)\,dx$$
$$= \frac{2\,b^3d^2x^3 + 6\,b^3cdx^2 + 6\,(bd^2x + bcd)\cos(bx+a)^2 + 3\,(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2)\cos(bx+a)\sin(bx}{12\,b^3}$$

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 +
3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a)
 + 3*(2*b^3*c^2 - b*d^2)*x)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(85) = 170.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.78

$$\int (c+dx)^2 \cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^2x\sin^2(a+bx)}{2} + \frac{c^2x\cos^2(a+bx)}{2} + \frac{cdx^2\sin^2(a+bx)}{2} + \frac{cdx^2\cos^2(a+bx)}{2} + \frac{d^2x^3\sin^2(a+bx)}{6} + \frac{d^2x^3\cos^2(a+bx)}{6} + \frac{c^2\sin(a+bx)\cos}{2b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cos^2(a) \end{cases}$$

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sin(a + b*x)**2/2 + c**2*x*cos(a + b*x)**2/2 + c*d*x**2*s
in(a + b*x)**2/2 + c*d*x**2*cos(a + b*x)**2/2 + d**2*x**3*sin(a + b*x)**2/6
 + d**2*x**3*cos(a + b*x)**2/6 + c**2*sin(a + b*x)*cos(a + b*x)/(2*b) + c*d
*x*sin(a + b*x)*cos(a + b*x)/b + d**2*x**2*sin(a + b*x)*cos(a + b*x)/(2*b)
+ c*d*cos(a + b*x)**2/(2*b**2) - d**2*x*sin(a + b*x)**2/(4*b**2) + d**2*x*c
os(a + b*x)**2/(4*b**2) - d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)
), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cos(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(85) = 170$.

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.34

$$\int (c+dx)^2 \cos^2(a+bx)\,dx$$

$$= \frac{6\,(2\,bx+2\,a+\sin(2\,bx+2\,a))c^2 - \frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))acd}{b} + \frac{6\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2d^2}{b^2} + \frac{6\left(2\,(bx+a)^2+2\,(bx+a\right.}{}$$

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^2 - 12*(2*b*x + 2*a + sin(2*b*x
+ 2*a))*a*c*d/b + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*d^2/b^2 + 6*(2*(b*
x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c*d/b - 6*(2*(b
*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a*d^2/b^2 + (4
*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b
*x + 2*a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int (c+dx)^2 \cos^2(a+bx)\,dx = \frac{1}{6}\,d^2x^3 + \frac{1}{2}\,cdx^2 + \frac{1}{2}\,c^2x + \frac{(bd^2x+bcd)\cos(2\,bx+2\,a)}{4\,b^3} + \frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\sin(2\,bx+2\,a)}{8\,b^3}$$

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x + 1/4*(b*d^2*x + b*c*d)*cos(2*b*x + 2
*a)/b^3 + 1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*sin(2*b*x + 2
*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.88

$$\begin{aligned}\int (c+dx)^2\cos^2(a+bx)\,dx = {} & x\left(\frac{c^2}{4}-\frac{d^2}{8\,b^2}\right)+x\left(\frac{c^2}{4}+\frac{d^2}{8\,b^2}\right)\\ & +\frac{d^2\,x^3}{6}-\frac{\sin\left(2\,a+2\,b\,x\right)\,\left(d^2-2\,b^2\,c^2\right)}{8\,b^3}\\ & -\frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{c^2}{2}-\frac{d^2}{4\,b^2}\right)}{2}\\ & +\frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{c^2}{2}+\frac{d^2}{4\,b^2}\right)}{2}\\ & +\frac{c\,d\,x^2}{2}+\frac{d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{4\,b}\\ & +\frac{c\,d\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^2}+\frac{c\,d\,x\,\sin\left(2\,a+2\,b\,x\right)}{2\,b}\end{aligned}$$

```
[In] int(cos(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x*(c^2/4 - d^2/(8*b^2)) + x*(c^2/4 + d^2/(8*b^2)) + (d^2*x^3)/6 - (sin(2*a
+ 2*b*x)*(d^2 - 2*b^2*c^2))/(8*b^3) - (x*cos(2*a + 2*b*x)*(c^2/2 - d^2/(4*b
^2)))/2 + (x*cos(2*a + 2*b*x)*(c^2/2 + d^2/(4*b^2)))/2 + (c*d*x^2)/2 + (d^2
*x^2*sin(2*a + 2*b*x))/(4*b) + (c*d*cos(2*a + 2*b*x))/(4*b^2) + (c*d*x*sin(
2*a + 2*b*x))/(2*b)
```

3.12 $\int (c+dx)\cos^2(a+bx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 55

$$\int (c+dx)\cos^2(a+bx)\,dx = \frac{cx}{2}+\frac{dx^2}{4}+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] 1/2*c*x+1/4*d*x^2+1/4*d*cos(b*x+a)^2/b^2+1/2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/
b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3391}

$$\int (c+dx)\cos^2(a+bx)\,dx = \frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

```
[In] Int[(c + d*x)*Cos[a + b*x]^2,x]
```

```
[Out] (c*x)/2 + (d*x^2)/4 + (d*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/(2*b)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\cos^2(a+bx)}{4b^2} + \frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b} + \frac{1}{2}\int (c+dx)\,dx \\ &= \frac{cx}{2} + \frac{dx^2}{4} + \frac{d\cos^2(a+bx)}{4b^2} + \frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\begin{aligned}&\int (c+dx)\cos^2(a+bx)\,dx \\ &= \frac{d\cos(2(a+bx)) + 2b(2ac + bx(2c+dx) + (c+dx)\sin(2(a+bx)))}{8b^2}\end{aligned}$$

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2,x]
```

```
[Out] (d*Cos[2*(a + b*x)] + 2*b*(2*a*c + b*x*(2*c + d*x) + (c + d*x)*Sin[2*(a + b
*x)]))/(8*b^2)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result
risch	$\frac{dx^2}{4} + \frac{cx}{2} + \frac{d\cos(2bx+2a)}{8b^2} + \frac{(dx+c)\sin(2bx+2a)}{4b}$
parallelrisch	$\frac{2b\sin(2bx+2a)(dx+c)+d\cos(2bx+2a)+(2dx^2+4cx)b^2-d}{8b^2}$
derivativedivides	$\frac{-\frac{da\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{d\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{(s}{b}}{b}$
default	$\frac{-\frac{da\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{d\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{(s}{b}}{b}$
norman	$\frac{\frac{c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+cx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{cx}{2}+\frac{dx^2}{4}-\frac{c\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{cx\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{d}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$

```
[In] int((d*x+c)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*x^2+1/2*c*x+1/8*d/b^2*cos(2*b*x+2*a)+1/4/b*(d*x+c)*sin(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (c+dx)\cos^2(a+bx)\,dx$$
$$= \frac{b^2dx^2+2\,b^2cx+d\cos\left(bx+a\right)^2+2\left(bdx+bc\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{4\,b^2}$$

```
[In] integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^2 + 2*b^2*c*x + d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(b*x + a
)*sin(b*x + a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(49) = 98.

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int (c+dx)\cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{cx\sin^2(a+bx)}{2}+\frac{cx\cos^2(a+bx)}{2}+\frac{dx^2\sin^2(a+bx)}{4}+\frac{dx^2\cos^2(a+bx)}{4}+\frac{c\sin(a+bx)\cos(a+bx)}{2b}+\frac{dx\sin(a+bx)\cos(a+bx)}{2b}+\frac{d\cos^2} \\ \left(cx+\frac{dx^2}{2}\right)\cos^2(a) \end{cases}$$

```
[In] integrate((d*x+c)*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sin(a + b*x)**2/2 + c*x*cos(a + b*x)**2/2 + d*x**2*sin(a + b
*x)**2/4 + d*x**2*cos(a + b*x)**2/4 + c*sin(a + b*x)*cos(a + b*x)/(2*b) + d
*x*sin(a + b*x)*cos(a + b*x)/(2*b) + d*cos(a + b*x)**2/(4*b**2), Ne(b, 0)),
 ((c*x + d*x**2/2)*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.64

$$\int (c+dx)\cos^2(a+bx)\,dx$$
$$= \frac{2\left(2\,bx+2\,a+\sin\left(2\,bx+2\,a\right)\right)c-\frac{2\left(2\,bx+2\,a+\sin(2\,bx+2\,a)\right)ad}{b}+\frac{\left(2\left(bx+a\right)^2+2\left(bx+a\right)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)d}{b}}{8\,b}$$

```
[In] integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*b*x + 2*a + sin(2*b*x + 2*a))*c - 2*(2*b*x + 2*a + sin(2*b*x + 2*
a))*a*d/b + (2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a
))*d/b)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int (c+dx)\cos^2(a+bx)\,dx = \frac{1}{4}\,dx^2 + \frac{1}{2}\,cx + \frac{d\cos(2\,bx+2\,a)}{8\,b^2} + \frac{(bdx+bc)\sin(2\,bx+2\,a)}{4\,b^2}$$

```
[In] integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*x^2 + 1/2*c*x + 1/8*d*cos(2*b*x + 2*a)/b^2 + 1/4*(b*d*x + b*c)*sin(2*
b*x + 2*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos^2(a+bx)\,dx = \frac{c\,x}{2} + \frac{d\,x^2}{4} + \frac{d\,\cos(2\,a+2\,b\,x)}{8\,b^2} + \frac{c\,\sin(2\,a+2\,b\,x)}{4\,b} + \frac{d\,x\,\sin(2\,a+2\,b\,x)}{4\,b}$$

```
[In] int(cos(a + b*x)^2*(c + d*x),x)
```

```
[Out] (c*x)/2 + (d*x^2)/4 + (d*cos(2*a + 2*b*x))/(8*b^2) + (c*sin(2*a + 2*b*x))/(
4*b) + (d*x*sin(2*a + 2*b*x))/(4*b)
```

3.13 $\int \frac{\cos^2(a+bx)}{c+dx}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{\log(c+dx)}{2d} - \frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] 1/2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d+1/2*ln(d*x+c)/d-1/2*Si(2*b*c/d+2*b
*x)*sin(2*a-2*b*c/d)/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3384, 3380, 3383}

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d} - \frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{\log(c+dx)}{2d}$$

```
[In] Int[Cos[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x]/
(2*d) - (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2(c+dx)} + \frac{\cos(2a+2bx)}{2(c+dx)}\right) dx \\
&= \frac{\log(c+dx)}{2d} + \frac{1}{2}\int \frac{\cos(2a+2bx)}{c+dx}\, dx \\
&= \frac{\log(c+dx)}{2d} + \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\, dx \\
&\quad - \frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\, dx \\
&= \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{\log(c+dx)}{2d} - \frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int \frac{\cos^2(a+bx)}{c+dx}\, dx \\
&= \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \log(c+dx) - \sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d}
\end{aligned}$$

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - Sin[2
*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.37

method	result	s
risch	$\frac{\ln(dx+c)}{2d} - \frac{e^{-\frac{2i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(2ixb+2ia-\frac{2i(ad-bc)}{d}\right)}{4d} - \frac{e^{\frac{2i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(-2ixb-2ia-\frac{2(-iad+ibc)}{d}\right)}{4d}$	1
derivativedivides	$\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{4}+\frac{b\ln(-ad+bc+d(bx+a))}{2d}}{b}$	1
default	$\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{4}+\frac{b\ln(-ad+bc+d(bx+a))}{2d}}{b}$	1

```
[In] int(cos(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(d*x+c)/d-1/4/d*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*x*b+2*I*a-2*I*(a*d-b*c
)/d)-1/4/d*exp(2*I*(a*d-b*c)/d)*Ei(1,-2*I*x*b-2*I*a-2*(-I*a*d+I*b*c)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \frac{\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right) - \sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right) + \log\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - sin(-2*(b*c -
a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + log(d*x + c))/d
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \int \frac{\cos^2(a+bx)}{c+dx}\,dx$$

```
[In] integrate(cos(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.09

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = -\frac{b\left(E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad}{d}\right.\right.}{4\,bd}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_inte
gral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) - b
*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integr
al_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) - 2*b
*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 610, normalized size of antiderivative = 7.82

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(d*x + c))*tan(a)^2*tan(b*c/d)^2 + real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)^2*tan(b*c/d) + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*ta
n(b*c/d) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) + 2*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*imag_part(cos_in
```

```
tegral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)*tan(b*c/d)^2 + 2*log(abs(d*x + c))*tan(a)^2 - real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)^2 - real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 2*log(ab
s(d*x + c))*tan(b*c/d)^2 - real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c
/d)^2 - real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*imag_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(a) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a) + 2*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 2*
log(abs(d*x + c)) + real_part(cos_integral(2*b*x + 2*b*c/d)) + real_part(co
s_integral(-2*b*x - 2*b*c/d)))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*ta
n(b*c/d)^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

```
[In] int(cos(a + b*x)^2/(c + d*x),x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x), x)
```

3.14 $\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{\cos^2(a+bx)}{d(c+dx)} - \frac{b\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} - \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

```
[Out] -cos(b*x+a)^2/d/(d*x+c)-b*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2-b*Ci(2*b*c
/d+2*b*x)*sin(2*a-2*b*c/d)/d^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3394, 12, 3384, 3380, 3383}

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{\cos^2(a+bx)}{d(c+dx)}$$

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] -(Cos[a + b*x]^2/(d*(c + d*x))) - (b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a
 - (2*b*c)/d])/d^2 - (b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x]
)/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^2(a+bx)}{d(c+dx)} + \frac{(2b)\int -\frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d} \\
&= -\frac{\cos^2(a+bx)}{d(c+dx)} - \frac{b\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d} \\
&= -\frac{\cos^2(a+bx)}{d(c+dx)} - \frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} - \frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} \\
&= -\frac{\cos^2(a+bx)}{d(c+dx)} - \frac{b\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} - \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{\frac{d\cos^2(a+bx)}{c+dx} + b\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) + b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] -(((d*Cos[a + b*x]^2)/(c + d*x) + b*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a
- (2*b*c)/d] + b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{1}{2d(dx+c)} + \frac{ib\,e^{-\frac{2i(ad-bc)}{d}}\,\text{Ei}_1\left(2ixb+2ia-\frac{2i(ad-bc)}{d}\right)}{2d^2} - \frac{ib\,e^{\frac{2i(ad-bc)}{d}}\,\text{Ei}_1\left(-2ixb-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^2} - \frac{(-2bxd-$
derivativedivides	$\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-ad+bc+d(bx+a))d} - \frac{2\left(-\frac{2\,\text{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d} - \frac{2\,\text{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{d}\right)}{4} - $
default	$\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-ad+bc+d(bx+a))d} - \frac{2\left(-\frac{2\,\text{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d} - \frac{2\,\text{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{d}\right)}{4} - $

```
[In] int(cos(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/(d*x+c)+1/2*I*b/d^2*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*x*b+2*I*a-2*I*(a*
d-b*c)/d)-1/2*I*b/d^2*exp(2*I*(a*d-b*c)/d)*Ei(1,-2*I*x*b-2*I*a-2*(-I*a*d+I*
b*c)/d)-1/4/d*(-2*b*d*x-2*b*c)/(-b*d*x-b*c)/(d*x+c)*cos(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{d\cos(bx+a)^2 + (bdx+bc)\,\mathrm{Ci}\left(\frac{2(bdx+bc)}{d}\right)\sin\left(-\frac{2(bc-ad)}{d}\right) + (bdx+bc)\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2(bdx+bc)}{d}\right)}{d^3x+cd^2}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(d*cos(b*x + a)^2 + (b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(
b*c - a*d)/d) + (b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
 b*c)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx$$

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{b^2\left(E_2\left(\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{2(-i\,bc-i\,(bx+a)d+i}{d}\right.\right.}{4\,(bcd+(bx+a)d^2-ad^2)b}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) +
 b^2*(I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_int
egral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) +
2*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(83) = 166$.

Time = 0.36 (sec) , antiderivative size = 534, normalized size of antiderivative = 6.43

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx =$$

$$-\frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\,\mathrm{Ci}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\,b^3c\,\mathrm{Ci}\left(\frac{2\left((dx+c)\left(b\right.\right.\right.}{}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(2*((
d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c -
a*d)/d) + 2*b^3*c*cos_integral(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
 c)) + b*c - a*d)/d)*sin(-2*(b*c - a*d)/d) - 2*a*b^2*d*cos_integral(2*((d*x
 + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c - a*d
)/d) - 2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-2*(b*c - a*
d)/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
- a*d)/d) - 2*b^3*c*cos(-2*(b*c - a*d)/d)*sin_integral(-2*((d*x + c)*(b - b
*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 2*a*b^2*d*cos(-2*(b*c - a*d
)/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c -
 a*d)/d) + b^2*d*cos(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) +
b^2*d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*d^4 -
a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int(cos(a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x)^2, x)
```

3.15 $\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

```
[Out] -b^2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^3-1/2*cos(b*x+a)^2/d/(d*x+c)^2+b^
2*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3+b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c
)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3395, 31, 3393, 3384, 3380, 3383}

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = -\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)} - \frac{\cos^2(a+bx)}{2d(c+dx)^2}$$

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] -1/2*Cos[a + b*x]^2/(d*(c + d*x)^2) - (b^2*Cos[2*a - (2*b*c)/d]*CosIntegral
[(2*b*c)/d + 2*b*x])/d^3 + (b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*x)) +
(b^2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\text{integral} = -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{b^2\int \frac{1}{c+dx}\,dx}{d^2} - \frac{(2b^2)\int \frac{\cos^2(a+bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\log(c+dx)}{d^3} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{(2b^2)\int\left(\frac{1}{2(c+dx)} + \frac{\cos(2a+2bx)}{2(c+dx)}\right)dx}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{b^2\int\frac{\cos(2a+2bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} - \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} + \frac{\left(b^2\sin\left(2a-\frac{2bc}{d}\right)\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2}$$

$$= -\frac{\cos^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int\frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = \frac{-2b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \frac{d\left(-d\cos^2(a+bx)+b(c+dx)\sin(2(a+bx))\right)}{(c+dx)^2} + 2b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+d}{d}\right.}{2d^3}$$

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (-2*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(-(d*Cos[a
 + b*x]^2) + b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sin[2*a - (
2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-ad+bc+d(bx+a))^2d}-\frac{2\sin(2bx+2a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}}{d}+\frac{\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)}{d}}{d}\right.}{4}}{b}$
default	$\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-ad+bc+d(bx+a))^2d}-\frac{2\sin(2bx+2a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}}{d}+\frac{\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)}{d}}{d}\right.}{4}}{b}$
risch	$-\frac{1}{4d(dx+c)^2}+\frac{b^2\mathrm{e}^{-\frac{2i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(2ixb+2ia-\frac{2i(ad-bc)}{d}\right)}{2d^3}+\frac{b^2\mathrm{e}^{\frac{2i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(-2ixb-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^3}+\frac{(-2b^2$

```
[In] int(cos(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*b^3*(-cos(2*b*x+2*a)/(-a*d+b*c+d*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/(
-a*d+b*c+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/
d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)-1/4*b^3/(-
a*d+b*c+d*(b*x+a))^2/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.57

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = -\frac{d^2\cos(bx+a)^2+2\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-2\left(bd^2x+bcd\right)\cos(bx+a$$

$$2\left(d^5x^2+2\,cd^4x+c^2d^3\right)$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*cos(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(
b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - 2*(b*d^2*x + b*c*d)*cos(b*x
 + a)*sin(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c -
a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx$$

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**3,x)
[Out] Integral(cos(a + b*x)**2/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.82

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = -\frac{b^3\left(E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^3\left(i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i}{d}\right.\right.}{4\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)\right.}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
[Out] -1/4*(b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_in
tegral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) +
 b^3*(I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_int
egral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) +
b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 5136, normalized size of antiderivative = 45.86

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
[Out] -1/2*(b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_i
```

```
ntegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x
^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2
- 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)
*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b^2*d^2*x^2*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*real_part(cos
_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b^2*d^2*x^2*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^
2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*
c/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*t
an(a)^2*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d*x*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^2*c*d*x*sin_integr
al(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*d^2*x^2*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^2*c^
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^
2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
) + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^
2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^
2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 2*b
^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c
/d) + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) +
 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan
(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*
tan(a)*tan(b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
a)^2*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*c*d*x*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*real_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos
```

```
_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integr
al(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b^2*c^2*imag_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*si
n_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*
x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b^2*d^2*x^2*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b^2*c*d*x*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^2*c*d*x*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^2*c*d*x*sin_integral(2*
(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b^2*d^2*x^2*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)^2 - b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2
*tan(a)^2 - b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*ta
n(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2
*tan(b*c/d) + 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*
c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*
c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b
*c/d) + 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)*tan(b*c/d) + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)
^2*tan(a)*tan(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^
2*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c
/d)^2 - b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2
- b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2
- b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2
 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2
 - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^
2 + 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b*d
^2*x*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*c^2*real_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b*d^2*x*tan(b*x)*tan(a)^2*tan(b*c/d)^2
 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^2*
c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 2*b^2*d^2*x^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b^2*d^2*x^2*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b^2*d^2*x^2*sin_integral(2*(b*d*x
 + b*c)/d)*tan(a) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(a) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(a) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) -
2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^2*c*d*x
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 + 2*b^2*d^2*x^2*imag_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b^2*d^2*x^2*imag_part(cos_
```

```
integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^2*d^2*x^2*sin_integral(2*(b*d*
x + b*c)/d)*tan(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(b*x)^2*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(b*c/d) + 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x
)^2*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a
)*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)
*tan(b*c/d) - 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*t
an(b*c/d) + 2*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*ta
n(b*c/d) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) -
2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^2*c
*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 2*b^2*c^2*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*b*c*d*tan(b*x)^2*tan(a)*t
an(b*c/d)^2 + 2*b*c*d*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + d^2*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)) + b
^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d)) + b^2*c^2*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b^2*c^2*real_part(cos_integral(-2
*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(a) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
a) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a) + 2*b*d^2*x*tan(b*x
)^2*tan(a) - b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - b^
2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 + 2*b*d^2*x*tan(b*
x)*tan(a)^2 + 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/
d) - 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 8*b
^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 4*b^2*c^2*real_part(c
os_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 4*b^2*c^2*real_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - b^2*c^2*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - b^2*c^2*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(b*c/d)^2 - 2*b*d^2*x*tan(b*x)*tan(b*c/d)^2 - 2*b*d^2*x*tan(a
)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)) + 2*b
^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d)) - 2*b^2*c^2*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b^2*c^2*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(a) - 4*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a) +
2*b*c*d*tan(b*x)^2*tan(a) + 2*b*c*d*tan(b*x)*tan(a)^2 + d^2*tan(b*x)^2*tan(
a)^2 + 2*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b^
2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^2*c^2*sin_
integral(2*(b*d*x + b*c)/d)*tan(b*c/d) - 2*b*c*d*tan(b*x)*tan(b*c/d)^2 - 2*
b*c*d*tan(a)*tan(b*c/d)^2 - 2*d^2*tan(b*x)*tan(a)*tan(b*c/d)^2 + b^2*c^2*re
al_part(cos_integral(2*b*x + 2*b*c/d)) + b^2*c^2*real_part(cos_integral(-2*
b*x - 2*b*c/d)) - 2*b*d^2*x*tan(b*x) - 2*b*d^2*x*tan(a) - 2*b*c*d*tan(b*x)
- 2*b*c*d*tan(a) - 2*d^2*tan(b*x)*tan(a) + d^2*tan(b*c/d)^2 + d^2)/(d^5*x^2
*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2*tan(a)^2*tan(b*c/d
)^2 + d^5*x^2*tan(b*x)^2*tan(a)^2 + d^5*x^2*tan(b*x)^2*tan(b*c/d)^2 + d^5*x
^2*tan(a)^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d
```

```
^4*x*tan(b*x)^2*tan(a)^2 + 2*c*d^4*x*tan(b*x)^2*tan(b*c/d)^2 + 2*c*d^4*x*ta
n(a)^2*tan(b*c/d)^2 + d^5*x^2*tan(b*x)^2 + d^5*x^2*tan(a)^2 + c^2*d^3*tan(b
*x)^2*tan(a)^2 + d^5*x^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(b*c/d)^2 + c
^2*d^3*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2 + 2*c*d^4*x*tan(a)^2 +
2*c*d^4*x*tan(b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(b*x)^2 + c^2*d^3*tan(a)^2 +
c^2*d^3*tan(b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

```
[In] int(cos(a + b*x)^2/(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x)^3, x)
```

3.16 $\int (c+dx)^4 \cos^3(a+bx)\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 225

$$\begin{aligned}\int (c+dx)^4 \cos^3(a+bx)\,dx = &-\frac{160d^3(c+dx)\cos(a+bx)}{9b^4} \\ &+\frac{8d(c+dx)^3\cos(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4} \\ &+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{488d^4\sin(a+bx)}{27b^5} \\ &-\frac{80d^2(c+dx)^2\sin(a+bx)}{9b^3}+\frac{2(c+dx)^4\sin(a+bx)}{3b} \\ &-\frac{4d^2(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{9b^3} \\ &+\frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b}-\frac{8d^4\sin^3(a+bx)}{81b^5}\end{aligned}$$

```
[Out] -160/9*d^3*(d*x+c)*cos(b*x+a)/b^4+8/3*d*(d*x+c)^3*cos(b*x+a)/b^2-8/27*d^3*(
d*x+c)*cos(b*x+a)^3/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)^3/b^2+488/27*d^4*sin(b*x
+a)/b^5-80/9*d^2*(d*x+c)^2*sin(b*x+a)/b^3+2/3*(d*x+c)^4*sin(b*x+a)/b-4/9*d^
2*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b^3+1/3*(d*x+c)^4*cos(b*x+a)^2*sin(b*x+
a)/b-8/81*d^4*sin(b*x+a)^3/b^5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2717, 2713}

$$\begin{aligned}\int (c+dx)^4 \cos^3(a+bx)\,dx = &-\frac{8d^4\sin^3(a+bx)}{81b^5} + \frac{488d^4\sin(a+bx)}{27b^5} \\ &- \frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4} - \frac{160d^3(c+dx)\cos(a+bx)}{9b^4} \\ &- \frac{80d^2(c+dx)^2\sin(a+bx)}{9b^3} \\ &- \frac{4d^2(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{9b^3} \\ &+ \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2} + \frac{8d(c+dx)^3\cos(a+bx)}{3b^2} \\ &+ \frac{2(c+dx)^4\sin(a+bx)}{3b} + \frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}\end{aligned}$$

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3,x]
```

```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])/
(3*b^2) - (8*d^3*(c + d*x)*Cos[a + b*x]^3)/(27*b^4) + (4*d*(c + d*x)^3*Cos[
a + b*x]^3)/(9*b^2) + (488*d^4*Sin[a + b*x])/(27*b^5) - (80*d^2*(c + d*x)^2
*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^4*Sin[a + b*x])/(3*b) - (4*d^2*(c + d
*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^3) + ((c + d*x)^4*Cos[a + b*x]^2*Si
n[a + b*x])/(3*b) - (8*d^4*Sin[a + b*x]^3)/(81*b^5)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
 - Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2} + \frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad + \frac{2}{3}\int (c+dx)^4\cos(a+bx)\,dx - \frac{(4d^2)\int (c+dx)^2\cos^3(a+bx)\,dx}{3b^2} \\
&= -\frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4} + \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2} \\
&\quad + \frac{2(c+dx)^4\sin(a+bx)}{3b} - \frac{4d^2(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{9b^3} \\
&\quad + \frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{(8d)\int (c+dx)^3\sin(a+bx)\,dx}{3b} \\
&\quad - \frac{(8d^2)\int (c+dx)^2\cos(a+bx)\,dx}{9b^2} + \frac{(8d^4)\int \cos^3(a+bx)\,dx}{27b^4} \\
&= \frac{8d(c+dx)^3\cos(a+bx)}{3b^2} - \frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4} \\
&\quad + \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2} - \frac{8d^2(c+dx)^2\sin(a+bx)}{9b^3} \\
&\quad + \frac{2(c+dx)^4\sin(a+bx)}{3b} - \frac{4d^2(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{9b^3} \\
&\quad + \frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{(8d^2)\int (c+dx)^2\cos(a+bx)\,dx}{b^2} \\
&\quad + \frac{(16d^3)\int (c+dx)\sin(a+bx)\,dx}{9b^3} - \frac{(8d^4)\,\text{Subst}\left(\int (1-x^2)\,dx, x, -\sin(a+bx)\right)}{27b^5} \\
&= -\frac{16d^3(c+dx)\cos(a+bx)}{9b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{3b^2} - \frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4} \\
&\quad + \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2} + \frac{8d^4\sin(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\sin(a+bx)}{9b^3} \\
&\quad + \frac{2(c+dx)^4\sin(a+bx)}{3b} - \frac{4d^2(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{9b^3} \\
&\quad + \frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{8d^4\sin^3(a+bx)}{81b^5} \\
&\quad + \frac{(16d^3)\int (c+dx)\sin(a+bx)\,dx}{b^3} + \frac{(16d^4)\int \cos(a+bx)\,dx}{9b^4}
\end{aligned}
$$

$$= -\frac{160d^3(c+dx)\cos(a+bx)}{9b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{3b^2} - \frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}$$
$$+ \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2} + \frac{56d^4\sin(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\sin(a+bx)}{9b^3}$$
$$+ \frac{2(c+dx)^4\sin(a+bx)}{3b} - \frac{4d^2(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{9b^3}$$
$$+ \frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{8d^4\sin^3(a+bx)}{81b^5} + \frac{(16d^4)\int\cos(a+bx)\,dx}{b^4}$$
$$= -\frac{160d^3(c+dx)\cos(a+bx)}{9b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{3b^2} - \frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4}$$
$$+ \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2} + \frac{488d^4\sin(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\sin(a+bx)}{9b^3}$$
$$+ \frac{2(c+dx)^4\sin(a+bx)}{3b} - \frac{4d^2(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{9b^3}$$
$$+ \frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{8d^4\sin^3(a+bx)}{81b^5}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.71

$$\int (c+dx)^4\cos^3(a+bx)\,dx$$
$$= \frac{972bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx) + 12bd(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx)) + 2}{}$$

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3,x]
```

```
[Out] (972*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 12*b*d*(c + d*
x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 243*b^4*c^4*Sin[a + b*x]
 - 2916*b^2*c^2*d^2*Sin[a + b*x] + 5832*d^4*Sin[a + b*x] + 972*b^4*c^3*d*x*
Sin[a + b*x] - 5832*b^2*c*d^3*x*Sin[a + b*x] + 1458*b^4*c^2*d^2*x^2*Sin[a +
 b*x] - 2916*b^2*d^4*x^2*Sin[a + b*x] + 972*b^4*c*d^3*x^3*Sin[a + b*x] + 24
3*b^4*d^4*x^4*Sin[a + b*x] + 27*b^4*c^4*Sin[3*(a + b*x)] - 36*b^2*c^2*d^2*S
in[3*(a + b*x)] + 8*d^4*Sin[3*(a + b*x)] + 108*b^4*c^3*d*x*Sin[3*(a + b*x)]
 - 72*b^2*c*d^3*x*Sin[3*(a + b*x)] + 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)] -
 36*b^2*d^4*x^2*Sin[3*(a + b*x)] + 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] + 27*
b^4*d^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.77

method	result
parallelrisch	$\frac{\left(27(dx+c)^4b^4-36d^2(dx+c)^2b^2+8d^4\right)\sin(3bx+3a)+36b(dx+c)d\left((dx+c)^2b^2-\frac{2d^2}{3}\right)\cos(3bx+3a)+243\left((dx+c)^4b^4-12d^2$
risch	$\frac{3d\left(b^2d^3x^3+3b^2cd^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c\right)\cos(bx+a)}{b^4}+\frac{3\left(d^4x^4b^4+4b^4cd^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((d*x+c)^4*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/324*((27*(d*x+c)^4*b^4-36*d^2*(d*x+c)^2*b^2+8*d^4)*sin(3*b*x+3*a)+36*b*(d
*x+c)*d*((d*x+c)^2*b^2-2/3*d^2)*cos(3*b*x+3*a)+243*((d*x+c)^4*b^4-12*d^2*(d
*x+c)^2*b^2+24*d^4)*sin(b*x+a)+972*b*((d*x+c)*((d*x+c)^2*b^2-6*d^2)*cos(b*x
+a)+28/27*b^2*c^3-488/81*d^2*c)*d)/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.56

$$\int (c+dx)^4\cos^3(a+bx)\,dx$$

$$=\frac{12\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d-2\,bcd^3+(9\,b^3c^2d^2-2\,bd^4)x\right)\cos\left(bx+a\right)^3+72\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+\right.}{}$$

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*b^
3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 + 72*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^
2 + 3*b^3*c^3*d - 20*b*c*d^3 + (9*b^3*c^2*d^2 - 20*b*d^4)*x)*cos(b*x + a) +
 (54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 720*b^2*c^2*d^2 + 1456*
d^4 + 36*(9*b^4*c^2*d^2 - 20*b^2*d^4)*x^2 + (27*b^4*d^4*x^4 + 108*b^4*c*d^3
*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)
*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 72*(3*b^4*c^3*d -
 20*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(226) = 452$.

Time $= 0.73$ (sec) , antiderivative size $= 772$, normalized size of antiderivative $= 3.43$

$$\int (c+dx)^4 \cos^3(a+bx)\,dx$$

$$= \begin{cases} \frac{2c^4 \sin^3(a+bx)}{3b} + \frac{c^4 \sin(a+bx)\cos^2(a+bx)}{b} + \frac{8c^3 dx \sin^3(a+bx)}{3b} + \frac{4c^3 dx \sin(a+bx)\cos^2(a+bx)}{b} + \frac{4c^2 d^2 x^2 \sin^3(a+bx)}{b} + \frac{6c^2 d^2 x^2 \si \\ \left(c^4 x + 2c^3 dx^2 + 2c^2 d^2 x^3 + c d^3 x^4 + \frac{d^4 x^5}{5}\right) \cos^3(a) \end{cases}$$

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c**4*sin(a + b*x)**3/(3*b) + c**4*sin(a + b*x)*cos(a + b*x)**2
/b + 8*c**3*d*x*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)*cos(a + b*x
)**2/b + 4*c**2*d**2*x**2*sin(a + b*x)**3/b + 6*c**2*d**2*x**2*sin(a + b*x)
*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)**3/(3*b) + 4*c*d**3*x**3*si
n(a + b*x)*cos(a + b*x)**2/b + 2*d**4*x**4*sin(a + b*x)**3/(3*b) + d**4*x**
4*sin(a + b*x)*cos(a + b*x)**2/b + 8*c**3*d*sin(a + b*x)**2*cos(a + b*x)/(3
*b**2) + 28*c**3*d*cos(a + b*x)**3/(9*b**2) + 8*c**2*d**2*x*sin(a + b*x)**2
*cos(a + b*x)/b**2 + 28*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 8*c*d**3*x**
2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 28*c*d**3*x**2*cos(a + b*x)**3/(3*b**
2) + 8*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 28*d**4*x**3*cos(a
 + b*x)**3/(9*b**2) - 80*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 28*c**2*d**2*
sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 160*c*d**3*x*sin(a + b*x)**3/(9*b**
3) - 56*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 80*d**4*x**2*sin(a
 + b*x)**3/(9*b**3) - 28*d**4*x**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) -
160*c*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 488*c*d**3*cos(a + b*x)*
*3/(27*b**4) - 160*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 488*d**4*
x*cos(a + b*x)**3/(27*b**4) + 1456*d**4*sin(a + b*x)**3/(81*b**5) + 488*d**
4*sin(a + b*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x*
*2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cos(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(205) = 410$.

Time $= 0.33$ (sec) , antiderivative size $= 925$, normalized size of antiderivative $= 4.11$

$$\int (c+dx)^4 \cos^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/324*(108*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^4 - 432*(sin(b*x + a)^3 - 3
*sin(b*x + a))*a*c^3*d/b + 648*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*c^2*d^
```

```
2/b^2 - 432*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^3*c*d^3/b^3 + 108*(sin(b*x
+ a)^3 - 3*sin(b*x + a))*a^4*d^4/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) + 2
7*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*c^3*d/b + 10
8*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3
*a) + 27*cos(b*x + a))*a*c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) +
27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*a^2*c*d^3/b
^3 + 36*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b
*x + 3*a) + 27*cos(b*x + a))*a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a)
 + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*(
(b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*
a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81
*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x +
3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) +
81*((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*co
s(3*b*x + 3*a) + 243*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*
b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*
c*d^3/b^3 + 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 243*((b*x + a)^2 - 2
)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*
x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*
x - 2*a)*cos(3*b*x + 3*a) + 972*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) +
(27*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*sin(3*b*x + 3*a) + 243*((b*x + a)^4 -
 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.56

$$\int (c+dx)^4 \cos^3(a+bx)\,dx$$

$$= \frac{(3\,b^3d^4x^3 + 9\,b^3cd^3x^2 + 9\,b^3c^2d^2x + 3\,b^3c^3d - 2\,bd^4x - 2\,bcd^3)\cos(3\,bx + 3\,a)}{27\,b^5}$$

$$+ \frac{3\,(b^3d^4x^3 + 3\,b^3cd^3x^2 + 3\,b^3c^2d^2x + b^3c^3d - 6\,bd^4x - 6\,bcd^3)\cos(bx + a)}{b^5}$$

$$+ \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 + 108\,b^4c^3dx + 27\,b^4c^4 - 36\,b^2d^4x^2 - 72\,b^2cd^3x - 36\,b^2c^2d^2 + 8\,d}{324\,b^5}$$

$$+ \frac{3\,(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^4c^3dx + b^4c^4 - 12\,b^2d^4x^2 - 24\,b^2cd^3x - 12\,b^2c^2d^2 + 24\,d^4)\sin(bx}{4\,b^5}$$

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b
*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + 3*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 +
1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c
```

```
^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*
d^4)*sin(3*b*x + 3*a)/b^5 + 3/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*
d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^
2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

Mupad [B] (verification not implemented)

Time = 15.57 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.36

$$
\begin{aligned}
\int (c+dx)^4 \cos^3(a+bx)\,dx = {} & \frac{2\sin(a+bx)^3\,(27\,b^4\,c^4 - 360\,b^2\,c^2\,d^2 + 728\,d^4)}{81\,b^5} \\
& - \frac{4\cos(a+bx)^3\,(122\,c\,d^3 - 21\,b^2\,c^3\,d)}{27\,b^4} \\
& + \frac{\cos(a+bx)^2\,\sin(a+bx)\,(27\,b^4\,c^4 - 252\,b^2\,c^2\,d^2 + 488\,d^4)}{27\,b^5} \\
& - \frac{8\,\cos(a+bx)\,\sin(a+bx)^2\,(20\,c\,d^3 - 3\,b^2\,c^3\,d)}{9\,b^4} \\
& + \frac{28\,d^4\,x^3\cos(a+bx)^3}{9\,b^2} \\
& - \frac{4\,x\cos(a+bx)^3\,(122\,d^4 - 63\,b^2\,c^2\,d^2)}{27\,b^4} \\
& + \frac{2\,d^4\,x^4\sin(a+bx)^3}{3\,b} - \frac{8\,x\sin(a+bx)^3\,(20\,c\,d^3 - 3\,b^2\,c^3\,d)}{9\,b^3} \\
& - \frac{4\,x^2\sin(a+bx)^3\,(20\,d^4 - 9\,b^2\,c^2\,d^2)}{9\,b^3} \\
& - \frac{2\,x^2\cos(a+bx)^2\,\sin(a+bx)\,(14\,d^4 - 9\,b^2\,c^2\,d^2)}{3\,b^3} \\
& + \frac{28\,c\,d^3\,x^2\cos(a+bx)^3}{3\,b^2} + \frac{d^4\,x^4\cos(a+bx)^2\,\sin(a+bx)}{b} \\
& + \frac{8\,d^4\,x^3\,\cos(a+bx)\,\sin(a+bx)^2}{3\,b^2} + \frac{8\,c\,d^3\,x^3\sin(a+bx)^3}{3\,b} \\
& - \frac{8\,x\,\cos(a+bx)\,\sin(a+bx)^2\,(20\,d^4 - 9\,b^2\,c^2\,d^2)}{9\,b^4} \\
& - \frac{4\,x\cos(a+bx)^2\,\sin(a+bx)\,(14\,c\,d^3 - 3\,b^2\,c^3\,d)}{3\,b^3} \\
& + \frac{4\,c\,d^3\,x^3\cos(a+bx)^2\,\sin(a+bx)}{b} \\
& + \frac{8\,c\,d^3\,x^2\,\cos(a+bx)\,\sin(a+bx)^2}{b^2}
\end{aligned}
$$

```
[In] int(cos(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] (2*sin(a + b*x)^3*(728*d^4 + 27*b^4*c^4 - 360*b^2*c^2*d^2))/(81*b^5) - (4*c
os(a + b*x)^3*(122*c*d^3 - 21*b^2*c^3*d))/(27*b^4) + (cos(a + b*x)^2*sin(a
```

```
+ b*x)*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(27*b^5) - (8*cos(a + b*x)
*sin(a + b*x)^2*(20*c*d^3 - 3*b^2*c^3*d))/(9*b^4) + (28*d^4*x^3*cos(a + b*x
)^3)/(9*b^2) - (4*x*cos(a + b*x)^3*(122*d^4 - 63*b^2*c^2*d^2))/(27*b^4) + (
2*d^4*x^4*sin(a + b*x)^3)/(3*b) - (8*x*sin(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3
*d))/(9*b^3) - (4*x^2*sin(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(9*b^3) - (2
*x^2*cos(a + b*x)^2*sin(a + b*x)*(14*d^4 - 9*b^2*c^2*d^2))/(3*b^3) + (28*c*
d^3*x^2*cos(a + b*x)^3)/(3*b^2) + (d^4*x^4*cos(a + b*x)^2*sin(a + b*x))/b +
 (8*d^4*x^3*cos(a + b*x)*sin(a + b*x)^2)/(3*b^2) + (8*c*d^3*x^3*sin(a + b*x
)^3)/(3*b) - (8*x*cos(a + b*x)*sin(a + b*x)^2*(20*d^4 - 9*b^2*c^2*d^2))/(9*
b^4) - (4*x*cos(a + b*x)^2*sin(a + b*x)*(14*c*d^3 - 3*b^2*c^3*d))/(3*b^3) +
 (4*c*d^3*x^3*cos(a + b*x)^2*sin(a + b*x))/b + (8*c*d^3*x^2*cos(a + b*x)*si
n(a + b*x)^2)/b^2
```

3.17 $\int (c+dx)^3 \cos^3(a+bx)\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 175

$$\begin{aligned}\int (c+dx)^3 \cos^3(a+bx)\,dx = &-\frac{40d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{b^2} \\ &- \frac{2d^3\cos^3(a+bx)}{27b^4} + \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2} \\ &- \frac{40d^2(c+dx)\sin(a+bx)}{9b^3} + \frac{2(c+dx)^3\sin(a+bx)}{3b} \\ &- \frac{2d^2(c+dx)\cos^2(a+bx)\sin(a+bx)}{9b^3} \\ &+ \frac{(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{3b}\end{aligned}$$

```
[Out] -40/9*d^3*cos(b*x+a)/b^4+2*d*(d*x+c)^2*cos(b*x+a)/b^2-2/27*d^3*cos(b*x+a)^3
/b^4+1/3*d*(d*x+c)^2*cos(b*x+a)^3/b^2-40/9*d^2*(d*x+c)*sin(b*x+a)/b^3+2/3*(
d*x+c)^3*sin(b*x+a)/b-2/9*d^2*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^3+1/3*(d*x+
c)^3*cos(b*x+a)^2*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{3392, 3377, 2718, 3391\}$

$$\int (c+dx)^3 \cos^3(a+bx)\,dx = -\frac{2d^3\cos^3(a+bx)}{27b^4} - \frac{40d^3\cos(a+bx)}{9b^4} - \frac{40d^2(c+dx)\sin(a+bx)}{9b^3} - \frac{2d^2(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^3} + \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2} + \frac{2d(c+dx)^2\cos(a+bx)}{b^2} + \frac{2(c+dx)^3\sin(a+bx)}{3b} + \frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3,x]
```

```
[Out] (-40*d^3*Cos[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Cos[a + b*x])/b^2 - (2*d^
3*Cos[a + b*x]^3)/(27*b^4) + (d*(c + d*x)^2*Cos[a + b*x]^3)/(3*b^2) - (40*d
^2*(c + d*x)*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^3*Sin[a + b*x])/(3*b) - (
2*d^2*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^3) + ((c + d*x)^3*Cos[a +
 b*x]^2*Sin[a + b*x])/(3*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2} + \frac{(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad + \frac{2}{3}\int (c+dx)^3\cos(a+bx)\,dx - \frac{(2d^2)\int (c+dx)\cos^3(a+bx)\,dx}{3b^2} \\
&= -\frac{2d^3\cos^3(a+bx)}{27b^4} + \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2} + \frac{2(c+dx)^3\sin(a+bx)}{3b} \\
&\quad - \frac{2d^2(c+dx)\cos^2(a+bx)\sin(a+bx)}{9b^3} + \frac{(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad - \frac{(2d)\int (c+dx)^2\sin(a+bx)\,dx}{b} - \frac{(4d^2)\int (c+dx)\cos(a+bx)\,dx}{9b^2} \\
&= \frac{2d(c+dx)^2\cos(a+bx)}{b^2} - \frac{2d^3\cos^3(a+bx)}{27b^4} + \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2} \\
&\quad - \frac{4d^2(c+dx)\sin(a+bx)}{9b^3} + \frac{2(c+dx)^3\sin(a+bx)}{3b} \\
&\quad - \frac{2d^2(c+dx)\cos^2(a+bx)\sin(a+bx)}{9b^3} + \frac{(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad - \frac{(4d^2)\int (c+dx)\cos(a+bx)\,dx}{b^2} + \frac{(4d^3)\int \sin(a+bx)\,dx}{9b^3} \\
&= -\frac{4d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{b^2} - \frac{2d^3\cos^3(a+bx)}{27b^4} \\
&\quad + \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2} - \frac{40d^2(c+dx)\sin(a+bx)}{9b^3} \\
&\quad + \frac{2(c+dx)^3\sin(a+bx)}{3b} - \frac{2d^2(c+dx)\cos^2(a+bx)\sin(a+bx)}{9b^3} \\
&\quad + \frac{(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{(4d^3)\int \sin(a+bx)\,dx}{b^3} \\
&= -\frac{40d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{b^2} - \frac{2d^3\cos^3(a+bx)}{27b^4} \\
&\quad + \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2} - \frac{40d^2(c+dx)\sin(a+bx)}{9b^3} + \frac{2(c+dx)^3\sin(a+bx)}{3b} \\
&\quad - \frac{2d^2(c+dx)\cos^2(a+bx)\sin(a+bx)}{9b^3} + \frac{(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.69

$$\int (c+dx)^3 \cos^3(a+bx)\,dx$$
$$= \frac{243d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+d(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+6b(c+dx)\left(-82d^2+15b\right.}{108b^4}$$

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3,x]
```

```
[Out] (243*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*(c + d*x
)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d
^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^4)
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.28

method	result
risch	$\frac{9d\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\cos(bx+a)}{4b^4}+\frac{3\left(b^2d^3x^3+3b^2cd^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c\right)\sin(bx+a)}{4b^3}+\frac{d\left(9x^2d^2\right.}{}$
parallelrisch	$\frac{-126b^2d^2\left(\frac{dx}{2}+c\right)x\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+54b(dx+c)\left((dx+c)^2b^2-\frac{14d^2}{3}\right)\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\left(\left(-27d^3x^2-54cd^2x+162c^2d\right)b^2-\right.}{}$
derivativedivides	$-\frac{a^3d^3\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b^3}+\frac{a^2cd^2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{b^2}+\frac{3a^2d^3\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}+\right.}{b^3}$
default	$-\frac{a^3d^3\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b^3}+\frac{a^2cd^2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{b^2}+\frac{3a^2d^3\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}+\right.}{b^3}$
norman	$\frac{d^3x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{126b^2c^2d-244d^3}{27b^4}+\frac{7d^3x^2}{3b^2}+\frac{\left(18b^2c^2d-28d^3\right)\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^4}+\frac{\left(72b^2c^2d-160d^3\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{9b^4}+\frac{14cd^2x}{3b^2}$

```
[In] int((d*x+c)^3*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 9/4*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^4*cos(b*x+a)+3/4/b^3*(b^2*d
^3*x^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^2)*sin(b*x+a)+1/
108*d*(9*b^2*d^2*x^2+18*b^2*c*d*x+9*b^2*c^2-2*d^2)/b^4*cos(3*b*x+3*a)+1/36/
b^3*(3*b^2*d^3*x^3+9*b^2*c*d^2*x^2+9*b^2*c^2*d*x+3*b^2*c^3-2*d^3*x-2*c*d^2)
*sin(3*b*x+3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int (c+dx)^3 \cos^3(a+bx)\,dx$$
$$= \frac{(9\,b^2d^3x^2 + 18\,b^2cd^2x + 9\,b^2c^2d - 2\,d^3)\cos(bx+a)^3 + 6\,(9\,b^2d^3x^2 + 18\,b^2cd^2x + 9\,b^2c^2d - 20\,d^3)\cos(bx+$$

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^3
 + 6*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 20*d^3)*cos(b*x + a) +
 3*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - 40*b*c*d^2 + (3*b^3*d^3*
x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)*x)*
cos(b*x + a)^2 + 2*(9*b^3*c^2*d - 20*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(173) = 346.

Time = 0.49 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.83

$$\int (c+dx)^3 \cos^3(a+bx)\,dx$$
$$= \begin{cases} \frac{2c^3\sin^3(a+bx)}{3b} + \frac{c^3\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2c^2dx\sin^3(a+bx)}{b} + \frac{3c^2dx\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2cd^2x^2\sin^3(a+bx)}{b} + \frac{3cd^2x^2\sin($ \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cos^3(a) \end{cases}$$

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c**3*sin(a + b*x)**3/(3*b) + c**3*sin(a + b*x)*cos(a + b*x)**2
/b + 2*c**2*d*x*sin(a + b*x)**3/b + 3*c**2*d*x*sin(a + b*x)*cos(a + b*x)**2
/b + 2*c*d**2*x**2*sin(a + b*x)**3/b + 3*c*d**2*x**2*sin(a + b*x)*cos(a + b
*x)**2/b + 2*d**3*x**3*sin(a + b*x)**3/(3*b) + d**3*x**3*sin(a + b*x)*cos(a
 + b*x)**2/b + 2*c**2*d*sin(a + b*x)**2*cos(a + b*x)/b**2 + 7*c**2*d*cos(a
+ b*x)**3/(3*b**2) + 4*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)/b**2 + 14*c*d*
*2*x*cos(a + b*x)**3/(3*b**2) + 2*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b*
*2 + 7*d**3*x**2*cos(a + b*x)**3/(3*b**2) - 40*c*d**2*sin(a + b*x)**3/(9*b*
*3) - 14*c*d**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 40*d**3*x*sin(a + b
*x)**3/(9*b**3) - 14*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 40*d**3
*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 122*d**3*cos(a + b*x)**3/(27*b**4)
, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a)
**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(161) = 322$.

Time = 0.28 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.06

$$\int (c+dx)^3 \cos^3(a+bx)\,dx =$$

$$-\frac{36\left(\sin(bx+a)^3 - 3\sin(bx+a)\right)c^3 - \frac{108\left(\sin(bx+a)^3-3\sin(bx+a)\right)ac^2d}{b} + \frac{108\left(\sin(bx+a)^3-3\sin(bx+a)\right)a^2cd^2}{b^2} - \frac{36\left(si}{}$$

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(36*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^3 - 108*(sin(b*x + a)^3 - 3*
sin(b*x + a))*a*c^2*d/b + 108*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*c*d^2/b
^2 - 36*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^3*d^3/b^3 - 9*(3*(b*x + a)*sin(
3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x +
a))*c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) + 27*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*sin(3*b*
x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*
a^2*d^3/b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a)
+ (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))
*c*d^2/b^2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) +
 (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*
a*d^3/b^3 - ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 243*((b*x + a)^2 - 2)*c
os(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*x +
 a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.32

$$\begin{aligned}
&\int (c+dx)^3 \cos^3(a+bx)\,dx \\
&= \frac{\left(9\,b^2d^3x^2 + 18\,b^2cd^2x + 9\,b^2c^2d - 2\,d^3\right)\cos\left(3\,bx+3\,a\right)}{108\,b^4} \\
&\quad + \frac{9\left(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3\right)\cos\left(bx+a\right)}{4\,b^4} \\
&\quad + \frac{\left(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + 9\,b^3c^2dx + 3\,b^3c^3 - 2\,bd^3x - 2\,bcd^2\right)\sin\left(3\,bx+3\,a\right)}{36\,b^4} \\
&\quad + \frac{3\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 6\,bd^3x - 6\,bcd^2\right)\sin\left(bx+a\right)}{4\,b^4}
\end{aligned}$$

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3*
a)/b^4 + 9/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)
/b^4 + 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 -
2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 3/4*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.08

$$\begin{aligned}\int (c+dx)^3\cos^3(a+bx)\,dx = {} & \frac{7\,d^3\,x^2\cos\left(a+b\,x\right)^3}{3\,b^2} - \frac{2\sin\left(a+b\,x\right)^3\left(20\,c\,d^2-3\,b^2\,c^3\right)}{9\,b^3} \\ & - \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(14\,c\,d^2-3\,b^2\,c^3\right)}{3\,b^3} \\ & - \frac{2\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\left(20\,d^3-9\,b^2\,c^2\,d\right)}{9\,b^4} \\ & - \frac{2\,x\sin\left(a+b\,x\right)^3\left(20\,d^3-9\,b^2\,c^2\,d\right)}{9\,b^3} \\ & - \frac{\cos\left(a+b\,x\right)^3\left(122\,d^3-63\,b^2\,c^2\,d\right)}{27\,b^4} \\ & + \frac{2\,d^3\,x^3\sin\left(a+b\,x\right)^3}{3\,b} + \frac{14\,c\,d^2\,x\cos\left(a+b\,x\right)^3}{3\,b^2} \\ & - \frac{x\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(14\,d^3-9\,b^2\,c^2\,d\right)}{3\,b^3} \\ & + \frac{d^3\,x^3\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{b} \\ & + \frac{2\,d^3\,x^2\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{b^2} + \frac{2\,c\,d^2\,x^2\sin\left(a+b\,x\right)^3}{b} \\ & + \frac{3\,c\,d^2\,x^2\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{b} \\ & + \frac{4\,c\,d^2\,x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{b^2}\end{aligned}$$

```
[In] int(cos(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] (7*d^3*x^2*cos(a + b*x)^3)/(3*b^2) - (2*sin(a + b*x)^3*(20*c*d^2 - 3*b^2*c^
3))/(9*b^3) - (cos(a + b*x)^2*sin(a + b*x)*(14*c*d^2 - 3*b^2*c^3))/(3*b^3)
- (2*cos(a + b*x)*sin(a + b*x)^2*(20*d^3 - 9*b^2*c^2*d))/(9*b^4) - (2*x*sin
(a + b*x)^3*(20*d^3 - 9*b^2*c^2*d))/(9*b^3) - (cos(a + b*x)^3*(122*d^3 - 63
*b^2*c^2*d))/(27*b^4) + (2*d^3*x^3*sin(a + b*x)^3)/(3*b) + (14*c*d^2*x*cos(
a + b*x)^3)/(3*b^2) - (x*cos(a + b*x)^2*sin(a + b*x)*(14*d^3 - 9*b^2*c^2*d)
)/(3*b^3) + (d^3*x^3*cos(a + b*x)^2*sin(a + b*x))/b + (2*d^3*x^2*cos(a + b*
x)*sin(a + b*x)^2)/b^2 + (2*c*d^2*x^2*sin(a + b*x)^3)/b + (3*c*d^2*x^2*cos(
a + b*x)^2*sin(a + b*x))/b + (4*c*d^2*x*cos(a + b*x)*sin(a + b*x)^2)/b^2
```

3.18 $\int (c+dx)^2 \cos^3(a+bx)\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 123

$$\begin{aligned}\int (c+dx)^2 \cos^3(a+bx)\,dx = &\frac{4d(c+dx)\cos(a+bx)}{3b^2} + \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} \\ &- \frac{14d^2\sin(a+bx)}{9b^3} + \frac{2(c+dx)^2\sin(a+bx)}{3b} \\ &+ \frac{(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2d^2\sin^3(a+bx)}{27b^3}\end{aligned}$$

```
[Out] 4/3*d*(d*x+c)*cos(b*x+a)/b^2+2/9*d*(d*x+c)*cos(b*x+a)^3/b^2-14/9*d^2*sin(b*
x+a)/b^3+2/3*(d*x+c)^2*sin(b*x+a)/b+1/3*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b
+2/27*d^2*sin(b*x+a)^3/b^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2717, 2713}

$$\begin{aligned}\int (c+dx)^2 \cos^3(a+bx)\,dx = &\frac{2d^2\sin^3(a+bx)}{27b^3} - \frac{14d^2\sin(a+bx)}{9b^3} + \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} \\ &+ \frac{4d(c+dx)\cos(a+bx)}{3b^2} + \frac{2(c+dx)^2\sin(a+bx)}{3b} \\ &+ \frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\end{aligned}$$

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3,x]
```

```
[Out] (4*d*(c + d*x)*Cos[a + b*x])/(3*b^2) + (2*d*(c + d*x)*Cos[a + b*x]^3)/(9*b^
2) - (14*d^2*Sin[a + b*x])/(9*b^3) + (2*(c + d*x)^2*Sin[a + b*x])/(3*b) + (
```

```
(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2*d^2*Sin[a + b*x]^3)/(27
*b^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} + \frac{(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad + \frac{2}{3}\int (c+dx)^2\cos(a+bx)\,dx - \frac{(2d^2)\int \cos^3(a+bx)\,dx}{9b^2} \\
&= \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} + \frac{2(c+dx)^2\sin(a+bx)}{3b} + \frac{(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad - \frac{(4d)\int (c+dx)\sin(a+bx)\,dx}{3b} + \frac{(2d^2)\,\text{Subst}\left(\int (1-x^2)\,dx, x, -\sin(a+bx)\right)}{9b^3} \\
&= \frac{4d(c+dx)\cos(a+bx)}{3b^2} + \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} - \frac{2d^2\sin(a+bx)}{9b^3} + \frac{2(c+dx)^2\sin(a+bx)}{3b} \\
&\quad + \frac{(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2d^2\sin^3(a+bx)}{27b^3} - \frac{(4d^2)\int \cos(a+bx)\,dx}{3b^2} \\
&= \frac{4d(c+dx)\cos(a+bx)}{3b^2} + \frac{2d(c+dx)\cos^3(a+bx)}{9b^2} - \frac{14d^2\sin(a+bx)}{9b^3} \\
&\quad + \frac{2(c+dx)^2\sin(a+bx)}{3b} + \frac{(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2d^2\sin^3(a+bx)}{27b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int (c+dx)^2 \cos^3(a+bx)\,dx$$
$$= \frac{162bd(c+dx)\cos(a+bx) + 6bd(c+dx)\cos(3(a+bx)) + 2(-82d^2 + 45b^2(c+dx)^2 + (-2d^2 + 9b^2(c+dx)$$

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3,x]
```

```
[Out] (162*b*d*(c + d*x)*Cos[a + b*x] + 6*b*d*(c + d*x)*Cos[3*(a + b*x)] + 2*(-82
*d^2 + 45*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*
Sin[a + b*x])/(108*b^3)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

method	result
risch	$\frac{3d(dx+c)\cos(bx+a)}{2b^2} + \frac{3\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\sin(bx+a)}{4b^3} + \frac{d(dx+c)\cos(3bx+3a)}{18b^2} + \frac{(9x^2d^2b^2+18b^2cdx+9b^2}{108}$
parallelrisch	$\frac{-42d^2\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)xb+\left(54(dx+c)^2b^2-84d^2\right)\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+108b\left(-\frac{dx}{6}+c\right)d\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\left(36(dx+c)^2b^2-15}{27b^3\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$
derivativedivides	$\frac{a^2d^2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b^2} - \frac{2acd\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b} - \frac{2a\,d^2\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}+\frac{2\cos}{b^2}\right.$
default	$\frac{a^2d^2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b^2} - \frac{2acd\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b} - \frac{2a\,d^2\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}+\frac{2\cos}{b^2}\right.$
norman	$\frac{4cd\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{28cd}{9b^2}+\frac{14d^2x}{9b^2}+\frac{4\left(9b^2c^2-38d^2\right)\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{27b^3}+\frac{2\left(9b^2c^2-14d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{9b^3}+\frac{2\left(9b^2c^2-14d^2\right)\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{9b^3}$

```
[In] int((d*x+c)^2*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*d*(d*x+c)*cos(b*x+a)/b^2+3/4*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^
3*sin(b*x+a)+1/18/b^2*d*(d*x+c)*cos(3*b*x+3*a)+1/108*(9*b^2*d^2*x^2+18*b^2*
c*d*x+9*b^2*c^2-2*d^2)/b^3*sin(3*b*x+3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int (c+dx)^2 \cos^3(a+bx)\,dx$$
$$= \frac{6\,(bd^2x+bcd)\cos(bx+a)^3 + 36\,(bd^2x+bcd)\cos(bx+a) + \left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2+(9\,b^2d^2x^2+\right.}{27\,b^3}$$

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 + 36*(b*d^2*x + b*c*d)*cos(b*x + a
) + (18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 + (9*b^2*d^2*x^2 + 18*b^2*c
*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 40*d^2)*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.

Time = 0.38 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.31

$$\int (c+dx)^2 \cos^3(a+bx)\,dx$$
$$= \begin{cases} \frac{2c^2\sin^3(a+bx)}{3b} + \frac{c^2\sin(a+bx)\cos^2(a+bx)}{b} + \frac{4cdx\sin^3(a+bx)}{3b} + \frac{2cdx\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2d^2x^2\sin^3(a+bx)}{3b} + \frac{d^2x^2\sin(a+bx}{b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cos^3(a) \end{cases}$$

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c**2*sin(a + b*x)**3/(3*b) + c**2*sin(a + b*x)*cos(a + b*x)**2
/b + 4*c*d*x*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)*cos(a + b*x)**2/b
 + 2*d**2*x**2*sin(a + b*x)**3/(3*b) + d**2*x**2*sin(a + b*x)*cos(a + b*x)*
*2/b + 4*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 14*c*d*cos(a + b*x)**3
/(9*b**2) + 4*d**2*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 14*d**2*x*cos(
a + b*x)**3/(9*b**2) - 40*d**2*sin(a + b*x)**3/(27*b**3) - 14*d**2*sin(a +
b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3
)*cos(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(111) = 222$.

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.17

$$\int (c+dx)^2 \cos^3(a+bx)\,dx =$$

$$-\frac{36\left(\sin(bx+a)^3 - 3\sin(bx+a)\right)c^2 - \frac{72\left(\sin(bx+a)^3-3\sin(bx+a)\right)acd}{b} + \frac{36\left(\sin(bx+a)^3-3\sin(bx+a)\right)a^2d^2}{b^2} - \frac{6\,(3\,(bx+a}$$

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(36*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^2 - 72*(sin(b*x + a)^3 - 3*s
in(b*x + a))*a*c*d/b + 36*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*d^2/b^2 - 6
*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*
a) + 27*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a
)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*a*d^2/b^2 - (6*(b*x +
a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(
3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\begin{aligned}\int (c+dx)^2 \cos^3(a+bx)\,dx = {} & \frac{(bd^2x+bcd)\cos(3\,bx+3\,a)}{18\,b^3} + \frac{3\,(bd^2x+bcd)\cos(bx+a)}{2\,b^3} \\ & + \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\sin(3\,bx+3\,a)}{108\,b^3} \\ & + \frac{3\,(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin(bx+a)}{4\,b^3}\end{aligned}$$

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 3/2*(b*d^2*x + b*c*d)*cos(b*x
 + a)/b^3 + 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*
b*x + 3*a)/b^3 + 3/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x
+ a)/b^3
```

Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.41

$$\int (c+dx)^2\cos^3(a+bx)\,dx = \frac{\frac{d^2\,x\,\cos(3\,a+3\,b\,x)}{18}+\frac{3\,c\,d\,\cos(a+b\,x)}{2}+\frac{c\,d\,\cos(3\,a+3\,b\,x)}{18}+\frac{3\,d^2\,x\,\cos(a+b\,x)}{2}}{b^2}$$
$$+\frac{\frac{3\,c^2\,\sin(a+b\,x)}{4}+\frac{c^2\,\sin(3\,a+3\,b\,x)}{12}+\frac{3\,d^2\,x^2\,\sin(a+b\,x)}{4}+\frac{d^2\,x^2\,\sin(3\,a+3\,b\,x)}{12}+\frac{3\,c\,d\,x\,\sin(a+b\,x)}{2}+\frac{c\,d\,x\,\sin(3\,a+3\,b\,x)}{6}}{b}$$
$$-\frac{3\,d^2\,\sin\left(a+b\,x\right)}{2\,b^3}-\frac{d^2\,\sin\left(3\,a+3\,b\,x\right)}{54\,b^3}$$

```
[In] int(cos(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] ((d^2*x*cos(3*a + 3*b*x))/18 + (3*c*d*cos(a + b*x))/2 + (c*d*cos(3*a + 3*b*
x))/18 + (3*d^2*x*cos(a + b*x))/2)/b^2 + ((3*c^2*sin(a + b*x))/4 + (c^2*sin
(3*a + 3*b*x))/12 + (3*d^2*x^2*sin(a + b*x))/4 + (d^2*x^2*sin(3*a + 3*b*x))
/12 + (3*c*d*x*sin(a + b*x))/2 + (c*d*x*sin(3*a + 3*b*x))/6)/b - (3*d^2*sin
(a + b*x))/(2*b^3) - (d^2*sin(3*a + 3*b*x))/(54*b^3)
```

3.19 $\int (c+dx)\cos^3(a+bx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 75

$$\int (c+dx)\cos^3(a+bx)\,dx = \frac{2d\cos(a+bx)}{3b^2} + \frac{d\cos^3(a+bx)}{9b^2} + \frac{2(c+dx)\sin(a+bx)}{3b} + \frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b}$$

```
[Out] 2/3*d*cos(b*x+a)/b^2+1/9*d*cos(b*x+a)^3/b^2+2/3*(d*x+c)*sin(b*x+a)/b+1/3*(d
*x+c)*cos(b*x+a)^2*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3391, 3377, 2718}

$$\int (c+dx)\cos^3(a+bx)\,dx = \frac{d\cos^3(a+bx)}{9b^2} + \frac{2d\cos(a+bx)}{3b^2} + \frac{2(c+dx)\sin(a+bx)}{3b} + \frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

```
[In] Int[(c + d*x)*Cos[a + b*x]^3,x]
```

```
[Out] (2*d*Cos[a + b*x])/(3*b^2) + (d*Cos[a + b*x]^3)/(9*b^2) + (2*(c + d*x)*Sin[
a + b*x])/(3*b) + ((c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\cos^3(a+bx)}{9b^2} + \frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b} + \frac{2}{3}\int (c+dx)\cos(a+bx)\,dx \\ &= \frac{d\cos^3(a+bx)}{9b^2} + \frac{2(c+dx)\sin(a+bx)}{3b} \\ &\quad + \frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{(2d)\int \sin(a+bx)\,dx}{3b} \\ &= \frac{2d\cos(a+bx)}{3b^2} + \frac{d\cos^3(a+bx)}{9b^2} + \frac{2(c+dx)\sin(a+bx)}{3b} + \frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.23$ (sec) , antiderivative size $= 52$, normalized size of antiderivative $= 0.69$

$$\begin{aligned}&\int (c+dx)\cos^3(a+bx)\,dx \\ &= \frac{27d\cos(a+bx) + d\cos(3(a+bx)) + 3b(c+dx)(9\sin(a+bx) + \sin(3(a+bx)))}{36b^2}\end{aligned}$$

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3,x]
```

```
[Out] (27*d*Cos[a + b*x] + d*Cos[3*(a + b*x)] + 3*b*(c + d*x)*(9*Sin[a + b*x] + S
in[3*(a + b*x)]))/(36*b^2)
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{3b(dx+c)\sin(3bx+3a)+d\cos(3bx+3a)+27(dx+c)b\sin(bx+a)+27d\cos(bx+a)+28d}{36b^2}$
risch	$\frac{3d\cos(bx+a)}{4b^2}+\frac{3(dx+c)\sin(bx+a)}{4b}+\frac{d\cos(3bx+3a)}{36b^2}+\frac{(dx+c)\sin(3bx+3a)}{12b}$
derivativedivides	$-\frac{da\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b}+\frac{c\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{d\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}+\frac{2\cos(bx+a)}{3}\right.}{b}$
default	$-\frac{da\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b}+\frac{c\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{d\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}+\frac{2\cos(bx+a)}{3}\right.}{b}$
norman	$\frac{\frac{2d\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{14d}{9b^2}+\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4c\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{2c\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{8d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^2}+\frac{2dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4dx\left(\right.}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$

```
[In] int((d*x+c)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(3*b*(d*x+c)*sin(3*b*x+3*a)+d*cos(3*b*x+3*a)+27*(d*x+c)*b*sin(b*x+a)+2
7*d*cos(b*x+a)+28*d)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int (c+dx)\cos^3(a+bx)\,dx$$
$$=\frac{d\cos(bx+a)^3+6\,d\cos(bx+a)+3\left(2\,bdx+(bdx+bc)\cos(bx+a)^2+2\,bc\right)\sin(bx+a)}{9\,b^2}$$

```
[In] integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(d*cos(b*x + a)^3 + 6*d*cos(b*x + a) + 3*(2*b*d*x + (b*d*x + b*c)*cos(b
*x + a)^2 + 2*b*c)*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.68

$$\int (c+dx)\cos^3(a+bx)\,dx$$

$$= \begin{cases} \frac{2c\sin^3(a+bx)}{3b} + \frac{c\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2dx\sin^3(a+bx)}{3b} + \frac{dx\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2d\sin^2(a+bx)\cos(a+bx)}{3b^2} + \frac{7d\cos^3(a+b} \\ \left(cx + \frac{dx^2}{2}\right)\cos^3(a) \end{cases}$$

```
[In] integrate((d*x+c)*cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*c*sin(a + b*x)**3/(3*b) + c*sin(a + b*x)*cos(a + b*x)**2/b + 2
*d*x*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)*cos(a + b*x)**2/b + 2*d*sin(a
 + b*x)**2*cos(a + b*x)/(3*b**2) + 7*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)),
 ((c*x + d*x**2/2)*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

$$\int (c+dx)\cos^3(a+bx)\,dx =$$

$$-\frac{12\left(\sin(bx+a)^3 - 3\sin(bx+a)\right)c - \frac{12\left(\sin(bx+a)^3 - 3\sin(bx+a)\right)ad}{b} - \frac{(3(bx+a)\sin(3bx+3a) + 27(bx+a)\sin(bx+a) + \cos}{b}}{36\,b}$$

```
[In] integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*(12*(sin(b*x + a)^3 - 3*sin(b*x + a))*c - 12*(sin(b*x + a)^3 - 3*sin(
b*x + a))*a*d/b - (3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a)
 + cos(3*b*x + 3*a) + 27*cos(b*x + a))*d/b)/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int (c+dx)\cos^3(a+bx)\,dx = \frac{d\cos(3\,bx+3\,a)}{36\,b^2} + \frac{3\,d\cos(bx+a)}{4\,b^2} + \frac{(bdx+bc)\sin(3\,bx+3\,a)}{12\,b^2} + \frac{3\,(bdx+bc)\sin(bx+a)}{4\,b^2}$$

```
[In] integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/36*d*cos(3*b*x + 3*a)/b^2 + 3/4*d*cos(b*x + a)/b^2 + 1/12*(b*d*x + b*c)*s
in(3*b*x + 3*a)/b^2 + 3/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int (c+dx)\cos^3(a+bx)\,dx = \frac{\frac{3c\sin(a+bx)}{4} + \frac{c\sin(3a+3bx)}{12} + \frac{dx\sin(3a+3bx)}{12} + \frac{3dx\sin(a+bx)}{4}}{b} + \frac{d\cos(3a+3bx)}{36b^2} + \frac{3d\cos(a+bx)}{4b^2}$$

```
[In] int(cos(a + b*x)^3*(c + d*x),x)
```

```
[Out] ((3*c*sin(a + b*x))/4 + (c*sin(3*a + 3*b*x))/12 + (d*x*sin(3*a + 3*b*x))/12
 + (3*d*x*sin(a + b*x))/4)/b + (d*cos(3*a + 3*b*x))/(36*b^2) + (3*d*cos(a +
 b*x))/(4*b^2)
```

3.20 $\int \frac{\cos^3(a+bx)}{c+dx}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx = \frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

```
[Out] 1/4*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d+3/4*Ci(b*c/d+b*x)*cos(a-b*c/d)/d-1
/4*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-3/4*Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3384, 3380, 3383}

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx = \frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

```
[In] Int[Cos[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/
d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (3*Sin[a - (b*c)/d]*SinIntegral[
```

```
(b*c)/d + b*x])/(4*d) - (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x
])/(4*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3\cos(a+bx)}{4(c+dx)} + \frac{\cos(3a+3bx)}{4(c+dx)} \right) dx \\
&= \frac{1}{4}\int \frac{\cos(3a+3bx)}{c+dx}\,dx + \frac{3}{4}\int \frac{\cos(a+bx)}{c+dx}\,dx \\
&= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx + \frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx \\
&\quad -\frac{1}{4}\sin\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx - \frac{1}{4}\left(3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx \\
&= \frac{3\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d} \\
&\quad -\frac{3\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx$$
$$= \frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right.}{4d}$$

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/d]*CosInte
gral[(3*b*(c + d*x))/d] - 3*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - Sin
[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{b\left(-\frac{3\,\text{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{3\,\text{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{12}+\frac{3b\left(-\frac{\text{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)}{d}\right.}{b}$
default	$\frac{b\left(-\frac{3\,\text{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{3\,\text{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{12}+\frac{3b\left(-\frac{\text{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)}{d}\right.}{b}$
risch	$-\frac{e^{-\frac{3i(ad-bc)}{d}}\,\text{Ei}_1\left(3ixb+3ia-\frac{3i(ad-bc)}{d}\right)}{8d}-\frac{3\,e^{-\frac{i(ad-bc)}{d}}\,\text{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{8d}-\frac{3\,e^{\frac{i(ad-bc)}{d}}\,\text{Ei}_1\left(-ixb-ia-\frac{-i}{8d}\right.}{8d}$

```
[In] int(cos(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*
b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)+3/4*b*(-Si(-b*x-a-(-a*d+b*c)
/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx$$
$$= \frac{\cos\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+3\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)-\sin\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-3\sin\left(-\frac{bc-a}{d}\right.}{4\,d}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d) + 3*cos(-(b*c -
a*d)/d)*cos_integral((b*d*x + b*c)/d) - sin(-3*(b*c - a*d)/d)*sin_integral(
3*(b*d*x + b*c)/d) - 3*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx = \int \frac{\cos^3(a+bx)}{c+dx}\,dx$$

```
[In] integrate(cos(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.30

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx =$$
$$-\frac{3\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\right.\right.}{}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integ
ral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(exp_
integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b*(-I*exp_in
tegral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -(I*b*
c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b*(-I*exp_integral_e(1
, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -3*(-I*b*c -
I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 6075, normalized size of antiderivative = 50.21

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part(cos_integral(b*x + b*c/
d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*imag_part
(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1
/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*imag_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*
a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(-b*x
 - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d) + 12*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1
/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - real_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos
_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/
d)^2 + 4*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x - 3*b*c/d)
```

```
)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + real_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
3*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3
*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x +
b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 6*imag_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*imag_part(cos_i
ntegral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*sin_in
tegral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2
 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(
3/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 - 6*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^
2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*imag_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*sin_integral((b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*imag_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a
)*tan(1/2*b*c/d)^2 - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*t
an(1/2*a)*tan(1/2*b*c/d)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*
x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*ima
g_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)*ta
n(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b
*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag
```

```
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + 6*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*
a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan
(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 3*real_part(cos_integral(b*x + b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2 + 4*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan
(1/2*a)^2*tan(3/2*b*c/d) + 4*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d))*t
an(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*real_part(cos_integral(-b*x - b*c/d))*tan(
3/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3
/2*a)^2*tan(3/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2
*a)^2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^
2*tan(3/2*b*c/d)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*t
an(3/2*b*c/d)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 + 12*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1
/2*a)*tan(1/2*b*c/d) + 12*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^
2*tan(1/2*a)*tan(1/2*b*c/d) + 12*real_part(cos_integral(b*x + b*c/d))*tan(1
/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*real_part(cos_integral(-b*x - b*
c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - real_part(cos_integral(3
*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*real_part(cos_integral(b
*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d
))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral(-b*x - b*c/d))*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(3*b*x + 3*b*c/d))*t
an(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - real_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*real_part(cos
_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*real_part(cos
_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_
integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part
(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 6*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a) - 12*sin_integral((b*d*x + b*c)/
d)*tan(3/2*a)^2*tan(1/2*a) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan
(3/2*a)*tan(1/2*a)^2 + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*
a)*tan(1/2*a)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2
 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) +
 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) -
```

```
4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d) + 2*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d) - 2*imag_part(c
os_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d) + 4*sin_integral
(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*
c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2 - 6*imag_part(cos_integral(b*x + b*c/d))*
tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*imag_part(cos_integral(-b*x - b*c/d))*tan(1
/2*a)*tan(3/2*b*c/d)^2 - 12*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/
2*b*c/d)^2 + 6*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*
c/d) - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)
+ 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d) - 6*imag_par
t(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_
integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*sin_integral((b*d*
x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*
c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*imag_part(cos_integral(-b*x - b*c
/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*sin_integral((b*d*x + b*c)/d)*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*t
an(3/2*a)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*ta
n(3/2*a)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*ta
n(1/2*b*c/d)^2 + 6*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*
b*c/d)^2 - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d
)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*imag
_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*im
ag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4
*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2 + 3*real_part(cos_integral(b*
x + b*c/d))*tan(3/2*a)^2 + 3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*
a)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2 + real_part(c
os_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2 - 3*real_part(cos_integral(b*x +
 b*c/d))*tan(1/2*a)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^
2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2 + 4*real_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 4*real_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d
))*tan(3/2*b*c/d)^2 + 3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d
)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 + 12*real_
part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 12*real_part(co
s_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 - 3*real_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*c/d)^2 - 3*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d
)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 - 2*imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*a) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)
- 6*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 6*imag_part(cos_integ
```

```
ral(-b*x - b*c/d))*tan(1/2*a) - 12*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)
 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*imag_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) + 4*sin_integral(3*(b*d*x +
b*c)/d)*tan(3/2*b*c/d) + 6*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c
/d) - 6*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) + 12*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*b*c/d) + real_part(cos_integral(3*b*x + 3*b*c/
d)) + 3*real_part(cos_integral(b*x + b*c/d)) + 3*real_part(cos_integral(-b*
x - b*c/d)) + real_part(cos_integral(-3*b*x - 3*b*c/d)))/(d*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/
d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 +
d*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*t
an(3/2*a)^2 + d*tan(1/2*a)^2 + d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

```
[In] int(cos(a + b*x)^3/(c + d*x),x)
```

```
[Out] int(cos(a + b*x)^3/(c + d*x), x)
```

3.21 $\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 200
Rubi [A] (verified) . . . 200
Mathematica [A] (verified) . . . 202
Maple [A] (verified) . . . 202
Fricas [A] (verification not implemented) . . . 203
Sympy [F] . . . 203
Maxima [C] (verification not implemented) . . . 204
Giac [B] (verification not implemented) . . . 204
Mupad [F(-1)] . . . 205

Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = -\frac{\cos^3(a+bx)}{d(c+dx)} - \frac{3b\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2} - \frac{3b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d^2} - \frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2} - \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

```
[Out] -cos(b*x+a)^3/d/(d*x+c)-3/4*b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2-3/4*b*cos(3*a-
3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2-3/4*b*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2
-3/4*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3394, 3384, 3380, 3383}

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = -\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{3b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2} - \frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2} - \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{\cos^3(a+bx)}{d(c+dx)}$$

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] -(Cos[a + b*x]^3/(d*(c + d*x))) - (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3
*a - (3*b*c)/d])/(4*d^2) - (3*b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/(4*d^2) - (3*b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) - (3*
b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(a+bx)}{d(c+dx)} + \frac{(3b)\int \left(-\frac{\sin(a+bx)}{4(c+dx)} - \frac{\sin(3a+3bx)}{4(c+dx)}\right) dx}{d} \\
&= -\frac{\cos^3(a+bx)}{d(c+dx)} - \frac{(3b)\int \frac{\sin(a+bx)}{c+dx}\, dx}{4d} - \frac{(3b)\int \frac{\sin(3a+3bx)}{c+dx}\, dx}{4d} \\
&= -\frac{\cos^3(a+bx)}{d(c+dx)} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\, dx}{4d} \\
&\quad - \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\, dx}{4d} - \frac{\left(3b\sin\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\, dx}{4d} \\
&\quad - \frac{\left(3b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\, dx}{4d}
\end{aligned}$$

$$= -\frac{\cos^3(a+bx)}{d(c+dx)} - \frac{3b\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2} - \frac{3b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d^2} - \frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2} - \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = -\frac{3d\cos(a+bx)+d\cos(3(a+bx))+3b(c+dx)\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+3b(c+dx)\,\text{CosInt}}{}$$

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] -1/4*(3*d*Cos[a + b*x] + d*Cos[3*(a + b*x)] + 3*b*(c + d*x)*CosIntegral[(3*
b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + 3*b*(c + d*x)*CosIntegral[b*(c/d + x
)]*Sin[a - (b*c)/d] + 3*b*c*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b
*d*x*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*c*Cos[3*a - (3*b*c)/d]
*SinIntegral[(3*b*(c + d*x))/d] + 3*b*d*x*Cos[3*a - (3*b*c)/d]*SinIntegral[
(3*b*(c + d*x))/d])/(d^2*(c + d*x))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{b^2\left(-\frac{3\cos(3bx+3a)}{(-ad+bc+d(bx+a))d}-\frac{3\left(-\frac{3\,\text{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}-\frac{3\,\text{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{d}\right)}{12}+\frac{}{b}$
default	$\frac{b^2\left(-\frac{3\cos(3bx+3a)}{(-ad+bc+d(bx+a))d}-\frac{3\left(-\frac{3\,\text{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}-\frac{3\,\text{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{d}\right)}{12}+\frac{}{b}$
risch	$\frac{3ib\,e^{-\frac{3i(ad-bc)}{d}}\,\text{Ei}_1\left(3ixb+3ia-\frac{3i(ad-bc)}{d}\right)}{8d^2}+\frac{3ib\,e^{-\frac{i(ad-bc)}{d}}\,\text{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{8d^2}-\frac{3ib\,e^{\frac{i(ad-bc)}{d}}\,\text{Ei}_1\left(-ixb-ia-\right.}{8d^2}$

```
[In] int(cos(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b^2*(-3*cos(3*b*x+3*a)/(-a*d+b*c+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a
-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3
*(-a*d+b*c)/d)/d)/d)+3/4*b^2*(-cos(b*x+a)/(-a*d+b*c+d*(b*x+a))/d-(-Si(-b*x-
a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d
)/d)/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.22

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = -\frac{4\,d\cos\left(bx+a\right)^3 + 3\left(bdx+bc\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right) + 3\left(bdx+bc\right)\mathrm{Ci}\left(\frac{3\left(bdx+bc\right)}{d}\right)\sin\left(-\frac{3\left(bc-ad\right)}{d}\right) + }{4\left(d^3x+cd^2\right)}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*cos(b*x + a)^3 + 3*(b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)*si
n(-(b*c - a*d)/d) + 3*(b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d)*sin(-3*
(b*c - a*d)/d) + 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*
x + b*c)/d) + 3*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c
)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.10

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = -\frac{3\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left($$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2*(
exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b^2*(-I*
exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(2,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(I*exp_integ
ral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x + a
)*d^2 - a*d^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(137) = 274$.

Time = 0.44 (sec) , antiderivative size = 1000, normalized size of antiderivative = 6.90

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(((d*
x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)
/d) + 3*b^3*c*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) +
 b*c - a*d)/d)*sin(-(b*c - a*d)/d) - 3*a*b^2*d*cos_integral(((d*x + c)*(b -
 b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)/d) + 3*(d*
x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(3*((d*x + c)*(b
 - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-3*(b*c - a*d)/d) + 3
*b^3*c*cos_integral(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
- a*d)/d)*sin(-3*(b*c - a*d)/d) - 3*a*b^2*d*cos_integral(3*((d*x + c)*(b -
b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-3*(b*c - a*d)/d) - 3*(d
*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c - a*d)/d)*sin_int
egral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*b
```

```
^3*c*cos(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*cos(-(b*c - a*d)/d)*sin_integral(-((
d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*(d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-3*(b*c - a*d)/d)*sin_integral(
-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - 3*b^3*c
*cos(-3*(b*c - a*d)/d)*sin_integral(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*cos(-3*(b*c - a*d)/d)*sin_integral(-
3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*b^2*d*
cos(-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + b^2*d*cos(-3*(d*x +
 c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x +
 c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int(cos(a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)^3/(c + d*x)^2, x)
```

3.22 $\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx$

Optimal result . . . 206
Rubi [A] (verified) . . . 206
Mathematica [A] (verified) . . . 209
Maple [A] (verified) . . . 209
Fricas [A] (verification not implemented) . . . 210
Sympy [F] . . . 210
Maxima [C] (verification not implemented) . . . 210
Giac [C] (verification not implemented) . . . 211
Mupad [F(-1)] . . . 284

Optimal result

Integrand size = 16, antiderivative size = 184

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = &-\frac{\cos^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}\\ &-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\\ &+\frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} + \frac{3b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}\\ &+\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\end{aligned}$$

```
[Out] -9/8*b^2*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^3-3/8*b^2*Ci(b*c/d+b*x)*cos(a
-b*c/d)/d^3-1/2*cos(b*x+a)^3/d/(d*x+c)^2+9/8*b^2*Si(3*b*c/d+3*b*x)*sin(3*a-
3*b*c/d)/d^3+3/8*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+3/2*b*cos(b*x+a)^2*sin(
b*x+a)/d^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used

$= \{3395, 3384, 3380, 3383, 3393\}$

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = -\frac{3b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3} - \frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3} + \frac{3b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3} + \frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)} - \frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] -1/2*Cos[a + b*x]^3/(d*(c + d*x)^2) - (3*b^2*Cos[a - (b*c)/d]*CosIntegral[(
b*c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d
+ 3*b*x])/(8*d^3) + (3*b*Cos[a + b*x]^2*Sin[a + b*x])/(2*d^2*(c + d*x)) + (
3*b^2*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
```

```
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(a+bx)}{2d(c+dx)^2} + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} \\
&\quad + \frac{(3b^2)\int \frac{\cos(a+bx)}{c+dx}\,dx}{d^2} - \frac{(9b^2)\int \frac{\cos^3(a+bx)}{c+dx}\,dx}{2d^2} \\
&= -\frac{\cos^3(a+bx)}{2d(c+dx)^2} + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} - \frac{(9b^2)\int \left(\frac{3\cos(a+bx)}{4(c+dx)} + \frac{\cos(3a+3bx)}{4(c+dx)}\right)\,dx}{2d^2} \\
&\quad + \frac{\left(3b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d^2} - \frac{\left(3b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d^2} \\
&= -\frac{\cos^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^3} + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} \\
&\quad - \frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3} - \frac{(9b^2)\int \frac{\cos(3a+3bx)}{c+dx}\,dx}{8d^2} - \frac{(27b^2)\int \frac{\cos(a+bx)}{c+dx}\,dx}{8d^2} \\
&= -\frac{\cos^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^3} \\
&\quad + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} - \frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3} \\
&\quad - \frac{\left(9b^2\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{8d^2} - \frac{\left(27b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{8d^2} \\
&\quad + \frac{\left(9b^2\sin\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{8d^2} + \frac{\left(27b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{8d^2} \\
&= -\frac{\cos^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3} \\
&\quad - \frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3} + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} \\
&\quad + \frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{6d\cos(bx)(-d\cos(a)+b(c+dx)\sin(a))+2d\cos(3bx)(-d\cos(3a)+3b(c+dx)\sin(3a))+6d(b(c+dx)c}{}$$

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (6*d*Cos[b*x]*(-(d*Cos[a]) + b*(c + d*x)*Sin[a]) + 2*d*Cos[3*b*x]*(-(d*Cos[
3*a]) + 3*b*(c + d*x)*Sin[3*a]) + 6*d*(b*(c + d*x)*Cos[a] + d*Sin[a])*Sin[b
*x] + 2*d*(3*b*(c + d*x)*Cos[3*a] + d*Sin[3*a])*Sin[3*b*x] - 6*b^2*(c + d*x
)^2*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*Cos[3*a - (3*b*c)/d]*Cos
Integral[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3
*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]))/(16*d^3*(c + d*x)^2)
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{b^3\left(-\frac{3\cos(3bx+3a)}{2(-ad+bc+d(bx+a))^2d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad}{d}\right.}{d}}{2d}\right.}{12}$
default	$\frac{b^3\left(-\frac{3\cos(3bx+3a)}{2(-ad+bc+d(bx+a))^2d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad}{d}\right.}{d}}{2d}\right.}{12}$
risch	$\frac{9b^2\mathrm{e}^{-\frac{3i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(3ixb+3ia-\frac{3i(ad-bc)}{d}\right)}{16d^3}+\frac{3b^2\mathrm{e}^{-\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{16d^3}+\frac{3b^2\mathrm{e}^{\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(-ixb-ia\right.}{16d^3}$

```
[In] int(cos(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/12*b^3*(-3/2*cos(3*b*x+3*a)/(-a*d+b*c+d*(b*x+a))^2/d-3/2*(-3*sin(3*b
*x+3*a)/(-a*d+b*c+d*(b*x+a))/d+3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-
a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)+3
```

```
/4*b^3*(-1/2*cos(b*x+a)/(-a*d+b*c+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-a*d+b*c
+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+
b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.57

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = -\frac{4\,d^2\cos(bx+a)^3 - 12\,(bd^2x+bcd)\cos(bx+a)^2\sin(bx+a) + 9\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{3\,(bc-ad}{d}\right.}{}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*d^2*cos(b*x + a)^3 - 12*(b*d^2*x + b*c*d)*cos(b*x + a)^2*sin(b*x +
a) + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*cos_inte
gral(3*(b*d*x + b*c)/d) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c
 - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 3*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)
/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx$$

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)**3/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.84

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = -\frac{3\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(\right.\right.}{}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3*(
exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b^3*(-I*
exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(3,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(I*exp_integ
ral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*
b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.42 (sec) , antiderivative size = 115446, normalized size of antiderivative = 627.42

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*(9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b
^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2
*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x
^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_p
art(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral(
(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x
 + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2
*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^
```

```
2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_
part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a
)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral
(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1
/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2
*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 +
12*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2
*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_p
art(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c
)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d) - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 +
3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(
cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
 + 36*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36
*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c
*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
```

```
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 72*b^2*c*d*x*
sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_
integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^
2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(
cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)
*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_p
art(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2
*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part
(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b
*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^
2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(co
s_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/
d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*r
eal_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_par
t(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan
```

```
(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d) + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 36*b^2*d^2*
x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x
+ b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c
/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*d^2*x^2*si
n_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)*tan(3/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)
^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 36*b^2*d^2*x^2*
sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3
*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*
c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*real_
part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2
*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b
*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/
d) + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*ima
g_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*sin_integral((b*d*x + b
*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*
b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*
imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral
(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*
```

```
x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)
^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^
2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag
_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^2*d^2
*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integral((b*d*x
 + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*
c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x
^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/
2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*
b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*
d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral
(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 18*b
^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a
)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x
+ b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2
*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^
2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_in
tegral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)
*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*
```

```
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/
2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2
*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integ
ral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/
d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^
2 + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2
*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*t
an(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 +
 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(
cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x
- 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*
b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*
d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(
3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^
2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*
d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integral((
b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)
```

```
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*c^
2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/
d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_
integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(
3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag
_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(
-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b
*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c^
2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a
)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(
3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x
 + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d
*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/
d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*
```

```
x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan
(1/2*a)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*
x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 36*b^2*c*d*x*imag_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d) + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*t
an(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 72*
b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 9*b^2*d^2*x^2*real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/
d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*
c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(
cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)*tan(3/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))
*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2
4*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*
c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan
(1/2*a)^2*tan(3/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(c
os_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(
cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*ta
n(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_pa
```

```
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(c
os_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b^2*d^2*x^2*real_part(cos_integral(b*x +
b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d
) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part
(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) -
24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integra
l(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b
^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integ
ral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b
^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*
b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)
^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_in
tegral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(-b*x -
b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12
*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integra
l((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(
3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b
^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b
*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c*d
```

```
*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b*d*x +
 b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(
cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*b^2
*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*
c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(
cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1
/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^
2*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 36
*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 72*b^2*c*d*x*sin_integra
l(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*
tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*t
an(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*re
al_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan
(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^
2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_i
ntegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 +
36*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(
```

```
1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*real
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3
*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*
tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*t
an(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 -
 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_
integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*re
al_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)
^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(
-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*ta
n(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b
^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2
*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_part(cos
_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/
2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_part(cos_integral(
-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 72*b^2*c*
d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b
*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(
1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 72*b^
2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_par
t(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*
c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)
*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
```

```
 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*ta
n(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_
integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x
)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^
2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x -
 b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*
b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral
(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3
*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_
integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*
sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b
*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_
integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*
a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integr
al((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b
*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_p
art(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2
*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
```

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3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2
*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(
-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/
2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)
/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*ta
n(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x +
b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral
(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan
(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(
3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*
b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a
)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(co
s_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*i
mag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a
```

```
)^2*tan(1/2*a) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) - 12*b^2*d^2*x^2*sin_integr
al((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) -
 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral
(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 -
 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)*tan(1/2*a)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*
b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 6*b^2*c*d
*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 18*b^2*c*d*x*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a
)^2*tan(1/2*a)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^2*x^2*
imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(3/2*b*c/d) - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*t
an(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^2*x^2*i
mag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(3/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*d
^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/
2*a)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 7
2*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d) + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 36*b^2*d^2*x^2*sin_in
tegral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^2*x^2*imag_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(
3/2*b*c/d) - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b^2*c^2*imag_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d) + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 36*b^2
*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*
a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^2*x^2*imag_part(cos_integral(3*
b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 -
 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 +
18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/
```

```
2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral
(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2
+ 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(
-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d
)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^
2 - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x
+ b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*d
^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*ta
n(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_
part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(
3/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*d^2*x^2*sin_integr
al((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
 - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*
tan(3/2*b*c/d)^2 - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real
_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*rea
l_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*d^2*
x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(
1/2*a)^2*tan(3/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 36*b^2*
d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a
)^2*tan(3/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/
d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 18*b^2*d^2*x^
2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1
/2*a)^2*tan(3/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*t
an(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c^2*imag_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c
/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos
_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^
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2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_in
tegral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c
/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*sin_integral((b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) + 24
*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_inte
gral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*t
an(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*
b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan
(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
 + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral((b*d*x +
b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*b^2*d^2
*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*
sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(
1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*d
^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x
- b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 1
2*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x +
 b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^
2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c
)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^
2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
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3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b
*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2
*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) -
6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*sin_integral((
b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*
c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_inte
gral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*
d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(
-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x
)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(
cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^
2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integra
l((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(
cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*
b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x
 - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b
^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*ta
n(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*
b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*
x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1
/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_p
art(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*ta
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n(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 18
*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integra
l(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d
)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3
*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^
2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(
-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2
- 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)
^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integral((b*d*
x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*b
^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x -
b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*d^
2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*
tan(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_par
t(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/
2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(b
*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*
b*c/d)^2 - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_int
egral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a
)*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*r
eal_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*re
al_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*
b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^
2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/
d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2
*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2
*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b
*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 36*b^2*d^2
*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan
```

```
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^
2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2
*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(co
s_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c
/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c
/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*
c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*re
al_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/
2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_part(cos_integral(
-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*
tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^
2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan
(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/
d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*d^2
*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b
*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x
 + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(
1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^
2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*
x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_inte
gral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*
b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_p
```

```
art(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1
/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan
(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*r
eal_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a
)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_part(cos_integral(-
3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan
(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1
/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*
c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*t
an(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_
integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2
*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_in
tegral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)
^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*
c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*ima
g_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b
*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/
d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(c
os_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*
b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*
b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/
2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(
3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*t
an(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_inte
```

```
gral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*
b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*
tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*
x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*
c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2
*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/
2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b
*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integral((b*d*x +
 b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^
2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c^2*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*d^2*x^
2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))
*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2
*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)
^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c^2*im
ag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d
)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
+ 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c^2*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)
^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan
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(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b
*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_par
t(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2
*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c
/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*
c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b
*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b
^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral
(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan
(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/
d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3
/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)
*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/
d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d
*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*
x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x
)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + 8*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 + 3*b^2*d^2*x^2*real_part
(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 + 3*
b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 - 12*b^2*c*d*x*imag_part(cos_in
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tegral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)
+ 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 9*b^2*d^2*x^2*rea
l_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2
*a)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(1/2*a)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 9*b^2*d^2*x^2*real_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2
- 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 36*b^2*c*d*x*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 - 72
*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)*tan(1/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/
d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*d^2*x^2*real_part(cos_
integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*d^2
*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b
*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*d^2*x^2*rea
l_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
- 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^2*c^2*real_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)
^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x -
b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^2*c^2
*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d) + 36*b^2*d^2
*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(3/2*a)*tan(3/2*b*c/d) - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3
*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 36*b^2
*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(3/2*b*c/d) - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)
/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c*d*
x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d) - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 72*b^2*
c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2
*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/
d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2
*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/
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d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2
*a)^2*tan(3/2*b*c/d) + 36*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b
^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 36*b^2*c*d*x*imag_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*
c/d) + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 72*b^2*c*d*x*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 36
*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*
a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c*d*x*imag_part(cos_integral(-3*b
*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 72
*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(3/2*b*c/d) - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_p
art(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)
^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x
- 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + 36*b^2*c*d*x*i
mag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a)*tan(3/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 72*b^2*c*d
*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)
*tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*
tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_
integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*b^2
*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*
tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*
tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_
integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3
*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)
^2*tan(3/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 9
*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x +
 b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*b^
2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 12*b^
2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 24*b^2*c*
d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*
```

```
tan(3/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(
cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*
b*c/d)^2 - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(b*
x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b^
2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)*tan(3/2*b*c/d)^2 - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x
^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*
b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral
(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x^
2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2
*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral
(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b
*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 -
9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 -
 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(
3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 36*b^2*c
*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan
(1/2*a)^2*tan(3/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 72*b^2*c
*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2
*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan
(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3
*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 -
3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*
c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-
```

```
b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9
*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x
 - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^
2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integr
al(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)
- 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b*
c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) + 12*b^2*d^
2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(1/2*a)*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/
d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*d^2*x^
2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_p
art(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1
/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_inte
gral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*ta
n(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 12*b^2*c*d*x*im
ag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)
^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_in
tegral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*
b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) -
 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan
(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d)
)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*im
ag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*sin_integral(
(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(
```

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3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12
*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)
/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^
2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*
c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^
2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*
c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^
2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/
2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(-b
*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)
^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(co
s_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12
*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d) + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) -
12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) -
24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c
/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c*d
*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 9*b^2*d^2*x^2*real_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c
/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*re
al_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1
```

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/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*t
an(1/2*b*c/d)^2 - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos
_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 -
3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a
)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_
integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 -
9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 9
*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*
real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*
a)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a
)^2*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*b^2*c*d*x*imag
_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*
tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)
^2 - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integral((b*d*x + b
*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*c*d
*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*a)*tan(1/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_i
ntegral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c
/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*ta
n(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan
(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^
2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan
(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan
(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^
2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*
tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_inte
```

```
gral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 9*b^
2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c
^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 36*b^2*c
*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan
(1/2*a)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 72*b^2*c
*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*t
an(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*
x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real
_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/
d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c
/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(c
os_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 -
 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*
a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x +
 b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*
c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/
d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*
tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 9*
b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*
c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan
(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 36*b^
2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-
3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/
d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*real_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/
d)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x
)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*real_part(c
os_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(
```

```
1/2*b*c/d)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan
(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*
tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2
*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b
*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)
*tan(1/2*b*c/d)^2 - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*
x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*t
an(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*ta
n(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 72*b^2*c*d*x*si
n_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*ta
n(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3
/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_
part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d
*x*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2
*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*
b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 +
 36*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/
2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_part(co
s_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2
*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c
/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/
d)^2 - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3
*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d
^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan
(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*
```

```
x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^
2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan
(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(co
s_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*im
ag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 72*b^2*
c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/
d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 36*b^2*c*
d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/
d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x
*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x +
b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b*
x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2
*b*x)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part
(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(3
/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*
b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(-b*x
- b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 24*
b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d)
```

```
)*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b^2*c*d*
x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1
/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^
2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*
c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 24*b^2*c*
d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2
*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan
(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b
*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
+ 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*
b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/
2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*
b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3
6*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d
^2*x*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 - 12*b*d^2*x*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*
a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(c
os_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
```

```
)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b
*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a) + 18*b^2*d^2*x^2*im
ag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3
/2*a) - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a) - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 + 6*b^2*c*d*x*real_part(cos_in
tegral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 + 6*b^2*c*d
*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(b*
x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a) + 6*b^2*d^2*x^2*imag_p
art(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a) -
12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(1/2*a) - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b
*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) - 12*b^2*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) - 6*b^2*d^2*x^2*im
ag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) +
 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*a) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 6*b^2*c^2*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 18*b^2*c*d*x*real_part(cos_integral(3*b*x
 + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 6*b^2*c*d*x*real_
part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2
- 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(1/2*a)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 18*b^2*d^2*x^2*imag_part(cos
_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 18*b^2
*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a
)*tan(1/2*a)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x
)^2*tan(3/2*a)*tan(1/2*a)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x +
 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 18*b^2*d^2*x^2*imag_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 -
36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*ta
n(1/2*a)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 18*b^2*c^2*imag_part(cos_inte
gral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)
^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)*tan(1/2*a)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*
b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 6*b^2*c*d*x*real_part(co
s_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 6*b^2*c
```

```
*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)^2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x
 + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 6*b^2*c*d*x*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 6*
b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)^2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + 18*b^2*d^2*x^2*imag_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d) - 18*b^2*d
^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral
(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)
 + 72*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^
2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2
*tan(3/2*b*c/d) - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*
x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(3*
b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^2*x^2
*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
3/2*b*c/d) - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(3/2*b*c/d) - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*c
^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(3/2*b*c/d) - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^2*x^2*im
ag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*
b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b
*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x +
b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^2*x^2*imag_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d
) - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/
d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 18*b^2*c^2*imag_part(cos_in
tegral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2
*b*c/d) - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c^2*sin_integral(3*(
b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) +
 72*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3
/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral(-3*
b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 72*
b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a
)*tan(1/2*a)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral(-3*b*x
- 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b^2*
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d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d) + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 36*b^2*d^2*x^2*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b^2*c^2*imag_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d) + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 36*b^2*c^2*sin_integra
l(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
) - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 18*b^2*c^2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) -
36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(3/2*b*c/d) - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part
(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2
+ 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_
part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d
)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)
^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c
)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 -
 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*ta
n(3/2*a)*tan(3/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*
tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/
d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 18
*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*
a)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*ta
n(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_inte
gral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c
*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*
tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d
*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/
2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integra
l(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*d^2*x^2*
imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c
/d)^2 - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2
*a)*tan(3/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*t
```

```
an(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*d^
2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d
)^2 - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(-b*x
 - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b
^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*
a)*tan(3/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*d^2*x^2
*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 6
*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d)
)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*c^2*sin_
integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*
c/d)^2 - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^
2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*
tan(3/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c
/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(c
os_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*
b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*
tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*
b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/
2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integra
l(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*
d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*t
an(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c^2*imag_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3
/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 36*b^2*c^2*sin_integra
l(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^
2 + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(
3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 +
36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/
2*a)^2*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)
*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x
```

```
+ 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*real_p
art(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 -
 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(1
/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)*
tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 8*d^2*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*d^2
*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*sin_integral((b*d*x +
b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_p
art(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) -
 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral(
b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*im
ag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c
/d) + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a
)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(co
s_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*b*c/d) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*
x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^
2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*c/
d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^2*c*d*x*re
al_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*
tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_
integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/
d) - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d
))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral
((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^
2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*
b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)
*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integ
ral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
 + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2
*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c
```

```
/d) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(
3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(b*x
 + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*
c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d
) + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*
a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*im
ag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c
)/d)*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
) - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*
tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_in
tegral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*sin_integral((b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d)
)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*sin_integra
l((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2
*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*sin_
integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x
 + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^
2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*
c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*
d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*re
al_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1
```

```
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*d^
2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_integral(
-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
- 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*
x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos
_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x +
b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 18*b^2*
c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 18*b^2*
c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 -
36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*ta
n(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 36*b^
2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2
*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2
*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral
(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d
*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*r
eal_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x -
b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/
d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*ta
```

```
n(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c
/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integr
al((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*b^2*d^2*
x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*
b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)
^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)
^2 - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)
/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*b^2*d^2*x
^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c
/d)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*ta
n(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*ta
n(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(b
*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^
2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan
(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_
integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d
)^2 - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)/
d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 18*b^2*
c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*
tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2
*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(
-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1
/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2
*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(
b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c
/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)
^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*
b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))
*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)
^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d
)^2 + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*t
```

```
an(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x
+ b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 12*b*c*
d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 -
18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*
a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x
*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/
2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 + 8*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x
^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)*
tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b
*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^
2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*
b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*ta
n(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/
2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integ
ral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2
*b*c/d)^2 + 72*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b
*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_part(c
os_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan
(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1
/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)
*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))
*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
- 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d
)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2
*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*
tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_i
ntegral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1
/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1
/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_inte
```

```
gral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b
^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(
1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*
b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*
tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^
2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/
2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b
*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integra
l(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 + 72*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*ta
n(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b
*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d
*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b
*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18
*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d)
)*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(
cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*
b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x
^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d)^2 + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(3*b*x
 + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2
*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x +
b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^
2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x
- 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3
6*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral
(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c
```

```
*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral
(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*c
*d*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + 12*b*c*d*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - 4*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d)
)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(co
s_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12
*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^
2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c^2*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(b*x +
 b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^
2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan
(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2
*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_
integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 - 24*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2
*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(1/2*b*x)^2*tan(3/2*a)^
2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos
_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1
/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 + 4*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*
b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/
d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan
(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*
b*c*d*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 8*d^2*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c
```

```
/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*d^2*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(1/2*b*x)*tan(3/2*a)^
2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*d^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*re
al_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2 + 3*b^
2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(
1/2*b*x)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*b*x)^2*tan(1/2*b*x)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a) + 36*b^2*c*d*x*imag_part(cos_
integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a) - 72*b
^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a) - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2*tan(3/2*a)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3
/2*b*x)^2*tan(3/2*a)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d)
)*tan(3/2*b*x)^2*tan(3/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2 + 3*b^2*d^2*x^2*real_par
t(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2 + 3*b^2*d^2*x^2*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2 - 9*b^2*d^2
*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2
 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 + 3*b^2*c^2*real_part(cos_integral(-b
*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 - 9*b^2*c^2*real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)
^2 - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(1/2*a) + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*t
an(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a) - 24*b^2*c*d*x*sin_integral((b*d*x
+ b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a) - 12*b^2*c*d*x*imag_part
(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 12*b^2
*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*a) - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 12*b^2*c*d*x*imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) - 24*b^2*c*d*x*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 12*b*d^2*x*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 9*b^2*d^2*x^2*real_pa
rt(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2 - 3*b^2*d^2*x
^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2 - 3*b^2
*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2
 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*a)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2
*b*x)^2*tan(1/2*a)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*t
```

```
an(1/2*b*x)^2*tan(1/2*a)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*
c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 9*b^2*c^2*real_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 3*b^2*c
^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1
/2*a)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(1/2*a)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 36*b^2*c*d*x*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 36*b^
2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)
*tan(1/2*a)^2 - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2
*tan(3/2*a)*tan(1/2*a)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*
c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 36*b^2*c*d*x*imag_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 - 72*b^2*
c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^
2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 - 9*b^
2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^
2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2
*a)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*ta
n(1/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*a)^2*tan(1/2*a)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*c^2*real_part(cos_integral(b
*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*c^2*real_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b
^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^
2*tan(1/2*a)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/
2*a)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*c^2*real_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^2*c^2*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2 + 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + 36*b
^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*b*c/d) - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*sin_integral(
3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d) + 36*b^2*d^
2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*ta
n(3/2*b*c/d) + 36*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan
(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d) + 36*b^2*d^2*
x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan
(3/2*b*c/d) + 36*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d) + 36*b^2*c^2*real_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2
*b*c/d) - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
```

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)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) - 72*b^2*c*d*x*sin
_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) - 3
6*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(3/2*b*c/d) + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) - 72*b^2*c*d*x*sin_integral(3
*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c*d*x
*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3
/2*b*c/d) - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*
b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b
*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c*d*x*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) -
 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(
1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*real_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x
^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/
2*b*c/d) + 36*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)
^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c^2*real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)
 + 36*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3
/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c^2*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 36*b^
2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d) + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 72*b^2*c*d*x*sin_integral(3*(b*d*x
+ b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 9*b^2*d^2*x^2*real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2 + 3*b^2*d^
2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2
+ 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/
2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*b*x)^2*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*
b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integ
ral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part
(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x^
2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2
 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 - 9*b^
2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))
*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 72*b
^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*
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b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*
x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b
*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 72*b^2*c*d*x*si
n_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 +
12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 9*b^
2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c
/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan
(3/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2
*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*b^2*c^2*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)
^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2
*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)^2*
tan(1/2*b*x)*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 3*b^2*c
^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2
*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*
b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(
cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*
b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*t
an(3/2*b*c/d)^2 - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2
*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*
c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 24*
b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b
*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d
)^2 - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/
2*a)*tan(3/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 24*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*d^2*x^2
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 9*b^
2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*
c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/
d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 9*b^
2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2
```

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*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*a)^2*tan
(3/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*
b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2
 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(3/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d)
)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 72*b^2*
c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 -
12*b*d^2*x*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*
c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c
/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*
b*d^2*x*tan(3/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*d^2*
x*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b^2*c*d*x*im
ag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*
c/d) - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c*d
*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*b*c/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) + 24*b^2*c*d
*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)
 + 12*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1
/2*a)*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))
*tan(3/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*d^2*
x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2
*b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(
-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12
*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*t
an(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/
2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x +
b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*
real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a
)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*
```

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b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)
 - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2
*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*t
an(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b*d*
x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_
part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
+ 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2
*a)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b
*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(b*x
 + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*imag_par
t(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 24
*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*
b*c/d) + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*sin_
integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) +
12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*
tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*sin_integral(
(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c*
d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)
/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part
(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12
*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/
2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(
cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2
*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a
)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*
c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a
)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 24*b^2*c*d*x*sin_integral((b*d*x + b*
c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 9*b^2*d^2*x^2*real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^
2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2
- 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*b*x)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*
```

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b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integ
ral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part
(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^
2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2
 + 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_in
tegral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + 9*b^
2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))
*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 72*b
^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*
b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*
x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(-3*b
*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 72*b^2*c*d*x*si
n_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 +
 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 9*b^
2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c
/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan
(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2
*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)
^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d)
)*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)^2*
tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^2*c
^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*
b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(
cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*
b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*t
an(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2
*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*
c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(c
os_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 24*
b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d
)^2 + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/
2*a)*tan(1/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*
```

```
tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integral((b*d*x
 + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*d^2*x^2
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 9*b^
2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 + 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/
d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 9*b^
2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*a)^2*tan
(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*
b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d)
)*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 72*b^2*
c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d
)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 -
12*b*d^2*x*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*
c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(
1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c
/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 12*
b*d^2*x*tan(3/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*
x*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*im
ag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)*tan(1/
2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2
*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*
x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*i
mag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1
/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/
2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d
*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*d^2*x^
2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/
2*b*c/d)^2 + 36*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3
/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_part(cos_integral(3
*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
```

```
+ 36*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3
/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_part(cos_integral(3
*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
+ 36*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3
/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral
(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c*
d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*t
an(1/2*b*c/d)^2 - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2
*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*
x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/
2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(
3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2
*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*
b*c/d)*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integ
ral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^
2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x +
b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_p
art(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*re
al_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x
- 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*t
an(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x
*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1
/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*x
 + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(3/
2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(1/2*
b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(c
os_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_inte
gral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12
*b*d^2*x*tan(3/2*b*x)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b
```

```
*d^2*x*tan(1/2*b*x)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b^2
*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*
a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integral((b*d*x + b
*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*
x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(1/2*b*x)
^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*a)^2*t
an(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b^
2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*
b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x
*tan(3/2*b*x)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*d^2*x*t
an(1/2*b*x)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan
(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_
part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2 + 6*b^2*c
*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2 + 6
*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2
*tan(1/2*b*x)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*t
an(3/2*b*x)^2*tan(3/2*a) + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a) - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a) - 18*b^2*d^2*x^2*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a) + 18*b^2*d^2*x^2*imag_part(cos_
integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a) - 36*b^2*d^2*x^2*sin_
integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a) - 18*b^2*c^2*imag_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a) +
 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a) - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*
x)^2*tan(1/2*b*x)^2*tan(3/2*a) - 18*b^2*c*d*x*real_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2 + 6*b^2*c*d*x*real_part(cos_integra
l(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2 + 6*b^2*c*d*x*real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2 - 18*b^2*c*d*x*real_part(c
os_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2 - 18*b^2*c*d*x*r
eal_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2 + 6*b^2
*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2 + 6
*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^
2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*t
an(3/2*a)^2 - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*
x)^2*tan(1/2*a) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3
/2*b*x)^2*tan(1/2*a) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2
*b*x)^2*tan(1/2*a) - 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan
(1/2*b*x)^2*tan(1/2*a) + 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*b*x)^2*tan(1/2*a) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*
```

```
tan(1/2*b*x)^2*tan(1/2*a) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a) + 6*b^2*c^2*imag_part(cos_integral
(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a) - 12*b^2*c^2*sin_i
ntegral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a) - 6*b^2*d
^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 6*b^2
*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a) - 12
*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a) - 6*b^2*
c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a) + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 6*b^2*c^2*imag_part(cos_integral(-
b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) - 12*b^2*c^2*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 12*b*c*d*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 18*b^2*c*d*x*real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2 - 6*b^2*c*d*x*re
al_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2 - 6*b^2*c*d*
x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2 + 18*b^
2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)
^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*t
an(1/2*a)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)
^2*tan(1/2*a)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2
*b*x)^2*tan(1/2*a)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 + 18*b^2*d^2*x^2*imag_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 - 36*b^2*d^2*x^2*sin_integral
(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2 - 18*b^2*c^2*imag_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 18*b^2*c^2*
imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2
*a)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a
)*tan(1/2*a)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/
2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 18*b^2*c^2*imag_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 - 36*b^2*c^2*sin_integr
al(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 12*b*c*d*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 - 18*b^2*c*d*x*real_part
(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 6*b^2*c*d*x*rea
l_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 6*b^2*c*d*x*r
eal_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 18*b^2*c*d
*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + 12
*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2 + 12*b*c*d*tan
(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + 8*d^2*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + 18*b^2*d^2*x^2*imag_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d) - 18*b^2*d^2*x^2*imag_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d) + 36*b^2
*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*b*c/d) + 18
```

```
*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/
2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2
*b*x)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan
(1/2*b*x)^2*tan(3/2*b*c/d) + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d) - 18*b^2*c^2*imag_part(c
os_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)
 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan
(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral
(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d) + 72*b^2*c*d*x
*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2
*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*
x)^2*tan(3/2*a)*tan(3/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^2*x^2*imag_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) - 36*b^2*d^2*x^2*sin_
integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d) - 18*b^2*c^2*imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/
d) + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*ta
n(3/2*a)^2*tan(3/2*b*c/d) - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(
3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) - 18*b^2*c^2*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*c^2*i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/
2*b*c/d) - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(3/2*b*c/d) + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c
/d))*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*sin_integral(
3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d) + 18*b^2*c^2*imag_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b
^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^
2*tan(3/2*b*c/d) + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^
2*tan(1/2*a)^2*tan(3/2*b*c/d) + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3
*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b^2*c^2*imag_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)
+ 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/
2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral(-3*b
*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b^2*c^2*imag_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) +
 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d) - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b^2*c*d*x*real_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x
*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2
```

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- 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(
3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*ta
n(1/2*b*x)^2*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_
integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 - 18*b^2*d^2*x^2*ima
g_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 + 36*b^2
*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2 + 18*b
^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*t
an(3/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 18*b^2*c^2*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 -
 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/
2*a)*tan(3/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*
b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*a)*tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integra
l(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_i
ntegral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c*d*x*real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 12*b*c*d
*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(3
/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 4*d^2*tan(3/2*b*x)^2
*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(cos
_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*d^2*x^2
*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2 - 6*b^2*c^2*imag
_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
 + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a
)*tan(3/2*b*c/d)^2 - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^
2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/
d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*c
^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 6*b
^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2
*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*ta
n(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/
2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)*tan(3/2*b*c/d)^2 - 24*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^
2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x +
 b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral
(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_
```

```
integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(3/
2*b*x)^2*tan(1/2*b*x)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)
*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*d^2*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 18*b^2*c^2*ima
g_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*t
an(3/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b
*c/d)^2 - 12*b*c*d*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2
- 8*d^2*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^
2 + 12*b*c*d*tan(3/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*d^
2*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(
1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*d^2*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_in
tegral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_par
t(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^
2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x
^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d) - 6*b
^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c
/d) + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b
*c/d) + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 12*b^2*c^2*sin_integral((b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*i
mag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) - 6*b^2*d^2
*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) + 12
*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d) + 6*
b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)
^2*tan(3/2*a)^2*tan(1/2*b*c/d) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*t
an(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_integra
l(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag
_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d
) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*
x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(
cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^
2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan
(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^
2*tan(1/2*a)*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(-b*x - b*
c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_
integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_par
t(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*
sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_
```

```
part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
+ 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)
^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d
))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c^
2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)
- 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*t
an(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 6*b^2*d^2*x^2*imag_part(cos_integ
ral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*d^2*x^2*imag_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x
^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^
2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)
*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^
2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c
/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos
_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 12*
b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(-b*x - b*
c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_
integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2
*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*ta
n(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral
(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 + 18*
b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*b
*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*t
an(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2
*b*x)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b
*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*
x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*b*c/d)^2 - 18*b^2*c^
2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/
```

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2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b
*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)
/d)*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 18*b
^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*
tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^
2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c
/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x
 + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(
3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*
x)*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 4*d^2*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_inte
gral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^2*d^2*x^2*imag_part(co
s_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*d^2*x^2*sin_
integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part
(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*
b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan
(1/2*b*c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan
(1/2*a)*tan(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*t
an(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^2*c^2*imag_part(cos_integra
l(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*c^2*si
n_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12
*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 6*b^2*c^
2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/
d)^2 - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2
*a)*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^
2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)*tan(1/2*b*c/d)^2 - 24*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan
(1/2*a)*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*
tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/
d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c*d*x*real_part(cos_integral(-b*x
 - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x
)^2*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)*tan(
1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*d^2*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x
 + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(1/
2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)
^2 - 12*b*c*d*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 8*d
```

```
^2*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 1
2*b*c*d*tan(3/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 4*d^2*tan
(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(1/2*b
*x)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*d^2*tan(1/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*d^2*x^2*imag_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b
^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
+ 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/
2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integ
ral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*
b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c
/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3
*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c
*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan
(1/2*b*c/d)^2 + 72*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(
3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2
*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(
1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(
3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b
*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^
2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)*
tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x +
b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 18*b^2*c*d*x*real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b
*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*c
*d*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*d^2*ta
n(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*c^2*
imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a
)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*
c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^
2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(1/2*b*x)^2*ta
n(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*d^2*tan(3/2*b*x)*tan(1/2*b*x
)^2*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)*ta
n(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 8*d^2*tan(3/2*b*x)^2*tan(3/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(1/2*b*x)*tan(3/2*a)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*d^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c
```

```
/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b^2*c^2*imag_part(cos
_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 12*
b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 - 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 24*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 + 12*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 - 12*b*c*d*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 24*d^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 - 12*b*c*d*tan(3/2*b*x)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - 8*d^2*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 1
2*b*c*d*tan(1/2*b*x)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 8*d^2
*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*c*d*t
an(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*d^2*tan(3/2*b*
x)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*d^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_
part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2 + 3*b^2*d^2*x^2*real_par
t(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2 + 3*b^2*d^2*x^2*real_part(cos_i
ntegral(-b*x - b*c/d))*tan(3/2*b*x)^2 + 9*b^2*d^2*x^2*real_part(cos_integra
l(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(
3*b*x + 3*b*c/d))*tan(1/2*b*x)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x
 + b*c/d))*tan(1/2*b*x)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c
/d))*tan(1/2*b*x)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(1/2*b*x)^2 + 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*b*x)^2*tan(1/2*b*x)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c
/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2 - 36*b^2*c*d*x*imag_part(cos_in
tegral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a) + 36*b^2*c*d*x*imag_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a) - 72*b^2*c*d*x*s
in_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(3/2*a) - 36*b^2*c*d*x*ima
g_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a) + 36*b^2*c*
d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a) - 7
2*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*a) + 12*
b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a) - 9*b^2*d^2*x^2*real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2 + 3*b^2*d^2*x^2*real_part(cos_i
ntegral(b*x + b*c/d))*tan(3/2*a)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(-
b*x - b*c/d))*tan(3/2*a)^2 - 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*a)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))
*tan(3/2*b*x)^2*tan(3/2*a)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d
))*tan(3/2*b*x)^2*tan(3/2*a)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*
c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)^2 - 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/
2*b*x)*tan(3/2*a)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(1/2*b*x)^2*tan(3/2*a)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*
tan(1/2*b*x)^2*tan(3/2*a)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d
```

```
))*tan(1/2*b*x)^2*tan(3/2*a)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x -
3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)^2 + 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x
)^2*tan(3/2*a)^2 - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/
2*b*x)^2*tan(1/2*a) + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*b*x)^2*tan(1/2*a) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/
2*b*x)^2*tan(1/2*a) - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan
(1/2*b*x)^2*tan(1/2*a) + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))
*tan(1/2*b*x)^2*tan(1/2*a) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan
(1/2*b*x)^2*tan(1/2*a) + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a
) - 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*
a) + 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/
2*a) - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a) -
 12*b*d^2*x*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 12*b*d^2*x*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a) + 9*b^2*d^2*x^2*real_part(cos_integral(3*b*x +
3*b*c/d))*tan(1/2*a)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))
*tan(1/2*a)^2 - 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2
*a)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^
2 + 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1
/2*a)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan
(1/2*a)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*
tan(1/2*a)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*
b*x)^2*tan(1/2*a)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*a)^2 +
 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*a)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan
(1/2*a)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(1/2*a)^2 - 36
*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2
 + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/
2*a)^2 - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)
^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 - 12*b*d^2*x*tan(1/2
*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*c^2*real_part(cos_integral(b*x
+ b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x
 - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^2*c^2*real_part(cos_integral(-3*
b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + 12*b*d^2*x*tan(3/2*b*x)*tan(3/2
*a)^2*tan(1/2*a)^2 + 12*b*d^2*x*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2 + 36
*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*
b*c/d) - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x
)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b
*x)^2*tan(3/2*b*c/d) + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d)
)*tan(1/2*b*x)^2*tan(3/2*b*c/d) - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*sin_integral(3*(
b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*real_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 36*b^2*d^2*x^2*rea
```

```
l_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 36*b^2*c
^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*a)*tan(3
/2*b*c/d) + 36*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*
x)^2*tan(3/2*a)*tan(3/2*b*c/d) + 36*b^2*c^2*real_part(cos_integral(3*b*x +
3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d) + 36*b^2*c^2*real_part(c
os_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d) - 3
6*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b
*c/d) + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2
*tan(3/2*b*c/d) - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2
*tan(3/2*b*c/d) + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan
(1/2*a)^2*tan(3/2*b*c/d) - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b
*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*sin_integral(3*(b*d*x + b
*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c^2*real_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 36*b^2*c^2*real_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 9
*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2 + 3*
b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2 + 3*b^2*d
^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2 - 9*b^2*d^2*x
^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 9*b^2*c^2*r
eal_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2 + 3
*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c/d)
^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(3/2
*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x
)^2*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*b*c/d
)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan
(3/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)
^2*tan(3/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1
/2*b*x)^2*tan(3/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*
c/d))*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x
)^2*tan(3/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d)
)*tan(3/2*a)*tan(3/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x
- 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 + 72*b^2*c*d*x*sin_integral(3*(b*d*
x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(3/2
*a)*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^
2 + 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2
*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan
(3/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^
2*tan(3/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*ta
n(3/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)*tan(3/2*a)^2*tan(3/2*
b*c/d)^2 - 12*b*d^2*x*tan(1/2*b*x)*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 12*b^2*c
*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*
b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2
 - 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2 -
 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(1/2
*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*a)^2*tan(1/2*a)*ta
```

```
n(3/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2
*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan
(1/2*a)^2*tan(3/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d)
)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*
a)^2*tan(3/2*b*c/d)^2 + 12*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2*tan(3/2*b*c/d)
^2 - 12*b*d^2*x*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b^2*c*d*x*ima
g_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d) - 12*b^2*c*
d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d) + 2
4*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*c/d) + 1
2*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c
/d) - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan
(1/2*b*c/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan
(1/2*b*c/d) + 12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^
2*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3
/2*a)^2*tan(1/2*b*c/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2
*a)^2*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*
tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*
c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b
*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c
^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d) + 12*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan
(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(-b
*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_par
t(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x*ima
g_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 24*b^2*c*d
*x*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) + 12*b^2*c*d*x
*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*
b^2*c*d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))
*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 9*b^2*d^2*x^2*real_part(cos_i
ntegral(3*b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*b*c/d)^2 - 3*b^2*d^2*x^2*real_part(cos_integra
l(-b*x - b*c/d))*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integ
ral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos
_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 - 12*b*
d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*b*c/d)^2 + 9*b^2*c^2*real_part(co
s_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*re
```

```
al_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 3*b^2*
c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 +
 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2*tan(1/2
*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 36*b^
2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^
2 + 36*b^2*c*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1
/2*b*c/d)^2 - 72*b^2*c*d*x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1
/2*b*c/d)^2 + 12*b*d^2*x*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 12*b*
d^2*x*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_
integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_p
art(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*re
al_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 9*b^2*c
^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2
 + 12*b*d^2*x*tan(3/2*b*x)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(1/
2*b*x)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 12*b^2*c*d*x*imag_part(cos_integral(
b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*b^2*c*d*x*imag_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 24*b^2*c*d*x*sin_integral
((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)^2*t
an(1/2*a)*tan(1/2*b*c/d)^2 + 12*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d)^2 - 12*b*d^2*x*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 9*b^2*c^2*r
eal_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 3*b
^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 +
 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d
)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(
1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 12*b
*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*a)*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + 36*b^2*c*d*x*imag_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*c*d*x*imag_part(cos_inte
gral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 72*b^2*c*d*x*sin_
integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^2*c^2*re
al_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*
c/d)^2 + 36*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*
b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integra
l(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos
_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*c^2*real
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 1
2*b*d^2*x*tan(3/2*b*x)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(1
/2*b*x)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*a)*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2 + 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2 + 6*b
^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2 + 18*b^2*c*d*
x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2 + 18*b^2*c*d*x*r
eal_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2 + 6*b^2*c*d*x*real_p
```

```
art(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2 + 6*b^2*c*d*x*real_part(cos_i
ntegral(-b*x - b*c/d))*tan(1/2*b*x)^2 + 18*b^2*c*d*x*real_part(cos_integral
(-3*b*x - 3*b*c/d))*tan(1/2*b*x)^2 - 18*b^2*d^2*x^2*imag_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*a) + 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*a) - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*ta
n(3/2*a) - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)
^2*tan(3/2*a) + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/
2*b*x)^2*tan(3/2*a) - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*
x)^2*tan(3/2*a) - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1
/2*b*x)^2*tan(3/2*a) + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(1/2*b*x)^2*tan(3/2*a) - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan
(1/2*b*x)^2*tan(3/2*a) + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)
 - 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2 + 6*b^
2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2 + 6*b^2*c*d*x*rea
l_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2 - 18*b^2*c*d*x*real_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2 - 12*b*c*d*tan(3/2*b*x)^2*tan(1/
2*b*x)*tan(3/2*a)^2 + 12*b*c*d*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)^2 - 4
*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 - 6*b^2*d^2*x^2*imag_part(c
os_integral(b*x + b*c/d))*tan(1/2*a) + 6*b^2*d^2*x^2*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*a) - 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*t
an(1/2*a) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*t
an(1/2*a) + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*
tan(1/2*a) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/
2*a) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/
2*a) + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1
/2*a) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)
 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a) - 6*b^2*c^2*imag_part(c
os_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 6*b^2*c^2*imag_part(cos
_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a) - 12*b^2*c^2*sin_integral(
(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a) - 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*
a)^2*tan(1/2*a) - 24*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a
) + 12*b*c*d*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a) + 18*b^2*c*d*x*real_part
(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2 - 6*b^2*c*d*x*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b
*c/d))*tan(1/2*a)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*a)^2 - 1
2*b*c*d*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*d^2*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(1/2*a)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/
d))*tan(3/2*a)*tan(1/2*a)^2 + 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(3/2*a)*tan(1/2*a)^2 - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d
)*tan(3/2*a)*tan(1/2*a)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2
 - 12*b*c*d*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 - 8*d^2*tan(3/2*b*x)*tan
(1/2*b*x)^2*tan(3/2*a)*tan(1/2*a)^2 + 12*b*c*d*tan(3/2*b*x)*tan(3/2*a)^2*ta
n(1/2*a)^2 - 4*d^2*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + 12*b*c*d*tan(
1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)^2 + 4*d^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
```

```
1/2*a)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*
b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b
*c/d) + 36*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d) + 18*
b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(3/2*b*c
/d) - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*t
an(3/2*b*c/d) + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*t
an(3/2*b*c/d) + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2
*b*x)^2*tan(3/2*b*c/d) - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(1/2*b*x)^2*tan(3/2*b*c/d) + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/
d)*tan(1/2*b*x)^2*tan(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 72*b^2*c*d*x*real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - 18*b^2*c^2*imag_part(cos_
integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*c^2*imag_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d) - 36*b^2*c^2
*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d) + 18*b^2*c^2*i
mag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d) - 18*b^
2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)
 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d) -
 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2 + 6
*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2 + 6*b^2*c*
d*x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2 - 18*b^2*c*d*x*r
eal_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 12*b*c*d*tan(3/
2*b*x)^2*tan(1/2*b*x)*tan(3/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)*tan(1/2*b*x)
^2*tan(3/2*b*c/d)^2 - 8*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2
 + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*
c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*ta
n(3/2*b*c/d)^2 + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(
3/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 - 12*b*c
*d*tan(1/2*b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 - 8*d^2*tan(3/2*b*x)*tan(1/2*
b*x)^2*tan(3/2*a)*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)*tan(3/2*a)^2*tan
(3/2*b*c/d)^2 + 8*d^2*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 12*b*c
*d*tan(1/2*b*x)*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 8*d^2*tan(1/2*b*x)^2*tan(3/
2*a)^2*tan(3/2*b*c/d)^2 - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*ta
n(1/2*a)*tan(3/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))
*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan
(1/2*a)*tan(3/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d
)^2 - 24*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b
*c*d*tan(1/2*b*x)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 12*b*c*d*tan(3/2*a)^2*tan
(1/2*a)*tan(3/2*b*c/d)^2 - 24*d^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a)*tan(
3/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 8*d^2*
tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 12*b*c*d*tan(1/2*b*x)*tan(1/
2*a)^2*tan(3/2*b*c/d)^2 + 8*d^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^
2 - 12*b*c*d*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 8*d^2*tan(3/2*b*x)*
tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 8*d^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1
```

```
/2*b*c/d) - 6*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c
/d) + 12*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d) + 6*b^2*c
^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d) - 6*b
^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*c/d)
+ 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*c/d) +
6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*b*c/d
) - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
b*c/d) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*b*
c/d) + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*
b*c/d) - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1
/2*b*c/d) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b
*c/d) + 24*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/
2*b*c/d) + 24*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*ta
n(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*
tan(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)
^2*tan(1/2*b*c/d) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*t
an(1/2*b*c/d) + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*
b*c/d))*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))
*tan(1/2*b*c/d)^2 - 6*b^2*c*d*x*real_part(cos_integral(-b*x - b*c/d))*tan(1
/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/
2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/2*b*c/d)^2 - 12*b*c
*d*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 - 8*d^2*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_integral(-3*b
*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 - 36*b^2*c^2*sin_integral(3*(b*d
*x + b*c)/d)*tan(3/2*a)*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*
a)*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 -
 8*d^2*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a)*tan(1/2*b*c/d)^2 + 12*b*c*d*t
an(3/2*b*x)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 8*d^2*tan(3/2*b*x)^2*tan(3/2*a)
^2*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 -
 8*d^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^2*c^2*imag_part(c
os_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 6*b^2*c^2*imag_part
(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 12*b^2*c^2*sin_i
ntegral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x
)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 24*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)*tan(1/
2*a)*tan(1/2*b*c/d)^2 + 12*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2
 - 12*b*c*d*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 24*d^2*tan(1/2*b*x)*
tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(3/2*b*x)*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 - 8*d^2*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 +
12*b*c*d*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 8*d^2*tan(1/2*b*x)^2*
tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 - 8*d^2*tan(3/2*b*x)*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 8*d^
```

```
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 18*b^2*c^2*imag_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 18*b^2*c^2*imag_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 36*b^
2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 12*
b*c*d*tan(3/2*b*x)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*d^2*tan(3/2*b*x)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(1/2*b*x)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 - 4*d^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
- 12*b*c*d*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 8*d^2*tan(3/2*b*x
)*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*d^2*tan(3/2*a)^2*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 - 24*d^2*tan(1/2*b*x)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
- 4*d^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real
_part(cos_integral(3*b*x + 3*b*c/d)) + 3*b^2*d^2*x^2*real_part(cos_integral
(b*x + b*c/d)) + 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d)) + 9*b^
2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d)) + 9*b^2*c^2*real_part(c
os_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2 + 3*b^2*c^2*real_part(cos_inte
gral(b*x + b*c/d))*tan(3/2*b*x)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*b*x)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(3/2*b*x)^2 - 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*b*x) + 9*b^2*c^2*real
_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*b*x)^2 + 3*b^2*c^2*real_part(c
os_integral(b*x + b*c/d))*tan(1/2*b*x)^2 + 3*b^2*c^2*real_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x -
3*b*c/d))*tan(1/2*b*x)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*x)^2 - 36*b^2*
c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 36*b^2*c*d*x*im
ag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a) - 72*b^2*c*d*x*sin_integ
ral(3*(b*d*x + b*c)/d)*tan(3/2*a) + 12*b*d^2*x*tan(3/2*b*x)^2*tan(3/2*a) -
12*b*d^2*x*tan(1/2*b*x)^2*tan(3/2*a) - 9*b^2*c^2*real_part(cos_integral(3*b
*x + 3*b*c/d))*tan(3/2*a)^2 + 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d)
)*tan(3/2*a)^2 + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)
^2 - 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2 + 12*
b*d^2*x*tan(3/2*b*x)*tan(3/2*a)^2 - 12*b*d^2*x*tan(1/2*b*x)*tan(3/2*a)^2 -
12*b^2*c*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 12*b^2*c*d*x
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 24*b^2*c*d*x*sin_integr
al((b*d*x + b*c)/d)*tan(1/2*a) - 12*b*d^2*x*tan(3/2*b*x)^2*tan(1/2*a) + 12*
b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a) - 12*b*d^2*x*tan(3/2*a)^2*tan(1/2*a) + 9*
b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2 - 3*b^2*c^2*r
eal_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2 - 3*b^2*c^2*real_part(cos_
integral(-b*x - b*c/d))*tan(1/2*a)^2 + 9*b^2*c^2*real_part(cos_integral(-3*
b*x - 3*b*c/d))*tan(1/2*a)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*a)^2 + 12*b*
d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 - 12*b*d^2*x*tan(3/2*a)*tan(1/2*a)^2 + 36*b
^2*c*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 36*b^2*c
*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) + 72*b^2*c*d*
x*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d) + 36*b^2*c^2*real_part(cos
_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 36*b^2*c^2*real_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - 9*b^2*c^2*rea
```

```
l_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2 + 3*b^2*c^2*real_par
t(cos_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2 + 3*b^2*c^2*real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*b*c/d)^2 - 9*b^2*c^2*real_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*b*x)*tan(3/2*b*c/d)^
2 - 12*b*d^2*x*tan(1/2*b*x)*tan(3/2*b*c/d)^2 - 12*b*d^2*x*tan(3/2*a)*tan(3/
2*b*c/d)^2 - 12*b*d^2*x*tan(1/2*a)*tan(3/2*b*c/d)^2 + 12*b^2*c*d*x*imag_par
t(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 12*b^2*c*d*x*imag_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*b*c/d) + 24*b^2*c*d*x*sin_integral((b*d*x + b
*c)/d)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan
(1/2*a)*tan(1/2*b*c/d) + 12*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*a)*tan(1/2*b*c/d) + 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d
))*tan(1/2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(b*x + b*c/d))*tan(1/
2*b*c/d)^2 - 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)
^2 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 -
 12*b*d^2*x*tan(3/2*b*x)*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(1/2*b*x)*tan(1/2
*b*c/d)^2 - 12*b*d^2*x*tan(3/2*a)*tan(1/2*b*c/d)^2 - 12*b*d^2*x*tan(1/2*a)*
tan(1/2*b*c/d)^2 + 18*b^2*c*d*x*real_part(cos_integral(3*b*x + 3*b*c/d)) +
6*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d)) + 6*b^2*c*d*x*real_part(co
s_integral(-b*x - b*c/d)) + 18*b^2*c*d*x*real_part(cos_integral(-3*b*x - 3*
b*c/d)) - 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*b*x) - 12*b*c*d*tan(3/2*b*x)*tan(
1/2*b*x)^2 - 8*d^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2 - 18*b^2*c^2*imag_part(cos
_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 18*b^2*c^2*imag_part(cos_integral(
-3*b*x - 3*b*c/d))*tan(3/2*a) - 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*
tan(3/2*a) + 12*b*c*d*tan(3/2*b*x)^2*tan(3/2*a) - 12*b*c*d*tan(1/2*b*x)^2*t
an(3/2*a) - 8*d^2*tan(3/2*b*x)*tan(1/2*b*x)^2*tan(3/2*a) + 12*b*c*d*tan(3/2
*b*x)*tan(3/2*a)^2 + 8*d^2*tan(3/2*b*x)^2*tan(3/2*a)^2 - 12*b*c*d*tan(1/2*b
*x)*tan(3/2*a)^2 - 8*d^2*tan(1/2*b*x)^2*tan(3/2*a)^2 - 6*b^2*c^2*imag_part(
cos_integral(b*x + b*c/d))*tan(1/2*a) + 6*b^2*c^2*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a) - 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*
a) - 12*b*c*d*tan(3/2*b*x)^2*tan(1/2*a) - 24*d^2*tan(3/2*b*x)^2*tan(1/2*b*x
)*tan(1/2*a) + 12*b*c*d*tan(1/2*b*x)^2*tan(1/2*a) - 12*b*c*d*tan(3/2*a)^2*t
an(1/2*a) - 24*d^2*tan(1/2*b*x)*tan(3/2*a)^2*tan(1/2*a) - 12*b*c*d*tan(3/2*
b*x)*tan(1/2*a)^2 - 8*d^2*tan(3/2*b*x)^2*tan(1/2*a)^2 + 12*b*c*d*tan(1/2*b*
x)*tan(1/2*a)^2 + 8*d^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 12*b*c*d*tan(3/2*a)*t
an(1/2*a)^2 - 8*d^2*tan(3/2*b*x)*tan(3/2*a)*tan(1/2*a)^2 - 8*d^2*tan(3/2*a)
^2*tan(1/2*a)^2 + 18*b^2*c^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*b*c/d) - 18*b^2*c^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*
c/d) + 36*b^2*c^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d) - 12*b*c*d
*tan(3/2*b*x)*tan(3/2*b*c/d)^2 + 4*d^2*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2 - 12
*b*c*d*tan(1/2*b*x)*tan(3/2*b*c/d)^2 - 4*d^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^
2 - 12*b*c*d*tan(3/2*a)*tan(3/2*b*c/d)^2 - 8*d^2*tan(3/2*b*x)*tan(3/2*a)*ta
n(3/2*b*c/d)^2 + 4*d^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - 12*b*c*d*tan(1/2*a)*
tan(3/2*b*c/d)^2 - 24*d^2*tan(1/2*b*x)*tan(1/2*a)*tan(3/2*b*c/d)^2 - 4*d^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 6*b^2*c^2*imag_part(cos_integral(b*x + b*c/
d))*tan(1/2*b*c/d) - 6*b^2*c^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/
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2*b*c/d) + 12*b^2*c^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*c/d) - 12*b*c
*d*tan(3/2*b*x)*tan(1/2*b*c/d)^2 + 4*d^2*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 -
12*b*c*d*tan(1/2*b*x)*tan(1/2*b*c/d)^2 - 4*d^2*tan(1/2*b*x)^2*tan(1/2*b*c/d
)^2 - 12*b*c*d*tan(3/2*a)*tan(1/2*b*c/d)^2 - 8*d^2*tan(3/2*b*x)*tan(3/2*a)*
tan(1/2*b*c/d)^2 + 4*d^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 12*b*c*d*tan(1/2*a
)*tan(1/2*b*c/d)^2 - 24*d^2*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*b*c/d)^2 - 4*d^
2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 8*d^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 9*b^2*c^2*real_part(cos_integral(3*b*x + 3*b*c/d)) + 3*b^2*c^2*real_part(c
os_integral(b*x + b*c/d)) + 3*b^2*c^2*real_part(cos_integral(-b*x - b*c/d))
 + 9*b^2*c^2*real_part(cos_integral(-3*b*x - 3*b*c/d)) - 12*b*d^2*x*tan(3/2
*b*x) - 12*b*d^2*x*tan(1/2*b*x) - 12*b*d^2*x*tan(3/2*a) - 12*b*d^2*x*tan(1/
2*a) - 12*b*c*d*tan(3/2*b*x) + 4*d^2*tan(3/2*b*x)^2 - 12*b*c*d*tan(1/2*b*x)
 - 4*d^2*tan(1/2*b*x)^2 - 12*b*c*d*tan(3/2*a) - 8*d^2*tan(3/2*b*x)*tan(3/2*
a) + 4*d^2*tan(3/2*a)^2 - 12*b*c*d*tan(1/2*a) - 24*d^2*tan(1/2*b*x)*tan(1/2
*a) - 4*d^2*tan(1/2*a)^2 + 8*d^2*tan(3/2*b*c/d)^2 + 8*d^2*tan(1/2*b*c/d)^2
+ 8*d^2)/(d^5*x^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x
^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + d^5*x^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c
*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^
2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + d^5*x^2*tan(3/2
*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/
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d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x)^2*tan(3/2*
a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2
*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(
1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1
/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + 2*c*d^4*x*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 + d^5*x^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*a)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + c^2*d^3*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + d^5*x^2*tan(3/2*b
*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(3/2*a)^2
*tan(3/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 +
c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x
^2*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2
 + c^2*d^3*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + c^2*
d^3*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(3/
2*a)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d
)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 +
 d^5*x^2*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x
)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(1/2*a)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*
c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2
+ c^2*d^3*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^5*x
^2*tan(3/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^
2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2
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*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d)^2 + d^5*x^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d
^3*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*
tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(1/2*a)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + 2*c
*d^4*x*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*
tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(3/2*a)^2*tan
(3/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*
c*d^4*x*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*
x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 +
 2*c*d^4*x*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^
2 + 2*c*d^4*x*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/
2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d
^5*x^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(3/2*a)^2
+ d^5*x^2*tan(1/2*b*x)^2*tan(3/2*a)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(3/2*a)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1/2*b*x
)^2*tan(1/2*a)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + d^5
*x^2*tan(3/2*a)^2*tan(1/2*a)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)^2 + c^2*d^3*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2 + d^5*x^2*tan(3/2
*b*x)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + c^2*d^
3*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(3/2*a)^2*tan
(3/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + c^2*
d^3*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + c^2*
d^3*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + c^2*d^3*tan(3/2*a)^2*tan
(1/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x^2*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 + d^5*
x^2*tan(1/2*b*x)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(1/2*b*c/d)^2 + d^5*x^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x)^2*tan(3/2*a)^2
*tan(1/2*b*c/d)^2 + d^5*x^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2
*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x)^2*tan(1/2*a)^2
*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d^
5*x^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(3/2*b*
c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2*tan(1/2*b
*c/d)^2 + c^2*d^3*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + c^2*d^3*
tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*t
```

```
an(1/2*b*x)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(3/2*a)^2 + 2*c*d^4*x*tan(1/2*b
*x)^2*tan(3/2*a)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*a)^2 + 2*c*d^4*x*tan(
1/2*b*x)^2*tan(1/2*a)^2 + 2*c*d^4*x*tan(3/2*a)^2*tan(1/2*a)^2 + 2*c*d^4*x*t
an(3/2*b*x)^2*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2
+ 2*c*d^4*x*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*a)^2*tan(3/2*
b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(1/2*b*
x)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x
*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*c*d^4*x*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + d^5*x^2*tan(3/2*b*x)^2 + d^5*x^2*tan(1/2*b*x)^2 + c^2*d^3*tan(3/2*b*x)^
2*tan(1/2*b*x)^2 + d^5*x^2*tan(3/2*a)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(3/2*a)
^2 + c^2*d^3*tan(1/2*b*x)^2*tan(3/2*a)^2 + d^5*x^2*tan(1/2*a)^2 + c^2*d^3*t
an(3/2*b*x)^2*tan(1/2*a)^2 + c^2*d^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + c^2*d^3*
tan(3/2*a)^2*tan(1/2*a)^2 + d^5*x^2*tan(3/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)
^2*tan(3/2*b*c/d)^2 + c^2*d^3*tan(1/2*b*x)^2*tan(3/2*b*c/d)^2 + c^2*d^3*tan
(3/2*a)^2*tan(3/2*b*c/d)^2 + c^2*d^3*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d^5*x^
2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*x)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(
1/2*b*x)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d
^3*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + c^2*d^3*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + 2*c*d^4*x*tan(3/2*b*x)^2 + 2*c*d^4*x*tan(1/2*b*x)^2 + 2*c*d^4*x*tan(3/2
*a)^2 + 2*c*d^4*x*tan(1/2*a)^2 + 2*c*d^4*x*tan(3/2*b*c/d)^2 + 2*c*d^4*x*tan
(1/2*b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(3/2*b*x)^2 + c^2*d^3*tan(1/2*b*x)^2 +
 c^2*d^3*tan(3/2*a)^2 + c^2*d^3*tan(1/2*a)^2 + c^2*d^3*tan(3/2*b*c/d)^2 + c
^2*d^3*tan(1/2*b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(c+d\,x\right)^3}\,dx$$

```
[In] int(cos(a + b*x)^3/(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)^3/(c + d*x)^3, x)
```

3.23 $\int x^3 \cos^4(a+bx)\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 172

$$\begin{aligned}\int x^3 \cos^4(a+bx)\,dx = &-\frac{45x^2}{128b^2}+\frac{3x^4}{32}-\frac{45\cos^2(a+bx)}{128b^4}+\frac{9x^2\cos^2(a+bx)}{16b^2}\\&-\frac{3\cos^4(a+bx)}{128b^4}+\frac{3x^2\cos^4(a+bx)}{16b^2}\\&-\frac{45x\cos(a+bx)\sin(a+bx)}{64b^3}+\frac{3x^3\cos(a+bx)\sin(a+bx)}{8b}\\&-\frac{3x\cos^3(a+bx)\sin(a+bx)}{32b^3}+\frac{x^3\cos^3(a+bx)\sin(a+bx)}{4b}\end{aligned}$$

```
[Out] -45/128*x^2/b^2+3/32*x^4-45/128*cos(b*x+a)^2/b^4+9/16*x^2*cos(b*x+a)^2/b^2-
3/128*cos(b*x+a)^4/b^4+3/16*x^2*cos(b*x+a)^4/b^2-45/64*x*cos(b*x+a)*sin(b*x
+a)/b^3+3/8*x^3*cos(b*x+a)*sin(b*x+a)/b-3/32*x*cos(b*x+a)^3*sin(b*x+a)/b^3+
1/4*x^3*cos(b*x+a)^3*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 30, 3391}

$$\begin{aligned}\int x^3 \cos^4(a+bx)\,dx = &-\frac{3\cos^4(a+bx)}{128b^4}-\frac{45\cos^2(a+bx)}{128b^4}-\frac{3x\sin(a+bx)\cos^3(a+bx)}{32b^3}\\&-\frac{45x\sin(a+bx)\cos(a+bx)}{64b^3}+\frac{3x^2\cos^4(a+bx)}{16b^2}\\&+\frac{9x^2\cos^2(a+bx)}{16b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}\\&+\frac{3x^3\sin(a+bx)\cos(a+bx)}{8b}-\frac{45x^2}{128b^2}+\frac{3x^4}{32}\end{aligned}$$

```
[In] Int[x^3*Cos[a + b*x]^4,x]
```

```
[Out] (-45*x^2)/(128*b^2) + (3*x^4)/32 - (45*Cos[a + b*x]^2)/(128*b^4) + (9*x^2*C
os[a + b*x]^2)/(16*b^2) - (3*Cos[a + b*x]^4)/(128*b^4) + (3*x^2*Cos[a + b*x
]^4)/(16*b^2) - (45*x*Cos[a + b*x]*Sin[a + b*x])/(64*b^3) + (3*x^3*Cos[a +
b*x]*Sin[a + b*x])/(8*b) - (3*x*Cos[a + b*x]^3*Sin[a + b*x])/(32*b^3) + (x^
3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3x^2\cos^4(a+bx)}{16b^2} + \frac{x^3\cos^3(a+bx)\sin(a+bx)}{4b} \\ &\quad + \frac{3}{4}\int x^3\cos^2(a+bx)\,dx - \frac{3\int x\cos^4(a+bx)\,dx}{8b^2} \\ &= \frac{9x^2\cos^2(a+bx)}{16b^2} - \frac{3\cos^4(a+bx)}{128b^4} + \frac{3x^2\cos^4(a+bx)}{16b^2} + \frac{3x^3\cos(a+bx)\sin(a+bx)}{8b} \\ &\quad - \frac{3x\cos^3(a+bx)\sin(a+bx)}{32b^3} + \frac{x^3\cos^3(a+bx)\sin(a+bx)}{4b} \\ &\quad + \frac{3\int x^3\,dx}{8} - \frac{9\int x\cos^2(a+bx)\,dx}{32b^2} - \frac{9\int x\cos^2(a+bx)\,dx}{8b^2}\end{aligned}$$

$$
\begin{aligned}
&= \frac{3x^4}{32} - \frac{45\cos^2(a+bx)}{128b^4} + \frac{9x^2\cos^2(a+bx)}{16b^2} - \frac{3\cos^4(a+bx)}{128b^4} + \frac{3x^2\cos^4(a+bx)}{16b^2} \\
&\quad - \frac{45x\cos(a+bx)\sin(a+bx)}{64b^3} + \frac{3x^3\cos(a+bx)\sin(a+bx)}{8b} \\
&\quad - \frac{3x\cos^3(a+bx)\sin(a+bx)}{32b^3} + \frac{x^3\cos^3(a+bx)\sin(a+bx)}{4b} - \frac{9\int x\,dx}{64b^2} - \frac{9\int x\,dx}{16b^2} \\
&= -\frac{45x^2}{128b^2} + \frac{3x^4}{32} - \frac{45\cos^2(a+bx)}{128b^4} + \frac{9x^2\cos^2(a+bx)}{16b^2} - \frac{3\cos^4(a+bx)}{128b^4} \\
&\quad + \frac{3x^2\cos^4(a+bx)}{16b^2} - \frac{45x\cos(a+bx)\sin(a+bx)}{64b^3} + \frac{3x^3\cos(a+bx)\sin(a+bx)}{8b} \\
&\quad - \frac{3x\cos^3(a+bx)\sin(a+bx)}{32b^3} + \frac{x^3\cos^3(a+bx)\sin(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.58

$$
\begin{aligned}
&\int x^3\cos^4(a+bx)\,dx \\
&= \frac{192(-1+2b^2x^2)\cos(2(a+bx)) + 3(-1+8b^2x^2)\cos(4(a+bx)) + 4bx(24b^3x^3 + 32(-3+2b^2x^2)\sin(2(a+
\end{aligned}
$$

```
[In] Integrate[x^3*Cos[a + b*x]^4,x]
```

```
[Out] (192*(-1 + 2*b^2*x^2)*Cos[2*(a + b*x)] + 3*(-1 + 8*b^2*x^2)*Cos[4*(a + b*x)
] + 4*b*x*(24*b^3*x^3 + 32*(-3 + 2*b^2*x^2)*Sin[2*(a + b*x)] + (-3 + 8*b^2*
x^2)*Sin[4*(a + b*x)]))/(1024*b^4)
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{(384x^2b^2-192)\cos(2bx+2a)+(24x^2b^2-3)\cos(4bx+4a)+(256x^3b^3-384bx)\sin(2bx+2a)+(32x^3b^3-12bx)\sin(4bx+4a)}{1024b^4}$
risch	$\frac{3x^4}{32} + \frac{3(8x^2b^2-1)\cos(4bx+4a)}{1024b^4} + \frac{x(8x^2b^2-3)\sin(4bx+4a)}{256b^3} + \frac{3(2x^2b^2-1)\cos(2bx+2a)}{16b^4} + \frac{x(2x^2b^2-3)\sin(2bx-}{8b^3}$
derivativedivides	$\frac{-a^3\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)+3a^2\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)\right.}{}$
default	$\frac{-a^3\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)+3a^2\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)\right.}{}$

```
[In] int(x^3*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1024*((384*b^2*x^2-192)*cos(2*b*x+2*a)+(24*b^2*x^2-3)*cos(4*b*x+4*a)+(256
*b^3*x^3-384*b*x)*sin(2*b*x+2*a)+(32*b^3*x^3-12*b*x)*sin(4*b*x+4*a)+96*x^4*
b^4+195)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

$$\int x^3 \cos^4(a+bx)\,dx$$
$$= \frac{12\,b^4x^4 + 3\,(8\,b^2x^2-1)\cos(bx+a)^4 - 45\,b^2x^2 + 9\,(8\,b^2x^2-5)\cos(bx+a)^2 + 2\,\left(2\,(8\,b^3x^3-3\,bx)\cos(bx+\right.}{128\,b^4}$$

```
[In] integrate(x^3*cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(12*b^4*x^4 + 3*(8*b^2*x^2 - 1)*cos(b*x + a)^4 - 45*b^2*x^2 + 9*(8*b^
2*x^2 - 5)*cos(b*x + a)^2 + 2*(2*(8*b^3*x^3 - 3*b*x)*cos(b*x + a)^3 + 3*(8*
b^3*x^3 - 15*b*x)*cos(b*x + a))*sin(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.47

$$\int x^3 \cos^4(a+bx)\,dx$$
$$= \begin{cases} \frac{3x^4\sin^4(a+bx)}{32} + \frac{3x^4\sin^2(a+bx)\cos^2(a+bx)}{16} + \frac{3x^4\cos^4(a+bx)}{32} + \frac{3x^3\sin^3(a+bx)\cos(a+bx)}{8b} + \frac{5x^3\sin(a+bx)\cos^3(a+bx)}{8b} - \frac{45x^2\,si}{1} \\ \frac{x^4\cos^4(a)}{4} \end{cases}$$

```
[In] integrate(x**3*cos(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**4*sin(a + b*x)**4/32 + 3*x**4*sin(a + b*x)**2*cos(a + b*x)*
*2/16 + 3*x**4*cos(a + b*x)**4/32 + 3*x**3*sin(a + b*x)**3*cos(a + b*x)/(8*
b) + 5*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 45*x**2*sin(a + b*x)**4/(1
28*b**2) - 9*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**2) + 51*x**2*cos(a
 + b*x)**4/(128*b**2) - 45*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) - 51*x*
sin(a + b*x)*cos(a + b*x)**3/(64*b**3) + 45*sin(a + b*x)**4/(256*b**4) - 51
*cos(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*cos(a)**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.76

$$\int x^3 \cos^4(a+bx)\,dx$$
$$= \frac{96\,(bx+a)^4 - 32\,(12\,bx + 12\,a + \sin(4\,bx+4\,a) + 8\,\sin(2\,bx+2\,a))a^3 + 24\,\left(24\,(bx+a)^2 + 4\,(bx+a)\,\sin\right.}{}$$

```
[In] integrate(x^3*cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/1024*(96*(b*x + a)^4 - 32*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x
 + 2*a))*a^3 + 24*(24*(b*x + a)^2 + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x
+ a)*sin(2*b*x + 2*a) + cos(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))*a^2 - 12*(3
2*(b*x + a)^3 + 4*(b*x + a)*cos(4*b*x + 4*a) + 64*(b*x + a)*cos(2*b*x + 2*a
) + (8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 32*(2*(b*x + a)^2 - 1)*sin(2*b*x
 + 2*a))*a + 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 192*(2*(b*x + a)^2 -
1)*cos(2*b*x + 2*a) + 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) + 12
8*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.63

$$\int x^3 \cos^4(a+bx)\,dx = \frac{3}{32}\,x^4 + \frac{3\,(8\,b^2x^2-1)\cos(4\,bx+4\,a)}{1024\,b^4} + \frac{3\,(2\,b^2x^2-1)\cos(2\,bx+2\,a)}{16\,b^4} + \frac{(8\,b^3x^3-3\,bx)\sin(4\,bx+4\,a)}{256\,b^4} + \frac{(2\,b^3x^3-3\,bx)\sin(2\,bx+2\,a)}{8\,b^4}$$

```
[In] integrate(x^3*cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/32*x^4 + 3/1024*(8*b^2*x^2 - 1)*cos(4*b*x + 4*a)/b^4 + 3/16*(2*b^2*x^2 -
1)*cos(2*b*x + 2*a)/b^4 + 1/256*(8*b^3*x^3 - 3*b*x)*sin(4*b*x + 4*a)/b^4 +
1/8*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.80

$$\int x^3 \cos^4(a+bx)\,dx$$

$$= \frac{\frac{3\sin(2\,a+2\,b\,x)^2}{512} - b^2\left(\frac{3\,x^2\left(2\sin(2\,a+2\,b\,x)^2-1\right)}{128} + \frac{3\,x^2\left(2\sin(a+b\,x)^2-1\right)}{8}\right) - b\left(\frac{3\,x\,\sin(2\,a+2\,b\,x)}{8} + \frac{3\,x\,\sin(4\,a+4\,b\,x)}{256}\right) + b^3\left(\frac{x^3}{}\right.}{b^4}$$

$$+ \frac{3\,x^4}{32}$$

```
[In] int(x^3*cos(a + b*x)^4,x)
```

```
[Out] ((3*sin(2*a + 2*b*x)^2)/512 - b^2*((3*x^2*(2*sin(2*a + 2*b*x)^2 - 1))/128 +
 (3*x^2*(2*sin(a + b*x)^2 - 1))/8) - b*((3*x*sin(2*a + 2*b*x))/8 + (3*x*sin
(4*a + 4*b*x))/256) + b^3*((x^3*sin(2*a + 2*b*x))/4 + (x^3*sin(4*a + 4*b*x)
)/32) + (3*sin(a + b*x)^2)/8)/b^4 + (3*x^4)/32
```

3.24 $\int x^2 \cos^4(a+bx)\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 134

$$\int x^2 \cos^4(a+bx)\,dx = -\frac{15x}{64b^2} + \frac{x^3}{8} + \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} - \frac{15\cos(a+bx)\sin(a+bx)}{64b^3} + \frac{3x^2\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin(a+bx)}{32b^3} + \frac{x^2\cos^3(a+bx)\sin(a+bx)}{4b}$$

```
[Out] -15/64*x/b^2+1/8*x^3+3/8*x*cos(b*x+a)^2/b^2+1/8*x*cos(b*x+a)^4/b^2-15/64*co
s(b*x+a)*sin(b*x+a)/b^3+3/8*x^2*cos(b*x+a)*sin(b*x+a)/b-1/32*cos(b*x+a)^3*s
in(b*x+a)/b^3+1/4*x^2*cos(b*x+a)^3*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3392, 30, 2715, 8}

$$\int x^2 \cos^4(a+bx)\,dx = -\frac{\sin(a+bx)\cos^3(a+bx)}{32b^3} - \frac{15\sin(a+bx)\cos(a+bx)}{64b^3} + \frac{x\cos^4(a+bx)}{8b^2} + \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b} + \frac{3x^2\sin(a+bx)\cos(a+bx)}{8b} - \frac{15x}{64b^2} + \frac{x^3}{8}$$

```
[In] Int[x^2*Cos[a + b*x]^4,x]
```

```
[Out] (-15*x)/(64*b^2) + x^3/8 + (3*x*Cos[a + b*x]^2)/(8*b^2) + (x*Cos[a + b*x]^4
)/(8*b^2) - (15*Cos[a + b*x]*Sin[a + b*x])/(64*b^3) + (3*x^2*Cos[a + b*x]*S
```

```
in[a + b*x])/(8*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(32*b^3) + (x^2*Cos[a +
b*x]^3*Sin[a + b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\cos^4(a+bx)}{8b^2} + \frac{x^2\cos^3(a+bx)\sin(a+bx)}{4b} \\
&\quad + \frac{3}{4}\int x^2\cos^2(a+bx)\,dx - \frac{\int \cos^4(a+bx)\,dx}{8b^2} \\
&= \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} + \frac{3x^2\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin(a+bx)}{32b^3} \\
&\quad + \frac{x^2\cos^3(a+bx)\sin(a+bx)}{4b} + \frac{3\int x^2\,dx}{8} - \frac{3\int \cos^2(a+bx)\,dx}{32b^2} - \frac{3\int \cos^2(a+bx)\,dx}{8b^2} \\
&= \frac{x^3}{8} + \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} - \frac{15\cos(a+bx)\sin(a+bx)}{64b^3} \\
&\quad + \frac{3x^2\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin(a+bx)}{32b^3} \\
&\quad + \frac{x^2\cos^3(a+bx)\sin(a+bx)}{4b} - \frac{3\int 1\,dx}{64b^2} - \frac{3\int 1\,dx}{16b^2}
\end{aligned}$$

$$= -\frac{15x}{64b^2} + \frac{x^3}{8} + \frac{3x\cos^2(a+bx)}{8b^2} + \frac{x\cos^4(a+bx)}{8b^2} - \frac{15\cos(a+bx)\sin(a+bx)}{64b^3}$$
$$+ \frac{3x^2\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin(a+bx)}{32b^3} + \frac{x^2\cos^3(a+bx)\sin(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

$$\int x^2\cos^4(a+bx)\,dx$$
$$= \frac{32b^3x^3 + 64bx\cos(2(a+bx)) + 4bx\cos(4(a+bx)) - 32\sin(2(a+bx)) + 64b^2x^2\sin(2(a+bx)) - \sin(4(a$$

```
[In] Integrate[x^2*Cos[a + b*x]^4,x]
```

```
[Out] (32*b^3*x^3 + 64*b*x*Cos[2*(a + b*x)] + 4*b*x*Cos[4*(a + b*x)] - 32*Sin[2*(
a + b*x)] + 64*b^2*x^2*Sin[2*(a + b*x)] - Sin[4*(a + b*x)] + 8*b^2*x^2*Sin[
4*(a + b*x)])/(256*b^3)
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{(64x^2b^2-32)\sin(2bx+2a)+(8x^2b^2-1)\sin(4bx+4a)+32b\left(x^2b^2+2\cos(2bx+2a)+\frac{\cos(4bx+4a)}{8}\right)x}{256b^3}$
risch	$\frac{x^3}{8} + \frac{x\cos(4bx+4a)}{64b^2} + \frac{(8x^2b^2-1)\sin(4bx+4a)}{256b^3} + \frac{x\cos(2bx+2a)}{4b^2} + \frac{(2x^2b^2-1)\sin(2bx+2a)}{8b^3}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-2a\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-3$
default	$\frac{a^2\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-2a\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-3$
norman	$\frac{x^3}{8}-\frac{17\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{32b^3}-\frac{9\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b^3}+\frac{9\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b^3}+\frac{17\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b^3}+\frac{17x}{64b^2}+\frac{x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{3x^3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}$

```
[In] int(x^2*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/256*((64*b^2*x^2-32)*sin(2*b*x+2*a)+(8*b^2*x^2-1)*sin(4*b*x+4*a)+32*b*(x^
2*b^2+2*cos(2*b*x+2*a)+1/8*cos(4*b*x+4*a))*x)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int x^2 \cos^4(a+bx)\,dx$$
$$= \frac{8\,b^3x^3 + 8\,bx\cos\left(bx+a\right)^4 + 24\,bx\cos\left(bx+a\right)^2 - 15\,bx + \left(2\left(8\,b^2x^2-1\right)\cos\left(bx+a\right)^3 + 3\left(8\,b^2x^2-5\right)\cos\right.}{64\,b^3}$$

```
[In] integrate(x^2*cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(8*b^3*x^3 + 8*b*x*cos(b*x + a)^4 + 24*b*x*cos(b*x + a)^2 - 15*b*x + (
2*(8*b^2*x^2 - 1)*cos(b*x + a)^3 + 3*(8*b^2*x^2 - 5)*cos(b*x + a))*sin(b*x
+ a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.56

$$\int x^2 \cos^4(a+bx)\,dx$$
$$= \begin{cases} \frac{x^3\sin^4\left(a+bx\right)}{8} + \frac{x^3\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{4} + \frac{x^3\cos^4\left(a+bx\right)}{8} + \frac{3x^2\sin^3\left(a+bx\right)\cos\left(a+bx\right)}{8b} + \frac{5x^2\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{8b} - \frac{15x\sin^4\left(a\right.}{64b^2} \\ \frac{x^3\cos^4\left(a\right)}{3} \end{cases}$$

```
[In] integrate(x**2*cos(b*x+a)**4,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**4/8 + x**3*sin(a + b*x)**2*cos(a + b*x)**2/4
+ x**3*cos(a + b*x)**4/8 + 3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 5*x*
*2*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 15*x*sin(a + b*x)**4/(64*b**2) - 3*
x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) + 17*x*cos(a + b*x)**4/(64*b**2
) - 15*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) - 17*sin(a + b*x)*cos(a + b*x
)**3/(64*b**3), Ne(b, 0)), (x**3*cos(a)**4/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.40

$$\int x^2 \cos^4(a+bx)\,dx$$
$$= \frac{32\left(bx+a\right)^3 + 8\left(12\,bx + 12\,a + \sin\left(4\,bx+4\,a\right) + 8\,\sin\left(2\,bx+2\,a\right)\right)a^2 - 4\left(24\left(bx+a\right)^2 + 4\left(bx+a\right)\sin\left(4\right.\right.}{}$$

```
[In] integrate(x^2*cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/256*(32*(b*x + a)^3 + 8*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x +
 2*a))*a^2 - 4*(24*(b*x + a)^2 + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a
)*sin(2*b*x + 2*a) + cos(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))*a + 4*(b*x + a
)*cos(4*b*x + 4*a) + 64*(b*x + a)*cos(2*b*x + 2*a) + (8*(b*x + a)^2 - 1)*si
n(4*b*x + 4*a) + 32*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

$$\int x^2 \cos^4(a+bx)\,dx = \frac{1}{8}x^3 + \frac{x\cos(4\,bx+4\,a)}{64\,b^2} + \frac{x\cos(2\,bx+2\,a)}{4\,b^2} + \frac{(8\,b^2x^2-1)\sin(4\,bx+4\,a)}{256\,b^3} + \frac{(2\,b^2x^2-1)\sin(2\,bx+2\,a)}{8\,b^3}$$

```
[In] integrate(x^2*cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/8*x^3 + 1/64*x*cos(4*b*x + 4*a)/b^2 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/256*
(8*b^2*x^2 - 1)*sin(4*b*x + 4*a)/b^3 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)
/b^3
```

Mupad [B] (verification not implemented)

Time = 14.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.78

$$\int x^2 \cos^4(a+bx)\,dx = \frac{x^3}{8} - \frac{\frac{\sin(2\,a+2\,b\,x)}{8} + \frac{\sin(4\,a+4\,b\,x)}{256} + b\left(\frac{x\left(2\sin(a+b\,x)^2-1\right)}{4} + \frac{x\left(2\sin(2\,a+2\,b\,x)^2-1\right)}{64}\right) - b^2\left(\frac{x^2\sin(2\,a+2\,b\,x)}{4} + \frac{x^2\sin(4\,a+4\,b\,x)}{32}\right)}{b^3}$$

```
[In] int(x^2*cos(a + b*x)^4,x)
```

```
[Out] x^3/8 - (sin(2*a + 2*b*x)/8 + sin(4*a + 4*b*x)/256 + b*((x*(2*sin(a + b*x)^
2 - 1))/4 + (x*(2*sin(2*a + 2*b*x)^2 - 1))/64) - b^2*((x^2*sin(2*a + 2*b*x)
)/4 + (x^2*sin(4*a + 4*b*x))/32))/b^3
```

3.25 $\int x \cos^4(a+bx)\,dx$

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Mathematica [A] (verified) . . . 297
Maple [A] (verified) . . . 297
Fricas [A] (verification not implemented) . . . 298
Sympy [A] (verification not implemented) . . . 298
Maxima [A] (verification not implemented) . . . 299
Giac [A] (verification not implemented) . . . 299
Mupad [B] (verification not implemented) . . . 300

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x\cos^4(a+bx)\,dx = \frac{3x^2}{16} + \frac{3\cos^2(a+bx)}{16b^2} + \frac{\cos^4(a+bx)}{16b^2} + \frac{3x\cos(a+bx)\sin(a+bx)}{8b} + \frac{x\cos^3(a+bx)\sin(a+bx)}{4b}$$

```
[Out] 3/16*x^2+3/16*cos(b*x+a)^2/b^2+1/16*cos(b*x+a)^4/b^2+3/8*x*cos(b*x+a)*sin(b
*x+a)/b+1/4*x*cos(b*x+a)^3*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3391, 30}

$$\int x\cos^4(a+bx)\,dx = \frac{\cos^4(a+bx)}{16b^2} + \frac{3\cos^2(a+bx)}{16b^2} + \frac{x\sin(a+bx)\cos^3(a+bx)}{4b} + \frac{3x\sin(a+bx)\cos(a+bx)}{8b} + \frac{3x^2}{16}$$

```
[In] Int[x*Cos[a + b*x]^4,x]
```

```
[Out] (3*x^2)/16 + (3*Cos[a + b*x]^2)/(16*b^2) + Cos[a + b*x]^4/(16*b^2) + (3*x*C
os[a + b*x]*Sin[a + b*x])/(8*b) + (x*Cos[a + b*x]^3*Sin[a + b*x])/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\cos^4(a+bx)}{16b^2} + \frac{x\cos^3(a+bx)\sin(a+bx)}{4b} + \frac{3}{4}\int x\cos^2(a+bx)\,dx \\ &= \frac{3\cos^2(a+bx)}{16b^2} + \frac{\cos^4(a+bx)}{16b^2} + \frac{3x\cos(a+bx)\sin(a+bx)}{8b} \\ &\quad + \frac{x\cos^3(a+bx)\sin(a+bx)}{4b} + \frac{3\int x\,dx}{8} \\ &= \frac{3x^2}{16} + \frac{3\cos^2(a+bx)}{16b^2} + \frac{\cos^4(a+bx)}{16b^2} \\ &\quad + \frac{3x\cos(a+bx)\sin(a+bx)}{8b} + \frac{x\cos^3(a+bx)\sin(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\begin{aligned}&\int x\cos^4(a+bx)\,dx \\ &= \frac{16\cos(2(a+bx)) + \cos(4(a+bx)) + 4bx(6bx + 8\sin(2(a+bx)) + \sin(4(a+bx)))}{128b^2}\end{aligned}$$

```
[In] Integrate[x*Cos[a + b*x]^4,x]
```

```
[Out] (16*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 4*b*x*(6*b*x + 8*Sin[2*(a + b*x)]
 + Sin[4*(a + b*x)]))/(128*b^2)
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{24x^2b^2+4x\sin(4bx+4a)b+32x\sin(2bx+2a)b+\cos(4bx+4a)+16\cos(2bx+2a)-17}{128b^2}$
risch	$\frac{3x^2}{16}+\frac{\cos(4bx+4a)}{128b^2}+\frac{x\sin(4bx+4a)}{32b}+\frac{\cos(2bx+2a)}{8b^2}+\frac{x\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{(bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{3(bx+a)^2}{16}+\frac{\left(2\left(\cos^2(bx+a)\right)+3\right)^2}{64}-a\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)}{4}\right.}{b^2}$
default	$\frac{(bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{3(bx+a)^2}{16}+\frac{\left(2\left(\cos^2(bx+a)\right)+3\right)^2}{64}-a\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)}{4}\right.}{b^2}$
norman	$\frac{\frac{3x^2}{16}+\frac{3x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{9x^2\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8}+\frac{3x^2\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{3x^2\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16}+\frac{5x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}-\frac{3x\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{4b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$

```
[In] int(x*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(24*x^2*b^2+4*x*sin(4*b*x+4*a)*b+32*x*sin(2*b*x+2*a)*b+cos(4*b*x+4*a)
+16*cos(2*b*x+2*a)-17)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int x\cos^4(a+bx)\,dx$$
$$=\frac{3\,b^2x^2+\cos(bx+a)^4+3\,\cos(bx+a)^2+2\left(2\,bx\cos(bx+a)^3+3\,bx\cos(bx+a)\right)\sin(bx+a)}{16\,b^2}$$

```
[In] integrate(x*cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*(3*b^2*x^2 + cos(b*x + a)^4 + 3*cos(b*x + a)^2 + 2*(2*b*x*cos(b*x + a)
^3 + 3*b*x*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.72

$$\int x\cos^4(a+bx)\,dx$$
$$=\begin{cases}\frac{3x^2\sin^4(a+bx)}{16}+\frac{3x^2\sin^2(a+bx)\cos^2(a+bx)}{8}+\frac{3x^2\cos^4(a+bx)}{16}+\frac{3x\sin^3(a+bx)\cos(a+bx)}{8b}+\frac{5x\sin(a+bx)\cos^3(a+bx)}{8b}-\frac{3\sin^4(a+}{32b^2}\\ \frac{x^2\cos^4(a)}{2}\end{cases}$$

```
[In] integrate(x*cos(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**2*sin(a + b*x)**4/16 + 3*x**2*sin(a + b*x)**2*cos(a + b*x)*
**2/8 + 3*x**2*cos(a + b*x)**4/16 + 3*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) +
 5*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 3*sin(a + b*x)**4/(32*b**2) + 5*c
os(a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*cos(a)**4/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int x \cos^4(a+bx)\,dx$$

$$= \frac{24\,(bx+a)^2 - 4\,(12\,bx + 12\,a + \sin(4\,bx+4\,a) + 8\,\sin(2\,bx+2\,a))a + 4\,(bx+a)\sin(4\,bx+4\,a) + 32\,(b}{128\,b^2}$$

```
[In] integrate(x*cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/128*(24*(b*x + a)^2 - 4*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x +
 2*a))*a + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a) + c
os(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x \cos^4(a+bx)\,dx = \frac{3}{16}\,x^2 + \frac{x\sin(4\,bx+4\,a)}{32\,b} + \frac{x\sin(2\,bx+2\,a)}{4\,b} + \frac{\cos(4\,bx+4\,a)}{128\,b^2} + \frac{\cos(2\,bx+2\,a)}{8\,b^2}$$

```
[In] integrate(x*cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/16*x^2 + 1/32*x*sin(4*b*x + 4*a)/b + 1/4*x*sin(2*b*x + 2*a)/b + 1/128*cos
(4*b*x + 4*a)/b^2 + 1/8*cos(2*b*x + 2*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int x\cos^4(a+bx)\,dx = \frac{3\,x^2}{16} - \frac{\frac{\sin(2\,a+2\,b\,x)^2}{64} - b\left(\frac{x\,\sin(2\,a+2\,b\,x)}{4} + \frac{x\,\sin(4\,a+4\,b\,x)}{32}\right) + \frac{\sin(a+b\,x)^2}{4}}{b^2}$$

```
[In] int(x*cos(a + b*x)^4,x)
```

```
[Out] (3*x^2)/16 - (sin(2*a + 2*b*x)^2/64 - b*((x*sin(2*a + 2*b*x))/4 + (x*sin(4*
a + 4*b*x))/32) + sin(a + b*x)^2/4)/b^2
```

3.26 $\int \frac{\cos^4(a+bx)}{x}\,dx$

Optimal result . . . 301
Rubi [A] (verified) . . . 301
Mathematica [A] (verified) . . . 302
Maple [A] (verified) . . . 303
Fricas [A] (verification not implemented) . . . 303
Sympy [A] (verification not implemented) . . . 303
Maxima [C] (verification not implemented) . . . 304
Giac [C] (verification not implemented) . . . 304
Mupad [F(-1)] . . . 305

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \frac{\cos^4(a+bx)}{x}\,dx = \frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx) + \frac{1}{8}\cos(4a)\,\text{CosIntegral}(4bx) + \frac{3\log(x)}{8} - \frac{1}{2}\sin(2a)\text{Si}(2bx) - \frac{1}{8}\sin(4a)\text{Si}(4bx)$$

```
[Out] 1/2*Ci(2*b*x)*cos(2*a)+1/8*Ci(4*b*x)*cos(4*a)+3/8*ln(x)-1/2*Si(2*b*x)*sin(2
*a)-1/8*Si(4*b*x)*sin(4*a)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3384, 3380, 3383}

$$\int \frac{\cos^4(a+bx)}{x}\,dx = \frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx) + \frac{1}{8}\cos(4a)\,\text{CosIntegral}(4bx) - \frac{1}{2}\sin(2a)\text{Si}(2bx) - \frac{1}{8}\sin(4a)\text{Si}(4bx) + \frac{3\log(x)}{8}$$

```
[In] Int[Cos[a + b*x]^4/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x])/2 + (Cos[4*a]*CosIntegral[4*b*x])/8 + (3*Log[
x])/8 - (Sin[2*a]*SinIntegral[2*b*x])/2 - (Sin[4*a]*SinIntegral[4*b*x])/8
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8x} + \frac{\cos(2a+2bx)}{2x} + \frac{\cos(4a+4bx)}{8x}\right) dx \\
&= \frac{3\log(x)}{8} + \frac{1}{8}\int \frac{\cos(4a+4bx)}{x}\, dx + \frac{1}{2}\int \frac{\cos(2a+2bx)}{x}\, dx \\
&= \frac{3\log(x)}{8} + \frac{1}{2}\cos(2a)\int \frac{\cos(2bx)}{x}\, dx + \frac{1}{8}\cos(4a)\int \frac{\cos(4bx)}{x}\, dx \\
&\quad - \frac{1}{2}\sin(2a)\int \frac{\sin(2bx)}{x}\, dx - \frac{1}{8}\sin(4a)\int \frac{\sin(4bx)}{x}\, dx \\
&= \frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx) + \frac{1}{8}\cos(4a)\,\text{CosIntegral}(4bx) \\
&\quad + \frac{3\log(x)}{8} - \frac{1}{2}\sin(2a)\text{Si}(2bx) - \frac{1}{8}\sin(4a)\text{Si}(4bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(a+bx)}{x}\, dx = \frac{1}{8}(4\cos(2a)\,\text{CosIntegral}(2bx) + \cos(4a)\,\text{CosIntegral}(4bx) + 3\log(x) - 4\sin(2a)\text{Si}(2bx) - \sin(4a)\text{Si}(4bx))$$

```
[In] Integrate[Cos[a + b*x]^4/x,x]
```

```
[Out] (4*Cos[2*a]*CosIntegral[2*b*x] + Cos[4*a]*CosIntegral[4*b*x] + 3*Log[x] - 4
*Sin[2*a]*SinIntegral[2*b*x] - Sin[4*a]*SinIntegral[4*b*x])/8
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{\mathrm{Si}(4bx)\sin(4a)}{8}+\frac{\mathrm{Ci}(4bx)\cos(4a)}{8}-\frac{\mathrm{Si}(2bx)\sin(2a)}{2}+\frac{\mathrm{Ci}(2bx)\cos(2a)}{2}+\frac{3\ln(bx)}{8}$
default	$-\frac{\mathrm{Si}(4bx)\sin(4a)}{8}+\frac{\mathrm{Ci}(4bx)\cos(4a)}{8}-\frac{\mathrm{Si}(2bx)\sin(2a)}{2}+\frac{\mathrm{Ci}(2bx)\cos(2a)}{2}+\frac{3\ln(bx)}{8}$
risch	$\frac{3\ln(x)}{8}+\frac{i\pi\ \mathrm{csgn}(bx)\mathrm{e}^{-4ia}}{16}-\frac{i\,\mathrm{Si}(4bx)\mathrm{e}^{-4ia}}{8}-\frac{\mathrm{e}^{-4ia}\,\mathrm{Ei}_1(-4ixb)}{16}+\frac{i\pi\ \mathrm{csgn}(bx)\mathrm{e}^{-2ia}}{4}-\frac{i\,\mathrm{Si}(2bx)\mathrm{e}^{-2ia}}{2}-\mathrm{e}^{-2}$

```
[In] int(cos(b*x+a)^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Si(4*b*x)*sin(4*a)+1/8*Ci(4*b*x)*cos(4*a)-1/2*Si(2*b*x)*sin(2*a)+1/2*C
i(2*b*x)*cos(2*a)+3/8*ln(b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(a+bx)}{x}\,dx = \frac{1}{8}\cos(4\,a)\,\mathrm{Ci}(4\,bx)+\frac{1}{2}\cos(2\,a)\,\mathrm{Ci}(2\,bx) - \frac{1}{8}\sin(4\,a)\,\mathrm{Si}(4\,bx)-\frac{1}{2}\sin(2\,a)\,\mathrm{Si}(2\,bx)+\frac{3}{8}\log(x)$$

```
[In] integrate(cos(b*x+a)^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*cos(4*a)*cos_integral(4*b*x) + 1/2*cos(2*a)*cos_integral(2*b*x) - 1/8*s
in(4*a)*sin_integral(4*b*x) - 1/2*sin(2*a)*sin_integral(2*b*x) + 3/8*log(x)
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{\cos^4(a+bx)}{x}\,dx = \frac{3\log(x)}{8}-\frac{\sin(2a)\,\mathrm{Si}(2bx)}{2}-\frac{\sin(4a)\,\mathrm{Si}(4bx)}{8} + \frac{\cos(2a)\,\mathrm{Ci}(2bx)}{2}+\frac{\cos(4a)\,\mathrm{Ci}(4bx)}{8}$$

```
[In] integrate(cos(b*x+a)**4/x,x)
```

```
[Out] 3*log(x)/8 - sin(2*a)*Si(2*b*x)/2 - sin(4*a)*Si(4*b*x)/8 + cos(2*a)*Ci(2*b*
x)/2 + cos(4*a)*Ci(4*b*x)/8
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{\cos^4(a+bx)}{x}\,dx = &-\frac{1}{16}\left(E_1(4i\,bx)+E_1(-4i\,bx)\right)\cos(4\,a)\\ &-\frac{1}{4}\left(E_1(2i\,bx)+E_1(-2i\,bx)\right)\cos(2\,a)\\ &+\frac{1}{16}\left(i\,E_1(4i\,bx)-i\,E_1(-4i\,bx)\right)\sin(4\,a)\\ &-\frac{1}{4}\left(-i\,E_1(2i\,bx)+i\,E_1(-2i\,bx)\right)\sin(2\,a)+\frac{3}{8}\log(bx)\end{aligned}$$

```
[In] integrate(cos(b*x+a)^4/x,x, algorithm="maxima")
```

```
[Out] -1/16*(exp_integral_e(1, 4*I*b*x) + exp_integral_e(1, -4*I*b*x))*cos(4*a) -
 1/4*(exp_integral_e(1, 2*I*b*x) + exp_integral_e(1, -2*I*b*x))*cos(2*a) +
1/16*(I*exp_integral_e(1, 4*I*b*x) - I*exp_integral_e(1, -4*I*b*x))*sin(4*a
) - 1/4*(-I*exp_integral_e(1, 2*I*b*x) + I*exp_integral_e(1, -2*I*b*x))*sin
(2*a) + 3/8*log(b*x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 428, normalized size of antiderivative = 7.25

$$\int \frac{\cos^4(a+bx)}{x}\,dx$$
$$= \frac{6\,\log(|x|)\tan(2\,a)^2\tan(a)^2-\Re(\mathrm{Ci}(4\,bx))\tan(2\,a)^2\tan(a)^2-4\,\Re(\mathrm{Ci}(2\,bx))\tan(2\,a)^2\tan(a)^2-4\,\Re(\mathrm{Ci}($$

```
[In] integrate(cos(b*x+a)^4/x,x, algorithm="giac")
```

```
[Out] 1/16*(6*log(abs(x))*tan(2*a)^2*tan(a)^2 - real_part(cos_integral(4*b*x))*ta
n(2*a)^2*tan(a)^2 - 4*real_part(cos_integral(2*b*x))*tan(2*a)^2*tan(a)^2 -
4*real_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a)^2 - real_part(cos_integ
ral(-4*b*x))*tan(2*a)^2*tan(a)^2 - 8*imag_part(cos_integral(2*b*x))*tan(2*a
)^2*tan(a) + 8*imag_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a) - 16*sin_i
ntegral(2*b*x)*tan(2*a)^2*tan(a) - 2*imag_part(cos_integral(4*b*x))*tan(2*a
)*tan(a)^2 + 2*imag_part(cos_integral(-4*b*x))*tan(2*a)*tan(a)^2 - 4*sin_in
tegral(4*b*x)*tan(2*a)*tan(a)^2 + 6*log(abs(x))*tan(2*a)^2 - real_part(cos_
integral(4*b*x))*tan(2*a)^2 + 4*real_part(cos_integral(2*b*x))*tan(2*a)^2 +
 4*real_part(cos_integral(-2*b*x))*tan(2*a)^2 - real_part(cos_integral(-4*b
```

```
*x))*tan(2*a)^2 + 6*log(abs(x))*tan(a)^2 + real_part(cos_integral(4*b*x))*t
an(a)^2 - 4*real_part(cos_integral(2*b*x))*tan(a)^2 - 4*real_part(cos_integ
ral(-2*b*x))*tan(a)^2 + real_part(cos_integral(-4*b*x))*tan(a)^2 - 2*imag_p
art(cos_integral(4*b*x))*tan(2*a) + 2*imag_part(cos_integral(-4*b*x))*tan(2
*a) - 4*sin_integral(4*b*x)*tan(2*a) - 8*imag_part(cos_integral(2*b*x))*tan
(a) + 8*imag_part(cos_integral(-2*b*x))*tan(a) - 16*sin_integral(2*b*x)*tan
(a) + 6*log(abs(x)) + real_part(cos_integral(4*b*x)) + 4*real_part(cos_inte
gral(2*b*x)) + 4*real_part(cos_integral(-2*b*x)) + real_part(cos_integral(-
4*b*x)))/(tan(2*a)^2*tan(a)^2 + tan(2*a)^2 + tan(a)^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(a+bx)}{x}\,dx = \int \frac{\cos\left(a+b\,x\right)^4}{x}\,dx$$

```
[In] int(cos(a + b*x)^4/x,x)
```

```
[Out] int(cos(a + b*x)^4/x, x)
```

3.27 $\int \frac{\cos^4(a+bx)}{x^2}\,dx$

Optimal result . . . 306
Rubi [A] (verified) . . . 306
Mathematica [A] (verified) . . . 308
Maple [A] (verified) . . . 308
Fricas [A] (verification not implemented) . . . 308
Sympy [F] . . . 309
Maxima [C] (verification not implemented) . . . 309
Giac [C] (verification not implemented) . . . 310
Mupad [F(-1)] . . . 313

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = -\frac{\cos^4(a+bx)}{x} - b\,\text{CosIntegral}(2bx)\sin(2a) - \frac{1}{2}b\,\text{CosIntegral}(4bx)\sin(4a) - b\cos(2a)\text{Si}(2bx) - \frac{1}{2}b\cos(4a)\text{Si}(4bx)$$

```
[Out] -cos(b*x+a)^4/x-b*cos(2*a)*Si(2*b*x)-1/2*b*cos(4*a)*Si(4*b*x)-b*Ci(2*b*x)*s
in(2*a)-1/2*b*Ci(4*b*x)*sin(4*a)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3394, 3384, 3380, 3383}

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = -b\sin(2a)\,\text{CosIntegral}(2bx) - \frac{1}{2}b\sin(4a)\,\text{CosIntegral}(4bx) - b\cos(2a)\text{Si}(2bx) - \frac{1}{2}b\cos(4a)\text{Si}(4bx) - \frac{\cos^4(a+bx)}{x}$$

```
[In] Int[Cos[a + b*x]^4/x^2,x]
```

```
[Out] -(Cos[a + b*x]^4/x) - b*CosIntegral[2*b*x]*Sin[2*a] - (b*CosIntegral[4*b*x]
*Sin[4*a])/2 - b*Cos[2*a]*SinIntegral[2*b*x] - (b*Cos[4*a]*SinIntegral[4*b*
x])/2
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^4(a+bx)}{x} + (4b)\int\left(-\frac{\sin(2a+2bx)}{4x} - \frac{\sin(4a+4bx)}{8x}\right)dx \\
&= -\frac{\cos^4(a+bx)}{x} - \frac{1}{2}b\int\frac{\sin(4a+4bx)}{x}\,dx - b\int\frac{\sin(2a+2bx)}{x}\,dx \\
&= -\frac{\cos^4(a+bx)}{x} - (b\cos(2a))\int\frac{\sin(2bx)}{x}\,dx - \frac{1}{2}(b\cos(4a))\int\frac{\sin(4bx)}{x}\,dx \\
&\quad - (b\sin(2a))\int\frac{\cos(2bx)}{x}\,dx - \frac{1}{2}(b\sin(4a))\int\frac{\cos(4bx)}{x}\,dx \\
&= -\frac{\cos^4(a+bx)}{x} - b\,\text{CosIntegral}(2bx)\sin(2a) \\
&\quad - \frac{1}{2}b\,\text{CosIntegral}(4bx)\sin(4a) - b\cos(2a)\text{Si}(2bx) - \frac{1}{2}b\cos(4a)\text{Si}(4bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx =$$
$$-\frac{3+4\cos(2(a+bx))+\cos(4(a+bx))+8bx\,\text{CosIntegral}(2bx)\sin(2a)+4bx\,\text{CosIntegral}(4bx)\sin(4a)+8}{8x}$$

```
[In] Integrate[Cos[a + b*x]^4/x^2,x]
```

```
[Out] -1/8*(3 + 4*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 8*b*x*CosIntegral[2*b*x]*
Sin[2*a] + 4*b*x*CosIntegral[4*b*x]*Sin[4*a] + 8*b*x*Cos[2*a]*SinIntegral[2
*b*x] + 4*b*x*Cos[4*a]*SinIntegral[4*b*x])/x
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

method	result
derivativedivides	$b\left(-\frac{\cos(4bx+4a)}{8bx}-\frac{\text{Si}(4bx)\cos(4a)}{2}-\frac{\text{Ci}(4bx)\sin(4a)}{2}-\frac{\cos(2bx+2a)}{2bx}-\text{Si}\left(2bx\right)\cos\left(2a\right)-\text{Ci}\left(2bx\right)\text{si}\right.$
default	$b\left(-\frac{\cos(4bx+4a)}{8bx}-\frac{\text{Si}(4bx)\cos(4a)}{2}-\frac{\text{Ci}(4bx)\sin(4a)}{2}-\frac{\cos(2bx+2a)}{2bx}-\text{Si}\left(2bx\right)\cos\left(2a\right)-\text{Ci}\left(2bx\right)\text{si}\right.$
risch	$\frac{4\,\mathrm{e}^{-2ia}\pi\ \text{csgn}(bx)bx+4i\mathrm{e}^{-2ia}\ \text{Ei}_1(-2ixb)bx-2ib\ \text{Ei}_1(-4ixb)\mathrm{e}^{4ia}x+2i\,\text{Ei}_1(-4ixb)\mathrm{e}^{-4ia}bx+2\,\mathrm{e}^{-4ia}\pi\ \text{csgn}(bx)bx-4ib\ \text{Ei}_1}{8x}$

```
[In] int(cos(b*x+a)^4/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/8*cos(4*b*x+4*a)/b/x-1/2*Si(4*b*x)*cos(4*a)-1/2*Ci(4*b*x)*sin(4*a)-1/
2*cos(2*b*x+2*a)/b/x-Si(2*b*x)*cos(2*a)-Ci(2*b*x)*sin(2*a)-3/8/b/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx =$$
$$-\frac{2\,\cos\left(bx+a\right)^4+bx\,\text{Ci}\left(4\,bx\right)\sin\left(4\,a\right)+2\,bx\,\text{Ci}\left(2\,bx\right)\sin\left(2\,a\right)+bx\cos\left(4\,a\right)\text{Si}\left(4\,bx\right)+2\,bx\cos\left(2\,a\right)\text{Si}\left(2\right.}{2\,x}$$

```
[In] integrate(cos(b*x+a)^4/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(b*x + a)^4 + b*x*cos_integral(4*b*x)*sin(4*a) + 2*b*x*cos_integ
ral(2*b*x)*sin(2*a) + b*x*cos(4*a)*sin_integral(4*b*x) + 2*b*x*cos(2*a)*sin
_integral(2*b*x))/x
```

Sympy [F]

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = \int \frac{\cos^4(a+bx)}{x^2}\,dx$$

```
[In] integrate(cos(b*x+a)**4/x**2,x)
[Out] Integral(cos(a + b*x)**4/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 717, normalized size of antiderivative = 10.86

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^4/x^2,x, algorithm="maxima")
[Out] 1/32*(((exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*cos(2*a)^
2 + (exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*
cos(4*a)^3 + ((-I*exp_integral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4*I*b*x
))*cos(2*a)^2 + (-I*exp_integral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4*I*b
*x))*sin(2*a)^2)*sin(4*a)^3 + 4*((exp_integral_e(2, 2*I*b*x) + exp_integral
_e(2, -2*I*b*x))*cos(2*a)^3 + (-I*exp_integral_e(2, 2*I*b*x) + I*exp_integr
al_e(2, -2*I*b*x))*sin(2*a)^3 + ((exp_integral_e(2, 2*I*b*x) + exp_integral
_e(2, -2*I*b*x))*cos(2*a) + 3)*sin(2*a)^2 + (exp_integral_e(2, 2*I*b*x) + e
xp_integral_e(2, -2*I*b*x))*cos(2*a) + 3*cos(2*a)^2 + ((-I*exp_integral_e(2
, 2*I*b*x) + I*exp_integral_e(2, -2*I*b*x))*cos(2*a)^2 - I*exp_integral_e(2
, 2*I*b*x) + I*exp_integral_e(2, -2*I*b*x))*sin(2*a))*cos(4*a)^2 + (4*(exp_
integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))*cos(2*a)^3 + 4*(-I*ex
p_integral_e(2, 2*I*b*x) + I*exp_integral_e(2, -2*I*b*x))*sin(2*a)^3 + 4*((
exp_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))*cos(2*a) + 3)*sin
(2*a)^2 + ((exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*cos(2
*a)^2 + (exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*sin(2*a)
^2)*cos(4*a) + 4*(exp_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))
*cos(2*a) + 12*cos(2*a)^2 + 4*((-I*exp_integral_e(2, 2*I*b*x) + I*exp_integ
ral_e(2, -2*I*b*x))*cos(2*a)^2 - I*exp_integral_e(2, 2*I*b*x) + I*exp_integ
ral_e(2, -2*I*b*x))*sin(2*a))*sin(4*a)^2 + ((exp_integral_e(2, 4*I*b*x) + e
xp_integral_e(2, -4*I*b*x))*cos(2*a)^2 + (exp_integral_e(2, 4*I*b*x) + exp_
integral_e(2, -4*I*b*x))*sin(2*a)^2)*cos(4*a) + (((-I*exp_integral_e(2, 4*I
*b*x) + I*exp_integral_e(2, -4*I*b*x))*cos(2*a)^2 + (-I*exp_integral_e(2, 4
*I*b*x) + I*exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*cos(4*a)^2 + (-I*exp_i
ntegral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4*I*b*x))*cos(2*a)^2 + (-I*exp
```

```
_integral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4*I*b*x))*sin(2*a)^2)*sin(4*
a))*b/((a*cos(2*a)^2 + a*sin(2*a)^2)*cos(4*a)^2 + (a*cos(2*a)^2 + a*sin(2*a
)^2)*sin(4*a)^2 - ((cos(2*a)^2 + sin(2*a)^2)*cos(4*a)^2 + (cos(2*a)^2 + sin
(2*a)^2)*sin(4*a)^2)*(b*x + a))
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 3220, normalized size of antiderivative = 48.79

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*
tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b
*x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^
2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2*t
an(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(
b*x)^2*tan(2*a)^2*tan(a)^2 - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*ta
n(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 2*b*x*real_part(cos_integral(4*b*
x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*real_part(cos_integra
l(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 + b*x*imag_part(cos_in
tegral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 2*b*x*imag_part(cos_int
egral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 2*b*x*imag_part(cos_inte
gral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integr
al(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 2*b*x*sin_integral(4*b*x)*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*ta
n(b*x)^2*tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b
*x)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x
)^2*tan(a)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*
tan(a)^2 - 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b
*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + b*x*imag_part(cos
_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*imag_part(cos_in
tegral(2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 - 2*b*x*imag_part(cos_integ
ral(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(
-4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2
*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(2*
a)^2*tan(a)^2 + b*x*imag_part(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)^2*ta
n(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^
2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 -
```

```
b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*
sin_integral(4*b*x)*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b
*x)*tan(b*x)^2*tan(2*a)^2*tan(a)^2 - 2*b*x*real_part(cos_integral(4*b*x))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a) - 2*b*x*real_part(cos_integral(-4*b*x))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*
b*x)^2*tan(b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(2*b*x)
^2*tan(b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*ta
n(2*a)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(2*
a)^2*tan(a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(b*x)^2*tan(2*a)^2*ta
n(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a) -
 2*b*x*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 - 2*b*
x*real_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*re
al_part(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*real_part
(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 - 4*tan(2*b*x)^2*tan(b*
x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*
tan(b*x)^2 - 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2 +
 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2 + b*x*imag_p
art(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2 - 2*b*x*sin_integral(4*b*
x)*tan(2*b*x)^2*tan(b*x)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x
)^2 + b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)^2 - 2*b*x*im
ag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(2*a)^2 + 2*b*x*imag_part(cos_
integral(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(-4*b
*x))*tan(2*b*x)^2*tan(2*a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2*tan(2
*a)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(2*a)^2 + b*x*imag_part(c
os_integral(4*b*x))*tan(b*x)^2*tan(2*a)^2 - 2*b*x*imag_part(cos_integral(2*
b*x))*tan(b*x)^2*tan(2*a)^2 + 2*b*x*imag_part(cos_integral(-2*b*x))*tan(b*x
)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)^2
+ 2*b*x*sin_integral(4*b*x)*tan(b*x)^2*tan(2*a)^2 - 4*b*x*sin_integral(2*b*
x)*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*
tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a)^2 - 2*b
*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(a)^2 + b*x*imag_part(co
s_integral(-4*b*x))*tan(2*b*x)^2*tan(a)^2 - 2*b*x*sin_integral(4*b*x)*tan(2
*b*x)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^2*tan(a)^2 - b*x*im
ag_part(cos_integral(4*b*x))*tan(b*x)^2*tan(a)^2 + 2*b*x*imag_part(cos_inte
gral(2*b*x))*tan(b*x)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*ta
n(b*x)^2*tan(a)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(a)^2
 - 2*b*x*sin_integral(4*b*x)*tan(b*x)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x
)*tan(b*x)^2*tan(a)^2 + b*x*imag_part(cos_integral(4*b*x))*tan(2*a)^2*tan(a
)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*a)^2*tan(a)^2 - 2*b*x*imag
_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integra
l(-4*b*x))*tan(2*a)^2*tan(a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*a)^2*tan(a
)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*a)^2*tan(a)^2 - 2*b*x*real_part(cos_i
ntegral(4*b*x))*tan(2*b*x)^2*tan(2*a) - 2*b*x*real_part(cos_integral(-4*b*x
))*tan(2*b*x)^2*tan(2*a) - 2*b*x*real_part(cos_integral(4*b*x))*tan(b*x)^2*
tan(2*a) - 2*b*x*real_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a) - 4*b*
```

```
x*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a) - 4*b*x*real_part(cos_
integral(-2*b*x))*tan(2*b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(2*b*x)
)*tan(b*x)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(
a) - 4*b*x*real_part(cos_integral(2*b*x))*tan(2*a)^2*tan(a) - 4*b*x*real_pa
rt(cos_integral(-2*b*x))*tan(2*a)^2*tan(a) + 8*tan(2*b*x)^2*tan(b*x)*tan(2*
a)^2*tan(a) - 3*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 - 2*b*x*real_part(cos_inte
gral(4*b*x))*tan(2*a)*tan(a)^2 - 2*b*x*real_part(cos_integral(-4*b*x))*tan(
2*a)*tan(a)^2 + 2*tan(2*b*x)*tan(b*x)^2*tan(2*a)*tan(a)^2 - 3*tan(b*x)^2*ta
n(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2 - 2*b*x
*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2 + 2*b*x*imag_part(cos_integral
(-2*b*x))*tan(2*b*x)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2 -
 2*b*x*sin_integral(4*b*x)*tan(2*b*x)^2 - 4*b*x*sin_integral(2*b*x)*tan(2*b
*x)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(b*x)^2 - 2*b*x*imag_part(cos
_integral(2*b*x))*tan(b*x)^2 + 2*b*x*imag_part(cos_integral(-2*b*x))*tan(b*
x)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(b*x)^2 - 2*b*x*sin_integral(
4*b*x)*tan(b*x)^2 - 4*b*x*sin_integral(2*b*x)*tan(b*x)^2 + b*x*imag_part(co
s_integral(4*b*x))*tan(2*a)^2 - 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*
a)^2 + 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*a)^2 - b*x*imag_part(cos
_integral(-4*b*x))*tan(2*a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*a)^2 - 4*b*
x*sin_integral(2*b*x)*tan(2*a)^2 - b*x*imag_part(cos_integral(4*b*x))*tan(a
)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(a)^2 - 2*b*x*imag_part(cos_i
ntegral(-2*b*x))*tan(a)^2 + b*x*imag_part(cos_integral(-4*b*x))*tan(a)^2 -
2*b*x*sin_integral(4*b*x)*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(a)^2 + t
an(2*b*x)^2*tan(b*x)^2 - 2*b*x*real_part(cos_integral(4*b*x))*tan(2*a) - 2*
b*x*real_part(cos_integral(-4*b*x))*tan(2*a) + 2*tan(2*b*x)*tan(b*x)^2*tan(
2*a) - 4*tan(2*b*x)^2*tan(2*a)^2 + tan(b*x)^2*tan(2*a)^2 - 4*b*x*real_part(
cos_integral(2*b*x))*tan(a) - 4*b*x*real_part(cos_integral(-2*b*x))*tan(a)
+ 8*tan(2*b*x)^2*tan(b*x)*tan(a) + 8*tan(b*x)*tan(2*a)^2*tan(a) + tan(2*b*x
)^2*tan(a)^2 - 4*tan(b*x)^2*tan(a)^2 + 2*tan(2*b*x)*tan(2*a)*tan(a)^2 + tan
(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(4*b*x)) - 2*b*x*imag_part(cos
_integral(2*b*x)) + 2*b*x*imag_part(cos_integral(-2*b*x)) + b*x*imag_part(c
os_integral(-4*b*x)) - 2*b*x*sin_integral(4*b*x) - 4*b*x*sin_integral(2*b*x
) - 3*tan(2*b*x)^2 + 2*tan(2*b*x)*tan(2*a) - 3*tan(2*a)^2 + 8*tan(b*x)*tan(
a) - 4)/(x*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + x*tan(2*b*x)^2*tan
(b*x)^2*tan(2*a)^2 + x*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + x*tan(2*b*x)^2*ta
n(2*a)^2*tan(a)^2 + x*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + x*tan(2*b*x)^2*tan(b
*x)^2 + x*tan(2*b*x)^2*tan(2*a)^2 + x*tan(b*x)^2*tan(2*a)^2 + x*tan(2*b*x)^
2*tan(a)^2 + x*tan(b*x)^2*tan(a)^2 + x*tan(2*a)^2*tan(a)^2 + x*tan(2*b*x)^2
 + x*tan(b*x)^2 + x*tan(2*a)^2 + x*tan(a)^2 + x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^4}{x^2}\,dx$$

```
[In] int(cos(a + b*x)^4/x^2,x)
[Out] int(cos(a + b*x)^4/x^2, x)
```

3.28 $\int \frac{\cos^4(a+bx)}{x^3}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = -\frac{\cos^4(a+bx)}{2x^2} - b^2\cos(2a)\,\text{CosIntegral}(2bx) - b^2\cos(4a)\,\text{CosIntegral}(4bx) + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} + b^2\sin(2a)\text{Si}(2bx) + b^2\sin(4a)\text{Si}(4bx)$$

```
[Out] -b^2*Ci(2*b*x)*cos(2*a)-b^2*Ci(4*b*x)*cos(4*a)-1/2*cos(b*x+a)^4/x^2+b^2*Si(
2*b*x)*sin(2*a)+b^2*Si(4*b*x)*sin(4*a)+2*b*cos(b*x+a)^3*sin(b*x+a)/x
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3395, 3393, 3384, 3380, 3383}

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = -b^2\cos(2a)\,\text{CosIntegral}(2bx) - b^2\cos(4a)\,\text{CosIntegral}(4bx) + b^2\sin(2a)\text{Si}(2bx) + b^2\sin(4a)\text{Si}(4bx) - \frac{\cos^4(a+bx)}{2x^2} + \frac{2b\sin(a+bx)\cos^3(a+bx)}{x}$$

```
[In] Int[Cos[a + b*x]^4/x^3,x]
```

```
[Out] -1/2*Cos[a + b*x]^4/x^2 - b^2*Cos[2*a]*CosIntegral[2*b*x] - b^2*Cos[4*a]*Co
sIntegral[4*b*x] + (2*b*Cos[a + b*x]^3*Sin[a + b*x])/x + b^2*Sin[2*a]*SinIn
tegral[2*b*x] + b^2*Sin[4*a]*SinIntegral[4*b*x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^4(a+bx)}{2x^2} + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} \\
&\quad + (6b^2)\int \frac{\cos^2(a+bx)}{x}\,dx - (8b^2)\int \frac{\cos^4(a+bx)}{x}\,dx \\
&= -\frac{\cos^4(a+bx)}{2x^2} + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} + (6b^2)\int \left(\frac{1}{2x} + \frac{\cos(2a+2bx)}{2x}\right)dx \\
&\quad - (8b^2)\int \left(\frac{3}{8x} + \frac{\cos(2a+2bx)}{2x} + \frac{\cos(4a+4bx)}{8x}\right)dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\cos^4(a+bx)}{2x^2} + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} - b^2\int \frac{\cos(4a+4bx)}{x}\,dx \\
&\quad + \left(3b^2\right)\int \frac{\cos(2a+2bx)}{x}\,dx - \left(4b^2\right)\int \frac{\cos(2a+2bx)}{x}\,dx \\
&= -\frac{\cos^4(a+bx)}{2x^2} + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} \\
&\quad + \left(3b^2\cos(2a)\right)\int \frac{\cos(2bx)}{x}\,dx - \left(4b^2\cos(2a)\right)\int \frac{\cos(2bx)}{x}\,dx \\
&\quad - \left(b^2\cos(4a)\right)\int \frac{\cos(4bx)}{x}\,dx - \left(3b^2\sin(2a)\right)\int \frac{\sin(2bx)}{x}\,dx \\
&\quad + \left(4b^2\sin(2a)\right)\int \frac{\sin(2bx)}{x}\,dx + \left(b^2\sin(4a)\right)\int \frac{\sin(4bx)}{x}\,dx \\
&= -\frac{\cos^4(a+bx)}{2x^2} - b^2\cos(2a)\,\mathrm{CosIntegral}(2bx) - b^2\cos(4a)\,\mathrm{CosIntegral}(4bx) \\
&\quad + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} + b^2\sin(2a)\mathrm{Si}(2bx) + b^2\sin(4a)\mathrm{Si}(4bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.32

$$
\int \frac{\cos^4(a+bx)}{x^3}\,dx = -\frac{3+4\cos(2(a+bx))+\cos(4(a+bx))+16b^2x^2\cos(2a)\,\mathrm{CosIntegral}(2bx)+16b^2x^2\cos(4a)\,\mathrm{CosIntegral}(4}{16x^2}
$$

```
[In] Integrate[Cos[a + b*x]^4/x^3,x]
```

```
[Out] -1/16*(3 + 4*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 16*b^2*x^2*Cos[2*a]*CosI
ntegral[2*b*x] + 16*b^2*x^2*Cos[4*a]*CosIntegral[4*b*x] - 8*b*x*Sin[2*(a +
b*x)] - 4*b*x*Sin[4*(a + b*x)] - 16*b^2*x^2*Sin[2*a]*SinIntegral[2*b*x] - 1
6*b^2*x^2*Sin[4*a]*SinIntegral[4*b*x])/x^2
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.38

method	result
derivativedivides	$b^2\left(-\frac{\cos(4bx+4a)}{16b^2x^2}+\frac{\sin(4bx+4a)}{4bx}+\mathrm{Si}\left(4bx\right)\sin\left(4a\right)-\mathrm{Ci}\left(4bx\right)\cos\left(4a\right)-\frac{\cos(2bx+2a)}{4b^2x^2}+\frac{\sin(2bx+$
default	$b^2\left(-\frac{\cos(4bx+4a)}{16b^2x^2}+\frac{\sin(4bx+4a)}{4bx}+\mathrm{Si}\left(4bx\right)\sin\left(4a\right)-\mathrm{Ci}\left(4bx\right)\cos\left(4a\right)-\frac{\cos(2bx+2a)}{4b^2x^2}+\frac{\sin(2bx+$
risch	$-\frac{8ie^{-2ia}\pi\ \mathrm{csgn}(bx)b^2x^2+8ie^{-4ia}\pi\ \mathrm{csgn}(bx)b^2x^2-16ie^{-4ia}\,\mathrm{Si}(4bx)b^2x^2-16ie^{-2ia}\,\mathrm{Si}(2bx)b^2x^2-8\,e^{4ia}\,\mathrm{Ei}_1(-4ixb)x^2b^2$

```
[In] int(cos(b*x+a)^4/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/16*cos(4*b*x+4*a)/b^2/x^2+1/4*sin(4*b*x+4*a)/b/x+Si(4*b*x)*sin(4*a)
-Ci(4*b*x)*cos(4*a)-1/4*cos(2*b*x+2*a)/b^2/x^2+1/2*sin(2*b*x+2*a)/b/x+Si(2*
b*x)*sin(2*a)-Ci(2*b*x)*cos(2*a)-3/16/x^2/b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = -\frac{2\,b^2x^2\cos(4\,a)\,\mathrm{Ci}(4\,bx) + 2\,b^2x^2\cos(2\,a)\,\mathrm{Ci}(2\,bx) - 4\,bx\cos(bx+a)^3\sin(bx+a) - 2\,b^2x^2\sin(4\,a)\,\mathrm{Si}(}{2\,x^2}$$

```
[In] integrate(cos(b*x+a)^4/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*cos(4*a)*cos_integral(4*b*x) + 2*b^2*x^2*cos(2*a)*cos_integ
ral(2*b*x) - 4*b*x*cos(b*x + a)^3*sin(b*x + a) - 2*b^2*x^2*sin(4*a)*sin_int
egral(4*b*x) - 2*b^2*x^2*sin(2*a)*sin_integral(2*b*x) + cos(b*x + a)^4)/x^2
```

Sympy [F]

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = \int \frac{\cos^4(a+bx)}{x^3}\,dx$$

```
[In] integrate(cos(b*x+a)**4/x**3,x)
```

```
[Out] Integral(cos(a + b*x)**4/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 790, normalized size of antiderivative = 8.78

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/32*(((exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*cos(2*a)
^2 + (exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)
*cos(4*a)^3 + ((-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*
```

```
x))*cos(2*a)^2 + (-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*
b*x))*sin(2*a)^2)*sin(4*a)^3 + 2*(2*(exp_integral_e(3, 2*I*b*x) + exp_integ
ral_e(3, -2*I*b*x))*cos(2*a)^3 + 2*(-I*exp_integral_e(3, 2*I*b*x) + I*exp_i
ntegral_e(3, -2*I*b*x))*sin(2*a)^3 + (2*(exp_integral_e(3, 2*I*b*x) + exp_i
ntegral_e(3, -2*I*b*x))*cos(2*a) + 3)*sin(2*a)^2 + 2*(exp_integral_e(3, 2*I
*b*x) + exp_integral_e(3, -2*I*b*x))*cos(2*a) + 3*cos(2*a)^2 + 2*((-I*exp_i
ntegral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*cos(2*a)^2 - I*exp_i
ntegral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*sin(2*a))*cos(4*a)^2
 + (4*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(3, -2*I*b*x))*cos(2*a)^3
 + 4*(-I*exp_integral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*sin(2*
a)^3 + 2*(2*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(3, -2*I*b*x))*cos(
2*a) + 3)*sin(2*a)^2 + ((exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*
I*b*x))*cos(2*a)^2 + (exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b
*x))*sin(2*a)^2)*cos(4*a) + 4*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(
3, -2*I*b*x))*cos(2*a) + 6*cos(2*a)^2 + 4*((-I*exp_integral_e(3, 2*I*b*x) +
 I*exp_integral_e(3, -2*I*b*x))*cos(2*a)^2 - I*exp_integral_e(3, 2*I*b*x) +
 I*exp_integral_e(3, -2*I*b*x))*sin(2*a))*sin(4*a)^2 + ((exp_integral_e(3,
4*I*b*x) + exp_integral_e(3, -4*I*b*x))*cos(2*a)^2 + (exp_integral_e(3, 4*I
*b*x) + exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)*cos(4*a) + (((-I*exp_integ
ral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*x))*cos(2*a)^2 + (-I*exp_int
egral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*x))*sin(2*a)^2)*cos(4*a)^2
 + (-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*x))*cos(2*a)
^2 + (-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I*b*x))*sin(2*
a)^2)*sin(4*a))*b^2/(((cos(2*a)^2 + sin(2*a)^2)*cos(4*a)^2 + (cos(2*a)^2 +
sin(2*a)^2)*sin(4*a)^2)*(b*x + a)^2 + (a^2*cos(2*a)^2 + a^2*sin(2*a)^2)*cos
(4*a)^2 + (a^2*cos(2*a)^2 + a^2*sin(2*a)^2)*sin(4*a)^2 - 2*((a*cos(2*a)^2 +
 a*sin(2*a)^2)*cos(4*a)^2 + (a*cos(2*a)^2 + a*sin(2*a)^2)*sin(4*a)^2)*(b*x
+ a))
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 3920, normalized size of antiderivative = 43.56

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^4/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2
*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(
b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(
-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 8*b^2*x^2*imag_part(
cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 8*b^2*x^2*
```

```
imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) +
 16*b^2*x^2*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) +
 8*b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*
tan(a)^2 - 8*b^2*x^2*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)*tan(a)^2 + 16*b^2*x^2*sin_integral(4*b*x)*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2
*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_part(cos_integral(-4*b*x))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x)
)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x
))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b
*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(-4
*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(
4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral
(2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integra
l(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-4*b*x))*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_int
egral(4*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_inte
gral(2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-2*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-4*b*x))*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 8*b^2*x^2*imag_part(cos_integ
ral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) - 8*b^2*x^2*imag_part(cos_inte
gral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) + 16*b^2*x^2*sin_integral(4*
b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) + 8*b^2*x^2*imag_part(cos_integral(2*
b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*
b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(2
*b*x)^2*tan(b*x)^2*tan(a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x))*tan(2*
b*x)^2*tan(2*a)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*b*x))*tan(2*
b*x)^2*tan(2*a)^2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(2*b*x)^2*tan(
2*a)^2*tan(a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x))*tan(b*x)^2*tan(2*a
)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(2*a)^
2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(b*x)^2*tan(2*a)^2*tan(a) + 8*
b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 - 8*b
^2*x^2*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)*tan(a)^2 + 16*
b^2*x^2*sin_integral(4*b*x)*tan(2*b*x)^2*tan(2*a)*tan(a)^2 + 8*b^2*x^2*imag
_part(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 - 8*b^2*x^2*imag_pa
rt(cos_integral(-4*b*x))*tan(b*x)^2*tan(2*a)*tan(a)^2 + 16*b^2*x^2*sin_inte
gral(4*b*x)*tan(b*x)^2*tan(2*a)*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral
(4*b*x))*tan(2*b*x)^2*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))
*tan(2*b*x)^2*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*
b*x)^2*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(2*b*x)^2*
tan(b*x)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*a)
^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(2*a)^2 - 4*b
^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(2*a)^2 + 4*b^2*x^2*
```

```
real_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_pa
rt(cos_integral(4*b*x))*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_int
egral(2*b*x))*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(-2*b
*x))*tan(b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(
b*x)^2*tan(2*a)^2 - 8*b*x*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 4*b^2
*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(a)^2 + 4*b^2*x^2*real_
part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_i
ntegral(-2*b*x))*tan(2*b*x)^2*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(-
4*b*x))*tan(2*b*x)^2*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x))*ta
n(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(b*x)^2*tan
(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(b*x)^2*tan(a)^2 - 4*b
^2*x^2*real_part(cos_integral(-4*b*x))*tan(b*x)^2*tan(a)^2 - 4*b*x*tan(2*b*
x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x)
)*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*a)^2
*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a)^2 +
 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(2*a)^2*tan(a)^2 - 8*b*x*tan(
2*b*x)^2*tan(b*x)*tan(2*a)^2*tan(a)^2 - 4*b*x*tan(2*b*x)*tan(b*x)^2*tan(2*a
)^2*tan(a)^2 + 8*b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(2*
a) - 8*b^2*x^2*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(2*a) + 16*b
^2*x^2*sin_integral(4*b*x)*tan(2*b*x)^2*tan(2*a) + 8*b^2*x^2*imag_part(cos_
integral(4*b*x))*tan(b*x)^2*tan(2*a) - 8*b^2*x^2*imag_part(cos_integral(-4*
b*x))*tan(b*x)^2*tan(2*a) + 16*b^2*x^2*sin_integral(4*b*x)*tan(b*x)^2*tan(2
*a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(a) - 8*b^2*
x^2*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(a) + 16*b^2*x^2*sin_in
tegral(2*b*x)*tan(2*b*x)^2*tan(a) + 8*b^2*x^2*imag_part(cos_integral(2*b*x)
)*tan(b*x)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integral(-2*b*x))*tan(b*x)^2*
tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(b*x)^2*tan(a) + 8*b^2*x^2*imag_
part(cos_integral(2*b*x))*tan(2*a)^2*tan(a) - 8*b^2*x^2*imag_part(cos_integ
ral(-2*b*x))*tan(2*a)^2*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(2*a)^2*
tan(a) + 8*b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*a)*tan(a)^2 - 8*b^2
*x^2*imag_part(cos_integral(-4*b*x))*tan(2*a)*tan(a)^2 + 16*b^2*x^2*sin_int
egral(4*b*x)*tan(2*a)*tan(a)^2 - 4*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a
)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2 - 4*b^2*x^2*rea
l_part(cos_integral(2*b*x))*tan(2*b*x)^2 - 4*b^2*x^2*real_part(cos_integral
(-2*b*x))*tan(2*b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(2*b*
x)^2 - 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(b*x)^2 - 4*b^2*x^2*real
_part(cos_integral(2*b*x))*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-2
*b*x))*tan(b*x)^2 - 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(b*x)^2 -
4*b*x*tan(2*b*x)^2*tan(b*x)^2*tan(2*a) + 4*b^2*x^2*real_part(cos_integral(4
*b*x))*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*a)^2 - 4
*b^2*x^2*real_part(cos_integral(-2*b*x))*tan(2*a)^2 + 4*b^2*x^2*real_part(c
os_integral(-4*b*x))*tan(2*a)^2 + 8*b*x*tan(2*b*x)^2*tan(b*x)*tan(2*a)^2 -
4*b*x*tan(2*b*x)*tan(b*x)^2*tan(2*a)^2 - 8*b*x*tan(2*b*x)^2*tan(b*x)^2*tan(
a) + 8*b*x*tan(2*b*x)^2*tan(2*a)^2*tan(a) - 8*b*x*tan(b*x)^2*tan(2*a)^2*tan
(a) - 4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(a)^2 + 4*b^2*x^2*real_pa
```

```
rt(cos_integral(2*b*x))*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x)
)*tan(a)^2 - 4*b^2*x^2*real_part(cos_integral(-4*b*x))*tan(a)^2 - 8*b*x*tan
(2*b*x)^2*tan(b*x)*tan(a)^2 + 4*b*x*tan(2*b*x)*tan(b*x)^2*tan(a)^2 - 4*b*x*
tan(2*b*x)^2*tan(2*a)*tan(a)^2 + 4*b*x*tan(b*x)^2*tan(2*a)*tan(a)^2 - 4*b*x
*tan(2*b*x)*tan(2*a)^2*tan(a)^2 - 8*b*x*tan(b*x)*tan(2*a)^2*tan(a)^2 + 8*b^
2*x^2*imag_part(cos_integral(4*b*x))*tan(2*a) - 8*b^2*x^2*imag_part(cos_int
egral(-4*b*x))*tan(2*a) + 16*b^2*x^2*sin_integral(4*b*x)*tan(2*a) + 8*b^2*x
^2*imag_part(cos_integral(2*b*x))*tan(a) - 8*b^2*x^2*imag_part(cos_integral
(-2*b*x))*tan(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(a) + 8*tan(2*b*x)^2*t
an(b*x)*tan(2*a)^2*tan(a) - 3*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 2*tan(2*b*
x)*tan(b*x)^2*tan(2*a)*tan(a)^2 - 3*tan(b*x)^2*tan(2*a)^2*tan(a)^2 - 4*b^2*
x^2*real_part(cos_integral(4*b*x)) - 4*b^2*x^2*real_part(cos_integral(2*b*x
)) - 4*b^2*x^2*real_part(cos_integral(-2*b*x)) - 4*b^2*x^2*real_part(cos_in
tegral(-4*b*x)) + 8*b*x*tan(2*b*x)^2*tan(b*x) + 4*b*x*tan(2*b*x)*tan(b*x)^2
 - 4*b*x*tan(2*b*x)^2*tan(2*a) + 4*b*x*tan(b*x)^2*tan(2*a) - 4*b*x*tan(2*b*
x)*tan(2*a)^2 + 8*b*x*tan(b*x)*tan(2*a)^2 + 8*b*x*tan(2*b*x)^2*tan(a) - 8*b
*x*tan(b*x)^2*tan(a) + 8*b*x*tan(2*a)^2*tan(a) + 4*b*x*tan(2*b*x)*tan(a)^2
 - 8*b*x*tan(b*x)*tan(a)^2 + 4*b*x*tan(2*a)*tan(a)^2 + tan(2*b*x)^2*tan(b*x)
^2 + 2*tan(2*b*x)*tan(b*x)^2*tan(2*a) - 4*tan(2*b*x)^2*tan(2*a)^2 + tan(b*x
)^2*tan(2*a)^2 + 8*tan(2*b*x)^2*tan(b*x)*tan(a) + 8*tan(b*x)*tan(2*a)^2*tan
(a) + tan(2*b*x)^2*tan(a)^2 - 4*tan(b*x)^2*tan(a)^2 + 2*tan(2*b*x)*tan(2*a)
*tan(a)^2 + tan(2*a)^2*tan(a)^2 + 4*b*x*tan(2*b*x) + 8*b*x*tan(b*x) + 4*b*x
*tan(2*a) + 8*b*x*tan(a) - 3*tan(2*b*x)^2 + 2*tan(2*b*x)*tan(2*a) - 3*tan(2
*a)^2 + 8*tan(b*x)*tan(a) - 4)/(x^2*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(
a)^2 + x^2*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + x^2*tan(2*b*x)^2*tan(b*x)^2
*tan(a)^2 + x^2*tan(2*b*x)^2*tan(2*a)^2*tan(a)^2 + x^2*tan(b*x)^2*tan(2*a)^
2*tan(a)^2 + x^2*tan(2*b*x)^2*tan(b*x)^2 + x^2*tan(2*b*x)^2*tan(2*a)^2 + x^
2*tan(b*x)^2*tan(2*a)^2 + x^2*tan(2*b*x)^2*tan(a)^2 + x^2*tan(b*x)^2*tan(a)
^2 + x^2*tan(2*a)^2*tan(a)^2 + x^2*tan(2*b*x)^2 + x^2*tan(b*x)^2 + x^2*tan(
2*a)^2 + x^2*tan(a)^2 + x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^4}{x^3}\,dx$$

```
[In] int(cos(a + b*x)^4/x^3,x)
```

```
[Out] int(cos(a + b*x)^4/x^3, x)
```

3.29 $\int (c+dx)^3 \sec(a+bx)\,dx$

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Rubi [A] (verified) . . . 323
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Optimal result

Integrand size = 14, antiderivative size = 205

$$\begin{aligned}\int (c+dx)^3 \sec(a+bx)\,dx = &-\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} \\ &+ \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ &- \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\ &- \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\ &+ \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ &- \frac{6id^3 \operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4} + \frac{6id^3 \operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4}\end{aligned}$$

```
[Out] -2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*
(b*x+a)))/b^2-3*I*d*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)
*polylog(3,-I*exp(I*(b*x+a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))
/b^3-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*
x+a)))/b^4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4266, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int (c+dx)^3 \sec(a+bx)\,dx = &-\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} \\ &- \frac{6id^3 \operatorname{PolyLog}\left(4, -ie^{i(a+bx)}\right)}{b^4} + \frac{6id^3 \operatorname{PolyLog}\left(4, ie^{i(a+bx)}\right)}{b^4} \\ &- \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} \\ &+ \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3} \\ &+ \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} \\ &- \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2}\end{aligned}$$

```
[In] Int[(c + d*x)^3*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(
a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 +
(6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4,
 (-I)*E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
```

```
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2 \log\left(1-ie^{i(a+bx)}\right)\,dx}{b} \\
&\quad + \frac{(3d)\int (c+dx)^2 \log\left(1+ie^{i(a+bx)}\right)\,dx}{b} \\
&= -\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{(6id^2)\int (c+dx) \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)\,dx}{b^2} \\
&\quad + \frac{(6id^2)\int (c+dx) \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} + \frac{(6d^3)\int \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)\,dx}{b^3} \\
&\quad - \frac{(6d^3)\int \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)\,dx}{b^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3} - \frac{(6id^3) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-ix)}{x}\, dx, x, e^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{(6id^3) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,ix)}{x}\, dx, x, e^{i(a+bx)}\right)}{b^4} \\
&= -\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{6id^3 \operatorname{PolyLog}\left(4, -ie^{i(a+bx)}\right)}{b^4} + \frac{6id^3 \operatorname{PolyLog}\left(4, ie^{i(a+bx)}\right)}{b^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.96

$$
\int (c+dx)^3 \sec(a+bx)\, dx = \\
-\frac{i\left(2b^3(c+dx)^3 \arctan\left(e^{i(a+bx)}\right) - 3d\left(b^2(c+dx)^2 \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right) + 2ibd(c+dx) \operatorname{PolyLog}\left(3, -i\right.\right.\right.}{}
$$

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x],x]
```

```
[Out] ((-I)*(2*b^3*(c + d*x)^3*ArcTan[E^(I*(a + b*x))] - 3*d*(b^2*(c + d*x)^2*Pol
yLog[2, (-I)*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a
 + b*x))] - 2*d^2*PolyLog[4, (-I)*E^(I*(a + b*x))]) + 3*d*(b^2*(c + d*x)^2*
PolyLog[2, I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, I*E^(I*(a +
b*x))] - 2*d^2*PolyLog[4, I*E^(I*(a + b*x))])))/b^4
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(180) = 360$.

Time = 1.73 (sec) , antiderivative size = 685, normalized size of antiderivative = 3.34

method	result
risch	$\frac{3c^2d\ln(1-ie^{i(bx+a)})x}{b}+\frac{3c^2d\ln(1-ie^{i(bx+a)})a}{b^2}-\frac{3ic^2d\ \mathrm{Li}_2(ie^{i(bx+a)})}{b^2}+\frac{3id^3\ \mathrm{Li}_2(-ie^{i(bx+a)})x^2}{b^2}-\frac{3id^3\ \mathrm{Li}_2(ie^{i(bx+a)})x^2}{b^2}+$

```
[In] int((d*x+c)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x-6*I/b^3*c*d^2*a^2*arctan(exp(I
*(b*x+a)))+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))-6*I/b^2*d^2*c*polylog(2,I
*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+3/b*c^2*d*ln(1-I*ex
p(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3*I/b^2*c^2*d*polylog(
2,I*exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2-3*I/b^2*d^
3*polylog(2,I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+3*
I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+
a)))*x+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+6/b^3*d^3*polylog(3,I*exp(I*(b*x+
a)))*x-1/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x
+a)))-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+6/b^3*d^2*c*polylog(3,I*exp(I*(b*x
+a)))-6/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))-2*I/b*c^3*arctan(exp(I*(b*x+
a)))-3/b^3*a^2*d^2*c*ln(1-I*exp(I*(b*x+a)))-3/b*d^2*c*ln(1+I*exp(I*(b*x+a))
)*x^2+3/b^3*a^2*d^2*c*ln(1+I*exp(I*(b*x+a)))+3/b*d^2*c*ln(1-I*exp(I*(b*x+a)
))*x^2-3/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,I*exp(I*(b*x+a)
))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(167) = 334$.

Time = 0.33 (sec) , antiderivative size = 970, normalized size of antiderivative = 4.73

$$\int (c+dx)^3 \sec(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d
^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)
) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a)
- sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(
```

```
-I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x +
 a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*
x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*pol
ylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)))/b^4
```

Sympy [F]

$$\int (c+dx)^3 \sec(a+bx)\,dx = \int (c+dx)^3 \sec\left(a+bx\right)\,dx$$

```
[In] integrate((d*x+c)**3*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(167) = 334$.

Time $= 0.48$ (sec) , antiderivative size $= 722$, normalized size of antiderivative $= 3.52$

$$\int (c+dx)^3 \sec(a+bx)\,dx$$

$$= \frac{2\,c^3 \log\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right) - \frac{6\,ac^2d\log(\sec(bx+a)+\tan(bx+a))}{b} + \frac{6\,a^2cd^2\log(\sec(bx+a)+\tan(bx+a))}{b^2} - \frac{2\,a^3d^3\log(s}{}$$

```
[In] integrate((d*x+c)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*log(sec(b*x + a) + tan(b*x + a)) - 6*a*c^2*d*log(sec(b*x + a) +
tan(b*x + a))/b + 6*a^2*c*d^2*log(sec(b*x + a) + tan(b*x + a))/b^2 - 2*a^3*
```

```
d^3*log(sec(b*x + a) + tan(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x
+ I*a)) - 12*I*d^3*polylog(4, -I*e^(I*b*x + I*a)) - 2*(I*(b*x + a)^3*d^3 +
3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^
2*d^3)*(b*x + a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) - 2*(I*(b*x + a)^
3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^
2 + I*a^2*d^3)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 6*(I*b
^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I
*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*a)) - 6*(-I*b^2*c^2*d + 2*I*a*b*c*d
^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*di
log(-I*e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^
2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*polylog(3, I*e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*polylog(3, -I*e^(I*b*x + I*a)))/b^3)/b
```

Giac [F]

$$\int (c+dx)^3 \sec(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)\,dx$$

```
[In] integrate((d*x+c)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\cos(a+b\,x)}\,dx$$

```
[In] int((c + d*x)^3/cos(a + b*x),x)
```

```
[Out] int((c + d*x)^3/cos(a + b*x), x)
```

3.30 $\int (c+dx)^2 \sec(a+bx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 137

$$\begin{aligned}\int (c+dx)^2 \sec(a+bx)\,dx = &-\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} \\ &+\frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ &-\frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\ &-\frac{2d^2\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}\end{aligned}$$

```
[Out] -2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+2*I*d*(d*x+c)*polylog(2,-I*exp(I*(b
*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-
I*exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4266, 2611, 2320, 6724}

$$\begin{aligned}\int (c+dx)^2 \sec(a+bx)\,dx = &-\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} \\ &-\frac{2d^2\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ &+\frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ &-\frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\end{aligned}$$

```
[In] Int[(c + d*x)^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog
[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))])/b^2 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog
[3, I*E^(I*(a + b*x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)\,dx}{b} + \frac{(2d)\int (c+dx)\log\left(1+ie^{i(a+bx)}\right)\,dx}{b}$$

$$
\begin{aligned}
&= -\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{(2id^2)\int \mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)\,dx}{b^2} + \frac{(2id^2)\int \mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{(2d^2)\,\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(2,-ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{(2d^2)\,\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(2,ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^3} \\
&= -\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{2d^2\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{2d^2\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.95

$$
\int (c+dx)^2 \sec(a+bx)\,dx = \\
-\frac{2i\left(b^2(c+dx)^2 \arctan\left(e^{i(a+bx)}\right) - d\left(b(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right) + id\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)\right) + d\right.}{b^3}
$$

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(b^2*(c + d*x)^2*ArcTan[E^(I*(a + b*x))] - d*(b*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))] + I*d*PolyLog[3, (-I)*E^(I*(a + b*x))]) + d*(b*(c +
 d*x)*PolyLog[2, I*E^(I*(a + b*x))] + I*d*PolyLog[3, I*E^(I*(a + b*x))])))/
b^3
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(120) = 240$.

Time = 1.47 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.86

method	result
risch	$\frac{4icda\arctan\left(e^{i(bx+a)}\right)}{b^2} - \frac{2ic^2\arctan\left(e^{i(bx+a)}\right)}{b} + \frac{a^2d^2\ln\left(1+ie^{i(bx+a)}\right)}{b^3} - \frac{d^2\ln\left(1+ie^{i(bx+a)}\right)x^2}{b} + \frac{d^2\ln\left(1-ie^{i(bx+a)}\right)x^2}{b} - \frac{2c$

```
[In] int((d*x+c)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-2*I/b*c^2*arctan(exp(I*(b*x+a)))+1/b^3
*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+1/b*d^2*
ln(1-I*exp(I*(b*x+a)))*x^2-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2*d^2*pol
ylog(3,I*exp(I*(b*x+a)))/b^3-2/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+2/b*c*d*ln(1-
I*exp(I*(b*x+a)))*x-2/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a-2*I/b^2*c*d*polylog(
2,I*exp(I*(b*x+a)))-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))+2/b^2*c*d*ln(1-I*e
xp(I*(b*x+a)))*a-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-2*I/b^2*d^2*polylog
(2,I*exp(I*(b*x+a)))*x+2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x+2*I/b^2*c
*d*polylog(2,-I*exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(111) = 222$.

Time = 0.32 (sec) , antiderivative size = 598, normalized size of antiderivative = 4.36

$$\int (c+dx)^2\sec(a+bx)\,dx =$$

$$-\frac{2\,d^2\mathrm{polylog}(3, i\,\cos(bx+a)+\sin(bx+a)) - 2\,d^2\mathrm{polylog}(3, i\,\cos(bx+a)-\sin(bx+a)) + 2\,d^2\mathrm{polylog}$$

```
[In] integrate((d*x+c)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(I*b*d^2*x + I*
b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*dilog
(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x
 + a) + sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x +
a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)
```

```
*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*
b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2
*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) -
 (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) +
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/b^
3
```

Sympy [F]

$$\int (c+dx)^2 \sec(a+bx)\,dx = \int (c+dx)^2 \sec\left(a+bx\right)\,dx$$

```
[In] integrate((d*x+c)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(111) = 222$.

Time $= 0.42$ (sec) , antiderivative size $= 402$, normalized size of antiderivative $= 2.93$

$$\int (c+dx)^2 \sec(a+bx)\,dx$$

$$= \frac{2\,c^2 \log\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right) - \frac{4\,acd\log(\sec(bx+a)+\tan(bx+a))}{b} + \frac{2\,a^2d^2\log(\sec(bx+a)+\tan(bx+a))}{b^2} + \frac{4\,d^2\mathrm{Li}_3(i\,e^{(i\,bx}}{}$$

```
[In] integrate((d*x+c)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^2*log(sec(b*x + a) + tan(b*x + a)) - 4*a*c*d*log(sec(b*x + a) + ta
n(b*x + a))/b + 2*a^2*d^2*log(sec(b*x + a) + tan(b*x + a))/b^2 + (4*d^2*pol
ylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*b*x + I*a)) - 2*(I*(b
*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*arctan2(cos(b*x + a), sin(
b*x + a) + 1) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*arc
tan2(cos(b*x + a), -sin(b*x + a) + 1) - 4*(I*b*c*d + I*(b*x + a)*d^2 - I*a*
d^2)*dilog(I*e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*di
log(-I*e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin
(b*x + a) + 1))/b^2)/b
```

Giac [F]

$$\int (c+dx)^2 \sec(a+bx)\,dx = \int (dx+c)^2 \sec(bx+a)\,dx$$

```
[In] integrate((d*x+c)^2*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\cos(a+b\,x)}\,dx$$

```
[In] int((c + d*x)^2/cos(a + b*x),x)
[Out] int((c + d*x)^2/cos(a + b*x), x)
```

3.31 $\int (c+dx)\sec(a+bx)\,dx$

Optimal result . . . 335
Rubi [A] (verified) . . . 335
Mathematica [A] (verified) . . . 336
Maple [A] (verified) . . . 337
Fricas [B] (verification not implemented) . . . 337
Sympy [F] . . . 338
Maxima [F] . . . 338
Giac [F] . . . 338
Mupad [F(-1)] . . . 338

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (c+dx)\sec(a+bx)\,dx = -\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

```
[Out] -2*I*(d*x+c)*arctan(exp(I*(b*x+a)))/b+I*d*polylog(2,-I*exp(I*(b*x+a)))/b^2-
I*d*polylog(2,I*exp(I*(b*x+a)))/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4266, 2317, 2438}

$$\int (c+dx)\sec(a+bx)\,dx = -\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

```
[In] Int[(c + d*x)*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

integral

$$= -\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-ie^{i(a+bx)}\right)\,dx}{b} + \frac{d\int \log\left(1+ie^{i(a+bx)}\right)\,dx}{b}$$

$$= -\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{(id)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^2} - \frac{(id)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^2}$$

$$= -\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int (c+dx)\sec(a+bx)\,dx = -\frac{2idx\arctan\left(e^{ia+ibx}\right)}{b} + \frac{c\text{arctanh}(\sin(a+bx))}{b} + \frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

```
[In] Integrate[(c + d*x)*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*a + I*b*x)])/b + (c*ArcTanh[Sin[a + b*x]])/b + (I*d
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))]
)/b^2
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{-\frac{da\ln(\sec(bx+a)+\tan(bx+a))}{b}+c\ln(\sec(bx+a)+\tan(bx+a))+\frac{d\left(-(bx+a)\ln\left(1+ie^{i(bx+a)}\right)+(bx+a)\ln\left(1-ie^{i(bx+a)}\right)+i\,\mathrm{dilog}\left(1\right.\right.}{b}}{b}$
default	$\frac{-\frac{da\ln(\sec(bx+a)+\tan(bx+a))}{b}+c\ln(\sec(bx+a)+\tan(bx+a))+\frac{d\left(-(bx+a)\ln\left(1+ie^{i(bx+a)}\right)+(bx+a)\ln\left(1-ie^{i(bx+a)}\right)+i\,\mathrm{dilog}\left(1\right.\right.}{b}}{b}$
risch	$-\frac{2ic\arctan\left(e^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1+ie^{i(bx+a)}\right)x}{b}-\frac{d\ln\left(1+ie^{i(bx+a)}\right)a}{b^2}+\frac{d\ln\left(1-ie^{i(bx+a)}\right)x}{b}+\frac{d\ln\left(1-ie^{i(bx+a)}\right)a}{b^2}$

```
[In] int((d*x+c)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/b*d*a*ln(sec(b*x+a)+tan(b*x+a))+c*ln(sec(b*x+a)+tan(b*x+a))+1/b*d*(
-(b*x+a)*ln(1+I*exp(I*(b*x+a)))+(b*x+a)*ln(1-I*exp(I*(b*x+a)))+I*dilog(1+I*
exp(I*(b*x+a)))-I*dilog(1-I*exp(I*(b*x+a)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.08

$$\int (c+dx)\sec(a+bx)\,dx$$
$$= \frac{-i\,d\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a))-i\,d\mathrm{Li}_2(i\,\cos(bx+a)-\sin(bx+a))+i\,d\mathrm{Li}_2(-i\,\cos(bx+a)+si}{}$$

```
[In] integrate((d*x+c)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I*c
os(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a*d)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) -
 sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)
- (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-
cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(
b*x + a) + I))/b^2
```

Sympy [F]

$$\int (c+dx)\sec(a+bx)\,dx = \int (c+dx)\sec(a+bx)\,dx$$

```
[In] integrate((d*x+c)*sec(b*x+a),x)
[Out] Integral((c + d*x)*sec(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\sec(a+bx)\,dx = \int (dx+c)\sec(bx+a)\,dx$$

```
[In] integrate((d*x+c)*sec(b*x+a),x, algorithm="maxima")
[Out] 1/2*(4*b*d*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x + 2*a)*
sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1), x) + c*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(
b*x + a) + 1) - c*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
)/b
```

Giac [F]

$$\int (c+dx)\sec(a+bx)\,dx = \int (dx+c)\sec(bx+a)\,dx$$

```
[In] integrate((d*x+c)*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sec(a+bx)\,dx = \int \frac{c+d\,x}{\cos(a+b\,x)}\,dx$$

```
[In] int((c + d*x)/cos(a + b*x),x)
[Out] int((c + d*x)/cos(a + b*x), x)
```

3.32 $\int \frac{\sec(a+bx)}{c+dx}\,dx$

Optimal result . . . 339
Rubi [N/A] . . . 339
Mathematica [N/A] . . . 340
Maple [N/A] (verified) . . . 340
Fricas [N/A] . . . 340
Sympy [N/A] . . . 340
Maxima [N/A] . . . 341
Giac [N/A] . . . 341
Mupad [N/A] . . . 341

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\sec(a+bx)}{c+dx}, x\right)$$

```
[Out] Unintegrable(sec(b*x+a)/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)}{c+dx}\,dx$$

```
[In] Int[Sec[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Sec[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sec(a+bx)}{c+dx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)}{c+dx}\,dx$$

```
[In] Integrate[Sec[a + b*x]/(c + d*x),x]
[Out] Integrate[Sec[a + b*x]/(c + d*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sec\left(bx+a\right)}{dx+c}dx$$

```
[In] int(sec(b*x+a)/(d*x+c),x)
[Out] int(sec(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)}{dx+c}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(a+bx\right)}{c+dx}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*x+c),x)
[Out] Integral(sec(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)}{dx+c}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)}{dx+c}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 13.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int(1/(cos(a + b*x)*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)*(c + d*x)), x)
```

3.33 $\int (c+dx)^3 \sec^2(a+bx)\,dx$

Optimal result . . . 342
Rubi [A] (verified) . . . 342
Mathematica [A] (verified) . . . 344
Maple [B] (verified) . . . 345
Fricas [B] (verification not implemented) . . . 345
Sympy [F] . . . 346
Maxima [B] (verification not implemented) . . . 346
Giac [F] . . . 347
Mupad [F(-1)] . . . 347

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = -\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4} + \frac{(c+dx)^3\tan(a+bx)}{b}$$

```
[Out] -I*(d*x+c)^3/b+3*d*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^2-3*I*d^2*(d*x+c)*pol
ylog(2,-exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+(d*x
+c)^3*tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4269, 3800, 2221, 2611, 2320, 6724}

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = \frac{3d^3\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4} - \frac{3id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{3d(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{i(c+dx)^3}{b}$$

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b + (3*d*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^2 -
 ((3*I)*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + (3*d^3*PolyLo
g[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*Tan[a + b*x])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\tan(a+bx)\,dx}{b} \\
&= -\frac{i(c+dx)^3}{b} + \frac{(c+dx)^3\tan(a+bx)}{b} + \frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx}{b} \\
&= -\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} \\
&\quad + \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{(6d^2)\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} \\
&\quad + \frac{(c+dx)^3\tan(a+bx)}{b} + \frac{(3id^3)\int \text{PolyLog}\left(2,-e^{2i(a+bx)}\right)\,dx}{b^3} \\
&= -\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} \\
&\quad + \frac{(c+dx)^3\tan(a+bx)}{b} + \frac{(3d^3)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^4} \\
&= -\frac{i(c+dx)^3}{b} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} \\
&\quad + \frac{3d^3\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4} + \frac{(c+dx)^3\tan(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\begin{aligned}
&\int (c+dx)^3\sec^2(a+bx)\,dx \\
&= \frac{-6ibd^2(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right) + 3d^3\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right) + 2b^2(c+dx)^2\left(-ib(c+dx) + 3d\log\right.}{2b^4}
\end{aligned}$$

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] + 3*d^3*PolyLog[3,
 -E^((2*I)*(a + b*x))] + 2*b^2*(c + d*x)^2*((-I)*b*(c + d*x) + 3*d*Log[1 +
E^((2*I)*(a + b*x))] + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(104) = 208$.

Time = 2.14 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.77

method	result
risch	$-\frac{6id^2cx^2}{b}+\frac{4id^3a^3}{b^4}-\frac{6id^2ca^2}{b^3}-\frac{3id^2c\,\mathrm{Li}_2\left(-e^{2i(bx+a)}\right)}{b^3}+\frac{12d^2ca\ln\left(e^{i(bx+a)}\right)}{b^3}+\frac{6d^2c\ln\left(e^{2i(bx+a)}+1\right)x}{b^2}+\frac{2i\left(d^3x^3+3c$

```
[In] int((d*x+c)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -6*I/b*d^2*c*x^2+4*I/b^4*d^3*a^3-6*I/b^3*d^2*c*a^2-3*I/b^3*d^2*c*polylog(2,
-exp(2*I*(b*x+a)))+12/b^3*d^2*c*a*ln(exp(I*(b*x+a)))+6/b^2*d^2*c*ln(exp(2*I
*(b*x+a))+1)*x+2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))+
1)-2*I/b*d^3*x^3+6*I/b^3*d^3*a^2*x-12*I/b^2*d^2*c*a*x-6/b^4*d^3*a^2*ln(exp(
I*(b*x+a)))-3*I/b^3*d^3*polylog(2,-exp(2*I*(b*x+a)))*x+3/b^2*d^3*ln(exp(2*I
*(b*x+a))+1)*x^2+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4-6/b^2*d*c^2*ln(ex
p(I*(b*x+a)))+3/b^2*d*c^2*ln(exp(2*I*(b*x+a))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(101) = 202$.

Time = 0.32 (sec) , antiderivative size = 790, normalized size of antiderivative = 6.93

$$\int (c+dx)^3 \sec^2(a+bx)\,dx$$

$$= \frac{6\,d^3\cos(bx+a)\,\mathrm{polylog}(3, i\cos(bx+a)+\sin(bx+a)) + 6\,d^3\cos(bx+a)\,\mathrm{polylog}(3, i\cos(bx+a)-\sin$$

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*
polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3,
-I*cos(b*x + a) - sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*d
ilog(I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a
)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x
+ a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos
(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a
) + I) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x +
a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
```

```
+ 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1
) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*lo
g(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*
a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*s
in(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-
cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b
^3*c^2*d*x + b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a))
```

Sympy [F]

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = \int (c+dx)^3 \sec^2(a+bx)\,dx$$

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(101) = 202$.

Time $= 0.43$ (sec) , antiderivative size $= 1059$, normalized size of antiderivative $= 9.29$

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*tan(b*x + a) - 6*a*c^2*d*tan(b*x + a)/b + 6*a^2*c*d^2*tan(b*x +
a)/b^2 - 2*a^3*d^3*tan(b*x + a)/b^3 + 3*((cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) - 6*((cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x +
2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b
*x + 2*a))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a) + 1)*b^2) + 3*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*
x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3) + 2*(6*((b*x + a)^2*d^3 + 2*
(b*c*d^2 - a*d^3)*(b*x + a) + ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x
```

```
 + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) -
4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) - 6*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3 + (b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*
b*x + 2*a) + (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin(2*b*x + 2*a))*dilo
g(-e^(2*I*b*x + 2*I*a)) - 3*(I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b
*x + a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) - ((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a
))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) -
3*(I*d^3*cos(2*b*x + 2*a) - d^3*sin(2*b*x + 2*a) + I*d^3)*polylog(3, -e^(2*
I*b*x + 2*I*a)) - 4*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^
2)*sin(2*b*x + 2*a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) -
2*I*b^3))/b
```

Giac [F]

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)^2\,dx$$

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\cos(a+b\,x)^2}\,dx$$

```
[In] int((c + d*x)^3/cos(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^3/cos(a + b*x)^2, x)
```

3.34 $\int (c+dx)^2 \sec^2(a+bx)\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 82

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = -\frac{i(c+dx)^2}{b} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\tan(a+bx)}{b}$$

```
[Out] -I*(d*x+c)^2/b+2*d*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^2-I*d^2*polylog(2,-exp(
2*I*(b*x+a)))/b^3+(d*x+c)^2*tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4269, 3800, 2221, 2317, 2438}

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = -\frac{id^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{i(c+dx)^2}{b}$$

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b + (2*d*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^2 - (
I*d^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^2*Tan[a + b*x])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{(2d)\int (c+dx)\tan(a+bx)\,dx}{b} \\
&= -\frac{i(c+dx)^2}{b} + \frac{(c+dx)^2\tan(a+bx)}{b} + \frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx}{b} \\
&= -\frac{i(c+dx)^2}{b} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} \\
&\quad + \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{(2d^2)\int \log\left(1+e^{2i(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{i(c+dx)^2}{b} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} \\
&\quad + \frac{(c+dx)^2\tan(a+bx)}{b} + \frac{(id^2)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{b^3}
\end{aligned}
$$

$$= -\frac{i(c+dx)^2}{b} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\tan(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int (c+dx)^2\sec^2(a+bx)\,dx = \frac{-id^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right) + b(c+dx)\left(-ib(c+dx) + 2d\log\left(1+e^{2i(a+bx)}\right) + b(c+dx)\tan(a+bx)\right)}{b^3}$$

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-I)*d^2*PolyLog[2, -E^((2*I)*(a + b*x))] + b*(c + d*x)*((-I)*b*(c + d*x)
+ 2*d*Log[1 + E^((2*I)*(a + b*x))] + b*(c + d*x)*Tan[a + b*x]))/b^3
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(76) = 152.

Time = 1.88 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.07

method	result
risch	$\frac{2i\left(x^2d^2+2cdx+c^2\right)}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)} - \frac{4dc\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^2} + \frac{2dc\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b^2} - \frac{2id^2x^2}{b} - \frac{4id^2ax}{b^2} - \frac{2id^2a^2}{b^3} + \frac{2d^2\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)x}{b^2} - \frac{id^2}{}$

```
[In] int((d*x+c)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)-4/b^2*d*c*ln(exp(I*(b*x+a)
))+2/b^2*d*c*ln(exp(2*I*(b*x+a))+1)-2*I/b*d^2*x^2-4*I/b^2*d^2*a*x-2*I/b^3*d
^2*a^2+2/b^2*d^2*ln(exp(2*I*(b*x+a))+1)*x-I*d^2*polylog(2,-exp(2*I*(b*x+a))
)/b^3+4/b^3*d^2*a*ln(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(73) = 146$.

Time $= 0.29$ (sec) , antiderivative size $= 450$, normalized size of antiderivative $= 5.49$

$$\int (c+dx)^2 \sec^2(a+bx)\,dx$$

$$= \frac{i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos(bx+a)+\sin(bx+a)) - i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos(bx+a)-\sin(bx+a)) - i}{}$$

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b*x +
a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b
*x + a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*
x + a)) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) +
I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) +
(b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*
d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d^2
*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x
 + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c*d - a
*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2
)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a))
```

Sympy [F]

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = \int (c+dx)^2 \sec^2(a+bx)\,dx$$

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(73) = 146$.

Time $= 0.49$ (sec) , antiderivative size $= 324$, normalized size of antiderivative $= 3.95$

$$\int (c+dx)^2 \sec^2(a+bx)\,dx$$

$$= \frac{2\,b^2c^2 + 2\,(bd^2x + bcd + (bd^2x + bcd)\cos(2\,bx + 2\,a) - (-i\,bd^2x - i\,bcd)\sin(2\,bx + 2\,a))\arctan(\sin(2\,bx}{}$$

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*b^2*c^2 + 2*(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (-I*
b*d^2*x - I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x +
2*a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(2*b*x + 2*a) - (d^2*cos(2*b*x
 + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(-e^(2*I*b*x + 2*I*a)) + (-I*b
*d^2*x - I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c
*d)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1) - 2*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x)*sin(2*b*x + 2*a))/(-I*
b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) - I*b^3)
```

Giac [F]

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = \int (dx+c)^2 \sec(bx+a)^2\,dx$$

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\cos(a+b\,x)^2}\,dx$$

```
[In] int((c + d*x)^2/cos(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^2/cos(a + b*x)^2, x)
```

3.35 $\int (c+dx)\sec^2(a+bx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 28

$$\int (c+dx)\sec^2(a+bx)\,dx = \frac{d\log(\cos(a+bx))}{b^2} + \frac{(c+dx)\tan(a+bx)}{b}$$

```
[Out] d*ln(cos(b*x+a))/b^2+(d*x+c)*tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4269, 3556}

$$\int (c+dx)\sec^2(a+bx)\,dx = \frac{d\log(\cos(a+bx))}{b^2} + \frac{(c+dx)\tan(a+bx)}{b}$$

```
[In] Int[(c + d*x)*Sec[a + b*x]^2,x]
```

```
[Out] (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(c+dx)\tan(a+bx)}{b} - \frac{d\int \tan(a+bx)\,dx}{b} \\ &= \frac{d\log(\cos(a+bx))}{b^2} + \frac{(c+dx)\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int (c+dx)\sec^2(a+bx)\,dx = \frac{d\log(\cos(a+bx))}{b^2} + \frac{c\tan(a+bx)}{b} + \frac{dx\tan(a+bx)}{b}$$

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2,x]
[Out] (d*Log[Cos[a + b*x]])/b^2 + (c*Tan[a + b*x])/b + (d*x*Tan[a + b*x])/b
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{-\frac{da\tan(bx+a)}{b}+c\tan(bx+a)+\frac{d((bx+a)\tan(bx+a)+\ln(\cos(bx+a)))}{b}}{b}$
default	$\frac{-\frac{da\tan(bx+a)}{b}+c\tan(bx+a)+\frac{d((bx+a)\tan(bx+a)+\ln(\cos(bx+a)))}{b}}{b}$
risch	$-\frac{2idx}{b}-\frac{2ida}{b^2}+\frac{2i(dx+c)}{b\left(e^{2i(bx+a)}+1\right)}+\frac{d\ln\left(e^{2i(bx+a)}+1\right)}{b^2}$
parallelrisch	$\frac{-d\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos(bx+a)+d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\cos(bx+a)+d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\cos(bx+a)+(dx+c)b\sin(bx+a$ $b^2\cos(bx+a)$
norman	$\frac{-\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b^2}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b^2}-\frac{d\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}$

```
[In] int((d*x+c)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/b*d*a*tan(b*x+a)+c*tan(b*x+a)+1/b*d*((b*x+a)*tan(b*x+a)+ln(cos(b*x+
a))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int (c+dx)\sec^2(a+bx)\,dx = \frac{d\cos(bx+a)\log(-\cos(bx+a)) + (bdx+bc)\sin(bx+a)}{b^2\cos(bx+a)}$$

```
[In] integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a)*log(-cos(b*x + a)) + (b*d*x + b*c)*sin(b*x + a))/(b^2*cos(b
*x + a))
```

Sympy [F]

$$\int (c+dx)\sec^2(a+bx)\,dx = \int (c+dx)\sec^2(a+bx)\,dx$$

```
[In] integrate((d*x+c)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(28) = 56.

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 5.68

$$\int (c+dx)\sec^2(a+bx)\,dx$$

$$= \frac{2c\tan(bx+a) - \frac{2ad\tan(bx+a)}{b} + \frac{\left(\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)\log\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2\right.}{\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)b}}{2b}$$

```
[In] integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*tan(b*x + a) - 2*a*d*tan(b*x + a)/b + ((cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(28) = 56$.

Time = 0.64 (sec) , antiderivative size = 1404, normalized size of antiderivative = 50.14

$$\int (c + dx) \sec^2(a + bx)\, dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*tan(1/2*a)^2
 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8
*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)
^4 + 8*tan(1/2*b*x)^3*tan(1/2*a) + 20*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1
/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 2*tan(1/2*b*x)^2 - 8*tan(1/2*b*x)*tan
(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)
^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/
2*b*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 +
1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b*c*tan(1/2*b*x)^2*tan(1/2*a) + 4*b*c*t
an(1/2*b*x)*tan(1/2*a)^2 - 4*b*d*x*tan(1/2*b*x) + d*log(4*(tan(1/2*b*x)^4*t
an(1/2*a)^4 - 2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*tan(1/2*b*x)^3*tan(1/2*a)^3
 - 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 8*tan(1/2*b*x)^3*tan(1/
2*a) + 20*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
/2*a)^4 - 2*tan(1/2*b*x)^2 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1
)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*
b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(
1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 - 4*b*d*x
*tan(1/2*a) + 4*d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*tan(1/2*b*x)^4*tan
(1/2*a)^2 - 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^2*tan(1/2*a)^4 +
 tan(1/2*b*x)^4 + 8*tan(1/2*b*x)^3*tan(1/2*a) + 20*tan(1/2*b*x)^2*tan(1/2*a
)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 2*tan(1/2*b*x)^2 - 8*tan
(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2
*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)
^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*ta
n(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + d*log(4*(tan(1/2*b*x)^4*tan(1/2*
a)^4 - 2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*ta
n(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 8*tan(1/2*b*x)^3*tan(1/2*a) +
20*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4
 - 2*tan(1/2*b*x)^2 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(
1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*
tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*a)^
4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 + 1))*tan(1/2*a)^2 - 4*b*c*tan(1/2*b*
x) - 4*b*c*tan(1/2*a) - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*tan(1/2*b*
x)^4*tan(1/2*a)^2 - 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^2*tan(1/
2*a)^4 + tan(1/2*b*x)^4 + 8*tan(1/2*b*x)^3*tan(1/2*a) + 20*tan(1/2*b*x)^2*t
```

```
an(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 2*tan(1/2*b*x)^2
 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*
a)^4 + 2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(
1/2*b*x)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^
2 + 2*tan(1/2*a)^2 + 1)))/(b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*tan(1/2*b*
x)^2 - 4*b^2*tan(1/2*b*x)*tan(1/2*a) - b^2*tan(1/2*a)^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int (c+dx)\sec^2(a+bx)\,dx = \frac{d\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+1\right)}{b^2} + \frac{(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)} - \frac{d\,x\,2\mathrm{i}}{b}$$

```
[In] int((c + d*x)/cos(a + b*x)^2,x)
```

```
[Out] (d*log(exp(a*2i)*exp(b*x*2i) + 1))/b^2 + ((c + d*x)*2i)/(b*(exp(a*2i + b*x*
2i) + 1)) - (d*x*2i)/b
```

3.36 $\int \frac{\sec^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 358
Rubi [N/A] . . . 358
Mathematica [N/A] . . . 359
Maple [N/A] (verified) . . . 359
Fricas [N/A] . . . 359
Sympy [N/A] . . . 359
Maxima [N/A] . . . 360
Giac [N/A] . . . 360
Mupad [N/A] . . . 360

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\sec^2(a+bx)}{c+dx}, x\right)$$

```
[Out] Unintegrable(sec(b*x+a)^2/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

```
[In] Int[Sec[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Sec[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

```
[In] Integrate[Sec[a + b*x]^2/(c + d*x),x]
[Out] Integrate[Sec[a + b*x]^2/(c + d*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(bx+a)}{dx+c}dx$$

```
[In] int(sec(b*x+a)^2/(d*x+c),x)
[Out] int(sec(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2}{dx+c}\,dx$$

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

```
[In] integrate(sec(b*x+a)**2/(d*x+c),x)
[Out] Integral(sec(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 290, normalized size of antiderivative = 18.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)^2}{dx+c}\,dx$$

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 2*((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c
*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(si
n(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*
c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^
2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + sin(2*b*x + 2
*a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x +
2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)^2}{dx+c}\,dx$$

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 13.70 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

```
[In] int(1/(cos(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^2*(c + d*x)), x)
```

3.37 $\int (c+dx)^3 \sec^3(a+bx)\,dx$

Optimal result . . . 361
Rubi [A] (verified) . . . 362
Mathematica [A] (verified) . . . 365
Maple [B] (verified) . . . 366
Fricas [B] (verification not implemented) . . . 367
Sympy [F] . . . 368
Maxima [B] (verification not implemented) . . . 368
Giac [F] . . . 370
Mupad [F(-1)] . . . 370

Optimal result

Integrand size = 16, antiderivative size = 337

$$\begin{aligned}\int (c+dx)^3 \sec^3(a+bx)\,dx = &-\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3}\\ &-\frac{i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4}\\ &+\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}\\ &-\frac{3id^3\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^4}\\ &-\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}\\ &-\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}\\ &+\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}\\ &-\frac{3id^3\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4}+\frac{3id^3\operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4}\\ &-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

```
[Out] -6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3-I*(d*x+c)^3*arctan(exp(I*(b*x+a
)))/b+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,
-I*exp(I*(b*x+a)))/b^2-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x
+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x
+a)))/b^3+3*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3-3*I*d^3*polylog(4,-
I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/2*d*(d*x+c)
^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4271, 4266, 2317, 2438, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int (c+dx)^3 \sec^3(a+bx)\,dx = &-\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} - \frac{i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b} \\ &+ \frac{3id^3\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4} - \frac{3id^3\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^4} \\ &- \frac{3id^3\,\text{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4} + \frac{3id^3\,\text{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4} \\ &- \frac{3d^2(c+dx)\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\ &+ \frac{3d^2(c+dx)\,\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ &+ \frac{3id(c+dx)^2\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} \\ &- \frac{3id(c+dx)^2\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} \\ &- \frac{3d(c+dx)^2\sec(a+bx)}{2b^2} + \frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}\end{aligned}$$

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^3,x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - (I*(c + d*x)^3*ArcTan[
E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + ((
(3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*P
olyLog[2, I*E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E
^(I*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 + (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d^3*PolyL
og[4, (-I)*E^(I*(a + b*x))])/b^4 + ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
)/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*
Tan[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
 - 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3d(c+dx)^2\sec(a+bx)}{2b^2} + \frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad + \frac{1}{2}\int (c+dx)^3\sec(a+bx)\,dx + \frac{(3d^2)\int (c+dx)\sec(a+bx)\,dx}{b^2} \\
&= -\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} - \frac{i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b} \\
&\quad - \frac{3d(c+dx)^2\sec(a+bx)}{2b^2} + \frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad - \frac{(3d)\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)\,dx}{2b} + \frac{(3d)\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)\,dx}{2b} \\
&\quad - \frac{(3d^3)\int \log\left(1-ie^{i(a+bx)}\right)\,dx}{b^3} + \frac{(3d^3)\int \log\left(1+ie^{i(a+bx)}\right)\,dx}{b^3} \\
&= -\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} - \frac{i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b} \\
&\quad + \frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3d(c+dx)^2\sec(a+bx)}{2b^2} + \frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad - \frac{(3id^2)\int (c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)\,dx}{b^2} \\
&\quad + \frac{(3id^2)\int (c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)\,dx}{b^2} \\
&\quad + \frac{(3id^3)\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{i(a+bx)}\right)}{b^4} - \frac{(3id^3)\operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{i(a+bx)}\right)}{b^4} \\
&= -\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} - \frac{i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b} \\
&\quad + \frac{3id^3\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4} + \frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3id^3\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^4} - \frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{3d(c+dx)^2\sec(a+bx)}{2b^2} + \frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad + \frac{(3d^3)\int \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)\,dx}{b^3} - \frac{(3d^3)\int \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)\,dx}{b^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} - \frac{i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b} \\
&\quad + \frac{3id^3\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4} + \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3id^3\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^4} - \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{3d(c+dx)^2\sec(a+bx)}{2b^2} + \frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad - \frac{(3id^3)\,\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(3,-ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{(3id^3)\,\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(3,ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^4} \\
&= -\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} - \frac{i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b} \\
&\quad + \frac{3id^3\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4} + \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3id^3\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^4} - \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{3id^3\,\mathrm{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4} + \frac{3id^3\,\mathrm{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{3d(c+dx)^2\sec(a+bx)}{2b^2} + \frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.92

$$
\begin{aligned}
&\int (c+dx)^3\sec^3(a+bx)\,dx \\
&= \frac{-2ib^3(c+dx)^3\arctan\left(e^{i(a+bx)}\right) - 6id^2\left(2b(c+dx)\arctan\left(e^{i(a+bx)}\right) - d\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right) + d\,\mathrm{PolyL}\right.}{}
\end{aligned}
$$

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^3,x]
```

```
[Out] ((-2*I)*b^3*(c + d*x)^3*ArcTan[E^(I*(a + b*x))] - (6*I)*d^2*(2*b*(c + d*x)*
ArcTan[E^(I*(a + b*x))] - d*PolyLog[2, (-I)*E^(I*(a + b*x))] + d*PolyLog[2,
 I*E^(I*(a + b*x))]) + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b
*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLo
g[4, (-I)*E^(I*(a + b*x))]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a
```

```
 + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))] - 2*d^2*PolyL
og[4, I*E^(I*(a + b*x))]) - 3*b^2*d*(c + d*x)^2*Sec[a + b*x] + b^3*(c + d*x
)^3*Sec[a + b*x]*Tan[a + b*x])/(2*b^4)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(293) = 586$.

Time = 1.95 (sec) , antiderivative size = 1127, normalized size of antiderivative = 3.34

method	result	size
risch	Expression too large to display	1127

```
[In] int((d*x+c)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I/b^2/(exp(2*I*(b*x+a))+1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp
(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))-d^3*x^3*b*exp(I*(b*x+a))+b*c^3*e
xp(3*I*(b*x+a))-3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(3*I*(b*x+a))-3
*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a))-b*c^3*exp(I*(b*x+a))
-3*I*c^2*d*exp(3*I*(b*x+a))-3*I*d^3*x^2*exp(I*(b*x+a))-6*I*c*d^2*x*exp(I*(b
*x+a))-3*I*c^2*d*exp(I*(b*x+a)))+I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+3/2/b
*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a+3/2/
b^3*a^2*d^2*c*ln(1+I*exp(I*(b*x+a)))-3/2/b^3*a^2*d^2*c*ln(1-I*exp(I*(b*x+a)
))-3/2/b*c*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+3/2/b*c*d^2*ln(1-I*exp(I*(b*x+a))
)*x^2-3/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-3/2/b*c^2*d*ln(1+I*exp(I*(b*x+
a)))*x-3/2*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+3/2*I/b^2*d^3*polylog(
2,-I*exp(I*(b*x+a)))*x^2+6*I/b^4*d^3*a*arctan(exp(I*(b*x+a)))+3/2*I/b^2*c^2
*d*polylog(2,-I*exp(I*(b*x+a)))-3/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))
-6*I/b^3*d^2*c*arctan(exp(I*(b*x+a)))-3*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a
)))+3*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-3*I/b^2*c*d^2*polylog(2,I*
exp(I*(b*x+a)))*x+3*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))+3*I*d^3*polylog(2,
-I*exp(I*(b*x+a)))/b^4-3/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x-1/2/b*d^3*l
n(1+I*exp(I*(b*x+a)))*x^3+1/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+3/b^3*d^3*po
lylog(3,I*exp(I*(b*x+a)))*x-3/b^3*d^3*ln(1+I*exp(I*(b*x+a)))*x-3/b^4*d^3*ln
(1+I*exp(I*(b*x+a)))*a+3/b^3*d^3*ln(1-I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*
exp(I*(b*x+a)))*a-1/2/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))-3/b^3*c*d^2*polylo
g(3,-I*exp(I*(b*x+a)))+1/2/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))+3/b^3*c*d^2*p
olylog(3,I*exp(I*(b*x+a)))+3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-I/b*c^3*
arctan(exp(I*(b*x+a)))-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3*I*d^3*poly
log(4,-I*exp(I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(273) = 546$.

Time $= 0.37$ (sec) , antiderivative size $= 1315$, normalized size of antiderivative $= 3.90$

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I
*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*cos
(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x +
a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^
2*c*d^2*x + I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + si
n(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + 2*I*d^3)*c
os(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*
I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a)
 + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*
d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*
b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*log(cos(b
*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*
d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I
) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 +
 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) +
sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*
b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(I
*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(
b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d +
 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*
c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2
*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^
2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(
b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos(b*x +
 a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos
(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3,
 -I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*pol
ylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
b^2*c^2*d)*cos(b*x + a) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
+ b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

Sympy [F]

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \int (c+dx)^3 \sec^3(a+bx)\,dx$$

```
[In] integrate((d*x+c)**3*sec(b*x+a)**3,x)
[Out] Integral((c + d*x)**3*sec(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3831 vs. $2(273) = 546$.

Time = 1.56 (sec) , antiderivative size = 3831, normalized size of antiderivative = 11.37

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="maxima")
[Out] -1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + lo
g(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log
(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x + a)
/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + lo
g(sin(b*x + a) - 1))/b^3 + 4*(2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)
*(b*x + a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(
b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(4*b
*x + 4*a) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*
(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(2*
b*x + 2*a) + (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(I*b*c*d^2 -
I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*
(b*x + a))*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^
3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 +
(I*a^2 + 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b
*x + a) + 1) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a) + ((
b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*
(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(
(b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I
*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x +
 a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x + a))*sin(
```

```
4*b*x + 4*a) + 2*(I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(I*b*c*d^
2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) +
 4*((b*x + a)^3*d^3 - 3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3 + 3*(b*c*
d^2 - (a + I)*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*(a + I)*b*c*d^2 + (a^2 +
2*I*a)*d^3)*(b*x + a))*cos(3*b*x + 3*a) - 4*((b*x + a)^3*d^3 + 3*I*b^2*c^2*
d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3 + 3*(b*c*d^2 - (a - I)*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*(a - I)*b*c*d^2 + (a^2 - 2*I*a)*d^3)*(b*x + a))*cos(b*x + a
) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d
^2 - a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 +
 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (
I*a^2 + 2*I)*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*
(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + (I*a^2 + 2*I)*d^3 + 2*(I
*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) -
 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2
- a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(b^2*c^2*d - 2*a
*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*cos(2*b*x + 2*a) - (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I
*a^2 - 2*I)*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*
(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 + (-I*a^2 - 2*I)*d^3 + 2*
(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a
)) - (-I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3
)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x +
a) + (-I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 - 3*(I*b*c*d^2 - I*a*d^3
)*(b*x + a)^2 - 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(
I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2
+ 2*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*
a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6
*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^
2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) - (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 - 3*
(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*
a^2 - 2*I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 -
3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 - 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-
I*a^2 - 2*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*(-I*(b*x + a)^3*d^3 - 6*I
*b*c*d^2 + 6*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2
*d + 2*I*a*b*c*d^2 + (-I*a^2 - 2*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*
x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b
^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b
*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(
```

```
b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - 12*(d^3*cos(4*b*x +
4*a) + 2*d^3*cos(2*b*x + 2*a) + I*d^3*sin(4*b*x + 4*a) + 2*I*d^3*sin(2*b*x
+ 2*a) + d^3)*polylog(4, I*e^(I*b*x + I*a)) + 12*(d^3*cos(4*b*x + 4*a) + 2*
d^3*cos(2*b*x + 2*a) + I*d^3*sin(4*b*x + 4*a) + 2*I*d^3*sin(2*b*x + 2*a) +
d^3)*polylog(4, -I*e^(I*b*x + I*a)) + 12*(I*b*c*d^2 + I*(b*x + a)*d^3 - I*a
*d^3 + (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*cos(4*b*x + 4*a) + 2*(I*b*c*
d^2 + I*(b*x + a)*d^3 - I*a*d^3)*cos(2*b*x + 2*a) - (b*c*d^2 + (b*x + a)*d^
3 - a*d^3)*sin(4*b*x + 4*a) - 2*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x
 + 2*a))*polylog(3, I*e^(I*b*x + I*a)) + 12*(-I*b*c*d^2 - I*(b*x + a)*d^3 +
 I*a*d^3 + (-I*b*c*d^2 - I*(b*x + a)*d^3 + I*a*d^3)*cos(4*b*x + 4*a) + 2*(-
I*b*c*d^2 - I*(b*x + a)*d^3 + I*a*d^3)*cos(2*b*x + 2*a) + (b*c*d^2 + (b*x +
 a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 2*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin
(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x + I*a)) + 4*(I*(b*x + a)^3*d^3 + 3*b^
2*c^2*d - 6*a*b*c*d^2 + 3*a^2*d^3 + 3*(I*b*c*d^2 + (-I*a + 1)*d^3)*(b*x + a
)^2 + 3*(I*b^2*c^2*d + 2*(-I*a + 1)*b*c*d^2 + (I*a^2 - 2*a)*d^3)*(b*x + a))
*sin(3*b*x + 3*a) + 4*(-I*(b*x + a)^3*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2 + 3*a
^2*d^3 + 3*(-I*b*c*d^2 + (I*a + 1)*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*(
I*a + 1)*b*c*d^2 + (-I*a^2 - 2*a)*d^3)*(b*x + a))*sin(b*x + a))/(-4*I*b^3*c
os(4*b*x + 4*a) - 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a) + 8*b^3
*sin(2*b*x + 2*a) - 4*I*b^3))/b
```

Giac [F]

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)^3\,dx$$

```
[In] integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^3*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \text{Hanged}$$

```
[In] int((c + d*x)^3/cos(a + b*x)^3,x)
[Out] \text{Hanged}
```

3.38 $\int (c+dx)^2 \sec^3(a+bx)\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 193

$$\begin{aligned}\int (c+dx)^2 \sec^3(a+bx)\,dx = &-\frac{i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{d^2 \operatorname{arctanh}(\sin(a+bx))}{b^3} \\ &+ \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ &- \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\ &- \frac{d^2\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{d^2\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ &- \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2 \sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

```
[Out] -I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+d^2*arctanh(sin(b*x+a))/b^3+I*d*(d*x+
c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))
/b^2-d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+d^2*polylog(3,I*exp(I*(b*x+a)))/b
^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{4271, 3855, 4266, 2611, 2320, 6724\}$

$$\int (c+dx)^2 \sec^3(a+bx)\,dx = -\frac{i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{d^2 \operatorname{arctanh}(\sin(a+bx))}{b^3} - \frac{d^2 \operatorname{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} + \frac{d^2 \operatorname{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3} + \frac{id(c+dx) \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{id(c+dx) \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2 \tan(a+bx)\sec(a+bx)}{2b}$$

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/
b^3 + (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*(c + d*x)
*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))]
)/b^3 + (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x]
)/b^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
 - 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad + \frac{1}{2}\int (c+dx)^2\sec(a+bx)\,dx + \frac{d^2\int \sec(a+bx)\,dx}{b^2} \\
&= -\frac{i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\operatorname{arctanh}(\sin(a+bx))}{b^3} \\
&\quad - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad - \frac{d\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)\,dx}{b} + \frac{d\int (c+dx)\log\left(1+ie^{i(a+bx)}\right)\,dx}{b} \\
&= -\frac{i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\operatorname{arctanh}(\sin(a+bx))}{b^3} \\
&\quad + \frac{id(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad - \frac{(id^2)\int \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)\,dx}{b^2} + \frac{(id^2)\int \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)\,dx}{b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{d^2 \operatorname{arctanh}(\sin(a+bx))}{b^3} \\
&\quad + \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2 \sec(a+bx)\tan(a+bx)}{2b} \\
&\quad - \frac{d^2 \mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(2,-ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^3} + \frac{d^2 \mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(2,ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{b^3} \\
&= -\frac{i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{d^2 \operatorname{arctanh}(\sin(a+bx))}{b^3} \\
&\quad + \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{d^2\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{d^2\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2 \sec(a+bx)\tan(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int (c+dx)^2 \sec^3(a+bx)\,dx \\
&= \frac{-2ib^2(c+dx)^2 \arctan\left(e^{i(a+bx)}\right) + 2d^2 \operatorname{arctanh}(\sin(a+bx)) + 2ibd(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right) - 2ibd(c}{}
\end{aligned}
$$

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-2*I)*b^2*(c + d*x)^2*ArcTan[E^(I*(a + b*x))] + 2*d^2*ArcTanh[Sin[a + b*x
]] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c +
d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))]
 + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 2*b*d*(c + d*x)*Sec[a + b*x] + b^2
*(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(174) = 348.

Time = 1.78 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.03

method	result
risch	$-\frac{2id^2 \arctan\left(e^{i(bx+a)}\right)}{b^3} + \frac{d^2 \operatorname{Li}_3\left(ie^{i(bx+a)}\right)}{b^3} - \frac{d^2 \operatorname{Li}_3\left(-ie^{i(bx+a)}\right)}{b^3} - \frac{i\left(x^2 d^2 b\, e^{3i(bx+a)} + 2cdxb\, e^{3i(bx+a)} + b\, c^2 e^{3i(bx+a)} - x^2 d^2 b\, e^{i}\right.$

```
[In] int((d*x+c)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/b^3*d^2*arctan(exp(I*(b*x+a)))+d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-d^2
*polylog(3,-I*exp(I*(b*x+a)))/b^3-I/b^2/(exp(2*I*(b*x+a))+1)^2*(x^2*d^2*b*e
xp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+a))+b*c^2*exp(3*I*(b*x+a))-x^2*d^2*b
*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(b*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))-b*c^2*e
xp(I*(b*x+a))-2*I*c*d*exp(3*I*(b*x+a))-2*I*d^2*x*exp(I*(b*x+a))-2*I*c*d*exp
(I*(b*x+a)))-1/2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2-1/b^2*c*d*ln(1+I*exp(I*(b
*x+a)))*a+1/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+1/b*c*d*ln(1-I*exp(I*(b*x+a)
))*x+2*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-I/b^2*d^2*polylog(2,I*exp(I*(b*x+
a)))*x+I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-I/b^3*d^2*a^2*arctan(exp(I*
(b*x+a)))+1/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a-1/b*c*d*ln(1+I*exp(I*(b*x+a)))
*x+1/2/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-1/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x
+a)))-I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))+I/b^2*c*d*polylog(2,-I*exp(I*(b
*x+a)))-I/b*c^2*arctan(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(165) = 330$.

Time $= 0.34$ (sec) , antiderivative size $= 795$, normalized size of antiderivative $= 4.12$

$$\int (c+dx)^2 \sec^3(a+bx)\,dx =$$

$$-\frac{2\,d^2\cos(bx+a)^2\,\mathrm{polylog}(3, i\cos(bx+a)+\sin(bx+a)) - 2\,d^2\cos(bx+a)^2\,\mathrm{polylog}(3, i\cos(bx+a) +$$

```
[In] integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^
2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x
+ a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*po
lylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*cos(b*x
+ a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*cos(b
*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*c
os(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c
*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b
*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)
 + (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) -
I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos
(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a
) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*l
og(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b
*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b
```

```
^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*si
n(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(
-cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b*d^2*x + b*c*d)*cos(b*x + a) - 2*
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a)^2)
```

Sympy [F]

$$\int (c+dx)^2 \sec^3(a+bx)\,dx = \int (c+dx)^2 \sec^3(a+bx)\,dx$$

```
[In] integrate((d*x+c)**2*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*sec(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1891 vs. $2(165) = 330$.

Time $= 0.66$ (sec) , antiderivative size $= 1891$, normalized size of antiderivative $= 9.80$

$$\int (c+dx)^2 \sec^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + lo
g(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(s
in(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(sin(
b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b^2 + 4*(2
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + ((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^
2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*
d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) + 2*(I*(b
*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(2*b*x + 2*a)
)*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d -
 a*d^2)*(b*x + a) + 2*d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
 + 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a
) + 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b
*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I
*a*d^2)*(b*x + a) + 2*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b
*x + a) + 1) + 4*((b*x + a)^2*d^2 - 2*I*b*c*d + 2*I*a*d^2 + 2*(b*c*d - (a +
 I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) - 4*((b*x + a)^2*d^2 + 2*I*b*c*d - 2*I
*a*d^2 + 2*(b*c*d - (a - I)*d^2)*(b*x + a))*cos(b*x + a) + 4*(b*c*d + (b*x
```

```
+ a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*
c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2
 - I*a*d^2)*sin(4*b*x + 4*a) + 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2
*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2 +
(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2
 - a*d^2)*cos(2*b*x + 2*a) - (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b
*x + 4*a) - 2*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*dilo
g(-I*e^(I*b*x + I*a)) - (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x +
a) - 2*I*d^2 + (-I*(b*x + a)^2*d^2 - 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*
d^2)*cos(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
 a) + 2*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a) + 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b
*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
sin(b*x + a) + 1) - (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) +
 2*I*d^2 + (I*(b*x + a)^2*d^2 - 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2)
*cos(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a
) - 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
 a) + 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*si
n(b*x + a) + 1) + 4*(I*d^2*cos(4*b*x + 4*a) + 2*I*d^2*cos(2*b*x + 2*a) - d^
2*sin(4*b*x + 4*a) - 2*d^2*sin(2*b*x + 2*a) + I*d^2)*polylog(3, I*e^(I*b*x
+ I*a)) + 4*(-I*d^2*cos(4*b*x + 4*a) - 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4
*b*x + 4*a) + 2*d^2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -I*e^(I*b*x + I*a)
) + 4*(I*(b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(I*b*c*d + (-I*a + 1)*d^2)
*(b*x + a))*sin(3*b*x + 3*a) + 4*(-I*(b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 +
2*(-I*b*c*d + (I*a + 1)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^2*cos(4*b*x +
 4*a) - 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) + 8*b^2*sin(2*b*x
 + 2*a) - 4*I*b^2))/b
```

Giac [F]

$$\int (c+dx)^2 \sec^3(a+bx)\,dx = \int (dx+c)^2 \sec{(bx+a)}^3\,dx$$

```
[In] integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec^3(a+bx)\,dx = \text{Hanged}$$

```
[In] int((c + d*x)^2/cos(a + b*x)^3,x)
[Out] \text{Hanged}
```

3.39 $\int (c+dx)\sec^3(a+bx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 117

$$\int (c+dx)\sec^3(a+bx)\,dx = -\frac{i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} - \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b}$$

```
[Out] -I*(d*x+c)*arctan(exp(I*(b*x+a)))/b+1/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/b^
2-1/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c)*
sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4270, 4266, 2317, 2438}

$$\int (c+dx)\sec^3(a+bx)\,dx = -\frac{i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} - \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

```
[In] Int[(c + d*x)*Sec[a + b*x]^3,x]
```

```
[Out] ((-I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + ((I/2)*d*PolyLog[2, (-I)*E^(I*
(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a +
 b*x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b} + \frac{1}{2}\int (c+dx)\sec(a+bx)\,dx \\
&= -\frac{i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad - \frac{d\int \log\left(1-ie^{i(a+bx)}\right)\,dx}{2b} + \frac{d\int \log\left(1+ie^{i(a+bx)}\right)\,dx}{2b} \\
&= -\frac{i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b} \\
&\quad + \frac{(id)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{2b^2} - \frac{(id)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{i(a+bx)}\right)}{2b^2}
\end{aligned}$$

$$= -\frac{i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}$$
$$- \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 389 vs. $2(117) = 234$.

Time = 4.17 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.32

$$\int (c+dx)\sec^3(a+bx)\,dx = \frac{c\operatorname{arctanh}(\sin(a+bx))}{2b}$$
$$+ \frac{d\left(\left(-a+\frac{\pi}{2}-bx\right)\left(\log\left(1-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\log\left(1+e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)-\left(-a+\frac{\pi}{2}\right)\log\left(\tan\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-\right.\right.\right.\right.}{2b^2}$$
$$+ \frac{dx}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}$$
$$- \frac{d\sin\left(\frac{bx}{2}\right)}{2b^2\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$
$$- \frac{dx}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}$$
$$+ \frac{d\sin\left(\frac{bx}{2}\right)}{2b^2\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)} + \frac{c\sec(a+bx)\tan(a+bx)}{2b}$$

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^3,x]
```

```
[Out] (c*ArcTanh[Sin[a + b*x]])/(2*b) + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a +
 Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-
a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2,
E^(I*(-a + Pi/2 - b*x))])))/(2*b^2) + (d*x)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[
a/2 + (b*x)/2])^2) - (d*Sin[(b*x)/2])/(2*b^2*(Cos[a/2] - Sin[a/2])*(Cos[a/2
 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (d*x)/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a
/2 + (b*x)/2])^2) + (d*Sin[(b*x)/2])/(2*b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2
+ (b*x)/2] + Sin[a/2 + (b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(98) = 196$.

Time $= 1.38$ (sec) , antiderivative size $= 267$, normalized size of antiderivative $= 2.28$

method	result
risch	$-\frac{i(bxd\,e^{3i(bx+a)}+bc\,e^{3i(bx+a)}-bxd\,e^{i(bx+a)}-bc\,e^{i(bx+a)}-id\,e^{3i(bx+a)}-id\,e^{i(bx+a)})}{b^2(e^{2i(bx+a)}+1)^2}-\frac{ic\arctan(e^{i(bx+a)})}{b}-\frac{d\ln(1+ie^{i(bx+a)})x}{2b}$

```
[In] int((d*x+c)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I/b^2/(exp(2*I*(b*x+a))+1)^2*(b*x*d*exp(3*I*(b*x+a))+b*c*exp(3*I*(b*x+a))-
b*x*d*exp(I*(b*x+a))-b*c*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))-I*d*exp(I*(b*x
+a)))-I/b*c*arctan(exp(I*(b*x+a)))-1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/2/b^2
*d*ln(1+I*exp(I*(b*x+a)))*a+1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/2/b^2*d*ln(1
-I*exp(I*(b*x+a)))*a+1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))-1/2*I/b^2*d*dilo
g(1-I*exp(I*(b*x+a)))+I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(93) = 186$.

Time $= 0.35$ (sec) , antiderivative size $= 435$, normalized size of antiderivative $= 3.72$

$$\int (c+dx)\sec^3(a+bx)\,dx$$

$$= \frac{-i\,d\cos(bx+a)^2\,\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a))-i\,d\cos(bx+a)^2\,\mathrm{Li}_2(i\,\cos(bx+a)-\sin(bx+a))+i}{}$$

```
[In] integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*cos(b*x
 + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*
cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - s
in(b*x + a)) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) +
 (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d
*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x +
a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)
*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*cos(b
*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*cos(b*x + a
)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) + 2*(b*d*x +
 b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

Sympy [F]

$$\int (c+dx)\sec^3(a+bx)\,dx = \int (c+dx)\sec^3(a+bx)\,dx$$

```
[In] integrate((d*x+c)*sec(b*x+a)**3,x)
[Out] Integral((c + d*x)*sec(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)\sec^3(a+bx)\,dx = \int (dx+c)\sec(bx+a)^3\,dx$$

```
[In] integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="maxima")
[Out] -1/4*(4*(d*cos(3*b*x + 3*a) + d*cos(b*x + a) - (b*d*x + b*c)*sin(3*b*x + 3*
a) + (b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + 4*(2*d*cos(2*b*x + 2*a)
 + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 8*(d*cos(b*x +
a) + (b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + 4*d*cos(b*x + a) - 4*(b
^2*d*cos(4*b*x + 4*a)^2 + 4*b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(4*b*x + 4*
a)^2 + 4*b^2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*d*sin(2*b*x + 2*a)
^2 + 4*b^2*d*cos(2*b*x + 2*a) + b^2*d + 2*(2*b^2*d*cos(2*b*x + 2*a) + b^2*d
)*cos(4*b*x + 4*a))*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*
x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - (b*c*cos(4*b*x + 4*a)^2 + 4*b*c*cos(2
*b*x + 2*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 4*b*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c + 2*(2*b*c*
cos(2*b*x + 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*sin(b*x + a) + 1) + (b*c*cos(4*b*x + 4*a)^2 + 4*b*c*cos(2*b*x + 2*a)
^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b
*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c + 2*(2*b*c*cos(2*b*x +
 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(
b*x + a) + 1) + 4*((b*d*x + b*c)*cos(3*b*x + 3*a) - (b*d*x + b*c)*cos(b*x +
 a) + d*sin(3*b*x + 3*a) + d*sin(b*x + a))*sin(4*b*x + 4*a) - 4*(b*d*x + b*
c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*sin(3*b*x + 3*
a) - 8*((b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))*sin(2*b*x + 2*a) + 4*(
b*d*x + b*c)*sin(b*x + a))/(b^2*cos(4*b*x + 4*a)^2 + 4*b^2*cos(2*b*x + 2*a)
^2 + b^2*sin(4*b*x + 4*a)^2 + 4*b^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b
^2*sin(2*b*x + 2*a)^2 + 4*b^2*cos(2*b*x + 2*a) + b^2 + 2*(2*b^2*cos(2*b*x +
 2*a) + b^2)*cos(4*b*x + 4*a))
```

Giac [F]

$$\int (c+dx)\sec^3(a+bx)\,dx = \int (dx+c)\sec\left(bx+a\right)^3\,dx$$

```
[In] integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sec^3(a+bx)\,dx = \text{Hanged}$$

```
[In] int((c + d*x)/cos(a + b*x)^3,x)
[Out] \text{Hanged}
```

3.40 $\int \frac{\sec^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 385
Rubi [N/A] . . . 385
Mathematica [N/A] . . . 386
Maple [N/A] (verified) . . . 386
Fricas [N/A] . . . 386
Sympy [N/A] . . . 386
Maxima [N/A] . . . 387
Giac [N/A] . . . 387
Mupad [N/A] . . . 387

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\sec^2(a+bx)}{c+dx}, x\right)$$

```
[Out] Unintegrable(sec(b*x+a)^2/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

```
[In] Int[Sec[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Sec[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

```
[In] Integrate[Sec[a + b*x]^2/(c + d*x),x]
[Out] Integrate[Sec[a + b*x]^2/(c + d*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(bx+a)}{dx+c}dx$$

```
[In] int(sec(b*x+a)^2/(d*x+c),x)
[Out] int(sec(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2}{dx+c}\,dx$$

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

```
[In] integrate(sec(b*x+a)**2/(d*x+c),x)
[Out] Integral(sec(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 290, normalized size of antiderivative = 18.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)^2}{dx+c}\,dx$$

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 2*((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c
*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(si
n(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*
c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^
2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + sin(2*b*x + 2
*a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x +
2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)^2}{dx+c}\,dx$$

```
[In] integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

```
[In] int(1/(cos(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^2*(c + d*x)), x)
```

3.41 $\int (c+dx)^{5/2}\cos(a+bx)\,dx$

Optimal result . . . 388
Rubi [A] (verified) . . . 388
Mathematica [C] (verified) . . . 391
Maple [A] (verified) . . . 391
Fricas [A] (verification not implemented) . . . 392
Sympy [F] . . . 392
Maxima [C] (verification not implemented) . . . 392
Giac [C] (verification not implemented) . . . 393
Mupad [F(-1)] . . . 394

Optimal result

Integrand size = 16, antiderivative size = 194

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{7/2}} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b}$$

```
[Out] 5/2*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2+(d*x+c)^(5/2)*sin(b*x+a)/b+15/8*d^(5/2)*
cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/
2)*Pi^(1/2)/b^(7/2)+15/8*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/4*d^2*sin(b*x+a)*(d
*x+c)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{3377, 3387, 3386, 3432, 3385, 3433\}$

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}}$$
$$+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3}$$
$$+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b}$$

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x],x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(2*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a -
 (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(7/2))
 + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[a - (b*c)/d])/(4*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(4*
b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/b
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(c+dx)^{5/2}\sin(a+bx)}{b} - \frac{(5d)\int (c+dx)^{3/2}\sin(a+bx)\,dx}{2b} \\
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b} - \frac{(15d^2)\int \sqrt{c+dx}\cos(a+bx)\,dx}{4b^2} \\
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3} \\
&\quad + \frac{(c+dx)^{5/2}\sin(a+bx)}{b} + \frac{(15d^3)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{8b^3} \\
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b} \\
&\quad + \frac{\left(15d^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{8b^3} + \frac{\left(15d^3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{8b^3} \\
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b} \\
&\quad + \frac{\left(15d^2\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{4b^3} \\
&\quad + \frac{\left(15d^2\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{4b^3} \\
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}} \\
&\quad + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{7/2}} \\
&\quad - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.64

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = -\frac{d^3 e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x],x]
```

```
[Out] -1/2*(d^3*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((-I)*b*(c + d
*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, (I*b*(c + d*
x))/d]))/(b^4*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\right.}{b}\right.}{d}$
default	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\right.}{b}\right.}{d}$

```
[In] int((d*x+c)^(5/2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2/b*d*(d*x+c)^(5/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-5/2/b*d*(-1/2/b*d*(
d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2)*si
n(b*(d*x+c)/d+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b
*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*
c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-a}{d}\right.}{8}$$

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="fricas")
[Out] 1/8*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 2*sqrt(d*x
 + c)*(10*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) + (4*b^3*d^2*x^2 + 8*b^3*c*d*x
 + 4*b^3*c^2 - 15*b*d^2)*sin(b*x + a)))/b^4
```

Sympy [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\cos\left(a+bx\right)dx$$

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a),x)
[Out] Integral((c + d*x)**(5/2)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.36

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \frac{\sqrt{2}\left(40\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{(dx+c)b-bc+ad}{d}\right) - 15\left(-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right) + (i-1)\sqrt{\pi}\right.\right.}{}$$

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="maxima")
[Out] 1/32*sqrt(2)*(40*sqrt(2)*(d*x + c)^(3/2)*b^2*d*cos(((d*x + c)*b - b*c + a*d
)/d) - 15*(-(I + 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I -
 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) - 15*((I - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) -
(I + 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(-I*b/d)) + 4*(4*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c
)*b*d^2)*sin(((d*x + c)*b - b*c + a*d)/d))/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 1246, normalized size of antiderivative = 6.42

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(8*(I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)))*c^3 + 6*c*d^2*((I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*
c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2
) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)
*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi
)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c
*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) +
 d^3*((-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*
d^3)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(-
4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x +
c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*s
qrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (I*sqrt
(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/
2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(4*I*(d*x + c)^(
5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d -
10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^
3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 12*(-I*sqrt(2)*sqrt(pi
)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b) + I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 2*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c +
I*a*d)/d)/b - 2*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)
*c^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\left(c+d\,x\right)^{5/2}\,dx$$

```
[In] int(cos(a + b*x)*(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)*(c + d*x)^(5/2), x)
```

3.42 $\int (c+dx)^{3/2}\cos(a+bx)\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 169

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = \frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{2b^{5/2}} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b}$$

```
[Out] (d*x+c)^(3/2)*sin(b*x+a)/b-3/4*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/4*d^(3/2)*Fres
nelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*P
i^(1/2)/b^(5/2)+3/2*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3387, 3386, 3432, 3385, 3433}

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = -\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}} + \frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b}$$

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x],x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(2*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b
*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*b^(5/2)) +
(3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[a - (b*c)/d])/(2*b^(5/2)) + ((c + d*x)^(3/2)*Sin[a + b*x])/b
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^{3/2}\sin(a+bx)}{b} - \frac{(3d)\int \sqrt{c+dx}\sin(a+bx)\,dx}{2b}$$

$$
\begin{aligned}
&= \frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b} - \frac{(3d^2)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{4b^2} \\
&= \frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b} \\
&\quad - \frac{\left(3d^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{4b^2} + \frac{\left(3d^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{4b^2} \\
&= \frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b} \\
&\quad - \frac{\left(3d\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{2b^2} \\
&\quad + \frac{\left(3d\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{2b^2} \\
&= \frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}} \\
&\quad + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{2b^{5/2}} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.72

$$
\int (c+dx)^{3/2}\cos(a+bx)\,dx = \frac{de^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} + \frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{5}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b^2}
$$

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x],x]
```

```
[Out] (d*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[5/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d] + (E^(((2*I)*b*c)/d)*Gamma[5/2, (I*b*(c + d*x))/d])/Sqrt[(
I*b*(c + d*x))/d]))/(2*b^2*E^((I*(b*c + a*d))/d))
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)-\sin\left(\frac{ad-bc}{d}\right)\right.}{4b\sqrt{\frac{b}{d}}}\right.}{b}}{d}$
default	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)-\sin\left(\frac{ad-bc}{d}\right)\right.}{4b\sqrt{\frac{b}{d}}}\right.}{b}}{d}$

```
[In] int((d*x+c)^(3/2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2/b*d*(d*x+c)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(
d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1
/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)
/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx =$$

$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\,(3\,b}{4\,b^3}$$

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(
2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_
sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*(3*b*d*co
s(b*x + a) + 2*(b^2*d*x + b^2*c)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\cos\left(a+bx\right)dx$$

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a),x)
[Out] Integral((c + d*x)**(3/2)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = \frac{\sqrt{2}\left(8\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+12\sqrt{2}\sqrt{dx+c}bd\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-3\left(-(i-1)\sqrt{}\right.\right.}{}$$

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="maxima")
[Out] 1/16*sqrt(2)*(8*sqrt(2)*(d*x + c)^(3/2)*b^2*sin(((d*x + c)*b - b*c + a*d)/d
) + 12*sqrt(2)*sqrt(d*x + c)*b*d*cos(((d*x + c)*b - b*c + a*d)/d) - 3*(-(I
 - 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^
2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 3*(
(I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)
*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/
b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 779, normalized size of antiderivative = 4.61

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx =$$

$$4\left(\frac{i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+d^2\left(\frac{i\sqrt{2}}{}\right.$$

$$-\frac{}{}$$

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(4*(I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)))*c^2 + d^2*((I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d -
 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2
*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)
*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(4*
b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d -
3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 4*(-
I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*I*sqrt(d*x + c)*d*e^((I*(d*x
+ c)*b - I*b*c + I*a*d)/d)/b - 2*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b)*c)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = \int \cos(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

```
[In] int(cos(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*(c + d*x)^(3/2), x)
```

3.43 $\int \sqrt{c+dx}\cos(a+bx)\,dx$

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Rubi [A] (verified) . . . 401
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Optimal result

Integrand size = 16, antiderivative size = 142

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{b}$$

```
[Out] -1/2*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+sin(b*x+a)
*(d*x+c)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3387, 3386, 3432, 3385, 3433}

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = -\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{b}$$

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x],x]
```

```
[Out] -((Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/P
i]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/b^(3/2) + (Sqrt[c + d*x]*Sin[a
 + b*x])/b
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx}\sin(a+bx)}{b} - \frac{d\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{2b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{b} - \frac{\left(d\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b} - \frac{\left(d\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{b} - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{b}$$
$$- \frac{\sin\left(a-\frac{bc}{d}\right)\text{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{b}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}}$$
$$- \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int \sqrt{c+dx}\cos(a+bx)\,dx$$
$$= \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right) + e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b^2\sqrt{c+dx}}$$

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x],x]
```

```
[Out] (d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d]
 + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d])
)/(2*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2b\sqrt{\frac{b}{d}}}}{d}$	144
default	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2b\sqrt{\frac{b}{d}}}}{d}$	144

```
[In] int((d*x+c)^(1/2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2/b*d*(d*x+c)^(1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^
(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = -\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\sqrt{dx+}}{2\,b^2}$$

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*s
qrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*sqrt(d*x + c)*b*s
in(b*x + a))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = \int \sqrt{c+dx}\cos\left(a+bx\right)dx$$

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.38

$$\int \sqrt{c+dx}\cos(a+bx)\,dx$$

$$= \frac{\sqrt{2}\left(4\sqrt{2}\sqrt{dx+c}b\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left(-(i+1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\right.\right.\right.}{}$$

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(4*sqrt(2)*sqrt(d*x + c)*b*sin(((d*x + c)*b - b*c + a*d)/d) + (
-(I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*
d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((I
 - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d*(
b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.00

$$\int \sqrt{c+dx}\cos(a+bx)\,dx =$$

$$-\frac{i\sqrt{2}\sqrt{\pi}(2\,bc+i\,d)d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{i\sqrt{2}\sqrt{\pi}(2\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}$$

$$-\frac{}{}$$

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*(I*sqrt(2)*sqrt(pi)*d*
erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(
pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c + 2*I*sq
rt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 2*I*sqrt(d*x + c)*d
*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

```
[In] int(cos(a + b*x)*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)*(c + d*x)^(1/2), x)
```

3.44 $\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result . . . 407
Rubi [A] (verified) . . . 407
Mathematica [C] (verified) . . . 409
Maple [A] (verified) . . . 409
Fricas [A] (verification not implemented) . . . 410
Sympy [F] . . . 410
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Giac [C] (verification not implemented) . . . 411
Mupad [F(-1)] . . . 411

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}} - \frac{\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/
2)*Pi^(1/2)/b^(1/2)/d^(1/2)-FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}} - \frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[In] Int[Cos[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx - \sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx \\
&= \frac{\left(2\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d} \\
&\quad - \frac{\left(2\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d} \\
&= \frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}} - \frac{\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

```
[In] Integrate[Cos[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] ((I/2)*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d*
x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c + d*
x))/d]))/(b*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$	100
default	$\frac{\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$	100

```
[In] int(cos(b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)
/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)}{b}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d))/b
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos\left(a+bx\right)}{\sqrt{c+dx}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)/sqrt(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx =$$
$$-\frac{\sqrt{2}\left(\left((i-1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right) + (i+1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right) + \left(-(i+1\right.}{4\,b}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*(((I - 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I +
1)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b
/d)) + (-(I + 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqr
t(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))
/b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.42

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx$$

$$= -\frac{\frac{i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)} - \frac{i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}}{2\,d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos(a+b\,x)}{\sqrt{c+d\,x}}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^(1/2), x)
```

3.45 $\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result . . . 412
Rubi [A] (verified) . . . 412
Mathematica [C] (verified) . . . 414
Maple [A] (verified) . . . 414
Fricas [A] (verification not implemented) . . . 415
Sympy [F] . . . 415
Maxima [C] (verification not implemented) . . . 415
Giac [F] . . . 416
Mupad [F(-1)] . . . 416

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{2\sqrt{b}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}}$$

```
[Out] -2*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^
(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(a-b*c/d)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*cos(b*x+a)/d
/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\sqrt{2\pi}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sqrt{2\pi}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(d*Sqrt[c + d*x]) - (2*Sqrt[b]*Sqrt[2*Pi]*Cos[a - (b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (2*Sqrt[b
]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (
b*c)/d])/d^(3/2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\text{integral} = -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{(2b)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{\left(2b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d} - \frac{\left(2b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{\left(4b\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{\left(4b\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{2\sqrt{b}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-ia}\left(e^{2ia-\frac{ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right) + e^{-ibx}\left(-1-e^{2i(a+bx)}+e^{\frac{ib(c+dx)}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},\right.\right.\right.}{d\sqrt{c+dx}}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (E^((2*I)*a - (I*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c +
 d*x))/d] + (-1 - E^((2*I)*(a + b*x)) + E^((I*b*(c + d*x))/d)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(d*E^(I*a)*Sqrt[c + d*
x])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}} - \frac{2b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\operatorname{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	140
default	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}} - \frac{2b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\operatorname{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\operatorname{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	140

```
[In] int(cos(b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/(d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/
d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(\right.\right.}{d^2x+cd}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*
d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) +
 sqrt(d*x + c)*cos(b*x + a))/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(a+bx\right)}{(c+dx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\left(\left(-(i+1)\,\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{i\,(dx+c)b}{d}\right)+(i-1)\,\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i-\right.\right.}{4\sqrt{dx+c}}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*((-(I + 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma
(-1/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-1/2
, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(-(b
*c - a*d)/d))*sqrt((d*x + c)*b/d)/(sqrt(d*x + c)*d)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)}{(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^{3/2}}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)/(c + d*x)^(3/2), x)
```

3.46 $\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result . . . 417
Rubi [A] (verified) . . . 417
Mathematica [C] (verified) . . . 419
Maple [A] (verified) . . . 420
Fricas [A] (verification not implemented) . . . 420
Sympy [F] . . . 421
Maxima [C] (verification not implemented) . . . 421
Giac [F] . . . 421
Mupad [F(-1)] . . . 422

Optimal result

Integrand size = 16, antiderivative size = 168

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{4b^{3/2}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{3d^{5/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}}$$

```
[Out] -2/3*cos(b*x+a)/d/(d*x+c)^(3/2)-4/3*b^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)+4/3*b^(3/2)
*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1
/2)*Pi^(1/2)/d^(5/2)+4/3*b*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{4\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{4\sqrt{2\pi}b^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(3*d*(c + d*x)^(3/2)) - (4*b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*
c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (
4*b^(3/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[a - (b*c)/d])/(3*d^(5/2)) + (4*b*Sin[a + b*x])/(3*d^2*Sqrt[c + d*x])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\text{integral} = -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} - \frac{(2b)\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{(4b^2)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(4b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(4b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(8b^2\cos\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{3d^3} + \frac{\left(8b^2\sin\left(a-\frac{bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{3d^3}$$

$$= -\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{4b^{3/2}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{3d^{5/2}} + \frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.13

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{e^{-ia}\left(-2ie^{2ia-\frac{ibc}{d}}\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx)) - 2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2}, -\frac{ib(c+dx)}{d}\right)\right) + \cdots}{6d^2(c+dx)^{3/2}}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] ((-2*I)*E^((2*I)*a - (I*b*c)/d)*(E^((I*b*(c + d*x))/d)*((-I)*d + 2*b*(c + d
*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-I)*b*(c + d*x))/d
]) + (-2*d + (4*I)*b*(c + d*x) - 4*d*E^((I*b*(c + d*x))/d)*((I*b*(c + d*x))
/d)^(3/2)*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(6*d^2*E^(I*a)*(c + d*x
)^(3/2))
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\dfrac{-\dfrac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-4b\left(-\dfrac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\dfrac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$
default	$\dfrac{-\dfrac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-4b\left(-\dfrac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\dfrac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$

```
[In] int(cos(b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/3/(d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^(1
/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d
-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-
b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.24

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{2\left(2\sqrt{2}(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-2\sqrt{2}(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2\right.}{3\left(d^4x^2+2\,cd^3x+\right.}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 2*sqrt
(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*(d*c
os(b*x + a) - 2*(b*d*x + b*c)*sin(b*x + a)))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2
)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**(5/2),x)
[Out] Integral(cos(a + b*x)/(c + d*x)**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.77

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{i\,(dx+c)b}{d}\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{i\,(dx+c)b}{d}\right.\right.}{4\left(dx+c\right)^{\frac{3}{2}}d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
[Out] -1/4*(((I - 1)*sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma
(-3/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-3/2
, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin(-(b
*c - a*d)/d))*((d*x + c)*b/d)^(3/2)/((d*x + c)^(3/2)*d)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos\left(bx+a\right)}{\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^{5/2}}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)/(c + d*x)^(5/2), x)
```

3.47 $\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 193

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{15d^{7/2}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}}$$

```
[Out] -2/5*cos(b*x+a)/d/(d*x+c)^(5/2)+4/15*b*sin(b*x+a)/d^2/(d*x+c)^(3/2)+8/15*b^
(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^2*cos(b*x
+a)/d^3/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{3378, 3387, 3386, 3432, 3385, 3433\}$

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{8\sqrt{2\pi}b^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(5*d*(c + d*x)^(5/2)) + (8*b^2*Cos[a + b*x])/(15*d^3*Sqrt
[c + d*x]) + (8*b^(5/2)*Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (8*b^(5/2)*Sqrt[2*Pi]*Fresnel
C[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(15*d^(7/2)
) + (4*b*Sin[a + b*x])/(15*d^2*(c + d*x)^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} - \frac{(2b)\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}\,dx}{5d} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{(4b^2)\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(8b^3)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{15d^3} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} \\
&\quad + \frac{\left(8b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{15d^3} + \frac{\left(8b^3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{15d^3} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}} \\
&\quad + \frac{\left(16b^3\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{15d^4} \\
&\quad + \frac{\left(16b^3\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{15d^4} \\
&= -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} \\
&\quad + \frac{8b^{5/2}\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{15d^{7/2}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.18

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{e^{-ia}\left(2e^{2ia}\left(-3d^2e^{ibx}+2be^{-\frac{ibc}{d}}(c+dx)\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\right.\right.\right.$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (2*E^((2*I)*a)*(-3*d^2*E^(I*b*x) + (2*b*(c + d*x)*(E^((I*b*(c + d*x))/d)*((
-I)*d + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((
-I)*b*(c + d*x))/d]))/E^((I*b*c)/d)) + (-6*d^2 + (4*I)*b*d*(c + d*x) + 8*b^
2*(c + d*x)^2 + 8*d^2*E^((I*b*(c + d*x))/d)*((I*b*(c + d*x))/d)^(5/2)*Gamma
[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(30*d^3*E^(I*a)*(c + d*x)^(5/2))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.14

method	result
derivativedivides	$-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}\right.}{5d}}{d}$
default	$-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}\right.}{5d}}{d}$

```
[In] int(cos(b*x+a)/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/5/(d*x+c)^(5/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-2/5*b/d*(-1/3/(d*x+c)^
(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^(1/2)*cos(b*(d*x+c)/
d+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(
```

```
2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2
^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.53

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{2\left(4\sqrt{2}(\pi b^2 d^3 x^3 + 3\,\pi b^2 c d^2 x^2 + 3\,\pi b^2 c^2 d x + \pi b^2 c^3)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\right.}\right.}{}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + p
i*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) + 4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*p
i*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c
)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*((4*b^2*d^2*x^2 + 8*b
^2*c*d*x + 4*b^2*c^2 - 3*d^2)*cos(b*x + a) + 2*(b*d^2*x + b*c*d)*sin(b*x +
a)))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos\left(a+bx\right)}{(c+dx)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**(7/2),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x)**(7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.67

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx =$$

$$-\frac{\left(\left(-(i+1)\,\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{i\,(dx+c)b}{d}\right)+(i-1)\,\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i-1)\,\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{i\,(dx+c}{d}\right.\right.}{4\,(dx+c)^{\frac{5}{2}}d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/4*((-(I + 1)*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamm
a(-5/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-5/
2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-(
b*c - a*d)/d))*((d*x + c)*b/d)^(5/2)/((d*x + c)^(5/2)*d)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos\left(bx+a\right)}{(dx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^{7/2}}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^(7/2),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^(7/2), x)
```

3.48 $\int (c+dx)^{5/2}\cos^2(a+bx)\,dx$

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Mupad [F(-1)] . . . 436

Optimal result

Integrand size = 18, antiderivative size = 231

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} + \frac{15d^{5/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{7/2}} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} - \frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}$$

```
[Out] -5/16*d*(d*x+c)^(3/2)/b^2+1/7*(d*x+c)^(7/2)/d+5/8*d*(d*x+c)^(3/2)*cos(b*x+a
)^2/b^2+1/2*(d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)/b+15/128*d^(5/2)*cos(2*a-2*
b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+
15/128*d^(5/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2
*b*c/d)*Pi^(1/2)/b^(7/2)-15/64*d^2*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

$= \{3392, 32, 3393, 3377, 3387, 3386, 3432, 3385, 3433\}$

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}$$
$$+\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}$$
$$-\frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}+\frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2}$$
$$+\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}-\frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}$$

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2,x]
```

```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) + (5*d*(c + d*x)^(3
/2)*Cos[a + b*x]^2)/(8*b^2) + (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^
(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2
*a - (2*b*c)/d])/(128*b^(7/2)) + ((c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]
)/(2*b) - (15*d^2*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(64*b^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

```
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad + \frac{1}{2}\int (c+dx)^{5/2}\,dx - \frac{(15d^2)\int \sqrt{c+dx}\cos^2(a+bx)\,dx}{16b^2} \\
&= \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad - \frac{(15d^2)\int \left(\frac{1}{2}\sqrt{c+dx} + \frac{1}{2}\sqrt{c+dx}\cos(2a+2bx)\right)\,dx}{16b^2} \\
&= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} \\
&\quad + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} - \frac{(15d^2)\int \sqrt{c+dx}\cos(2a+2bx)\,dx}{32b^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} \\
&\quad + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad - \frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3} + \frac{(15d^3)\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{128b^3} \\
&= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} \\
&\quad + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} - \frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3} \\
&\quad + \frac{\left(15d^3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{128b^3} + \frac{\left(15d^3\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{128b^3} \\
&= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} \\
&\quad + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} - \frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3} \\
&\quad + \frac{\left(15d^2\cos\left(2a-\frac{2bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{64b^3} \\
&\quad + \frac{\left(15d^2\sin\left(2a-\frac{2bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{64b^3} \\
&= -\frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} + \frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2} \\
&\quad + \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} \\
&\quad + \frac{15d^{5/2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{7/2}} \\
&\quad + \frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b} - \frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.65

$$
\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \frac{64(c+dx)^4 - \frac{7\sqrt{2}d^4e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)}{b^4} - \frac{7\sqrt{2}d^4e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(c+dx)}{d}\right)}{b^4}}{448d\sqrt{c+dx}}
$$

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2,x]
```

```
[Out] (64*(c + d*x)^4 - (7*Sqrt[2]*d^4*E^((2*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c +
d*x))/d]*Gamma[7/2, ((-2*I)*b*(c + d*x))/d])/b^4 - (7*Sqrt[2]*d^4*Sqrt[(I*b
*(c + d*x))/d]*Gamma[7/2, ((2*I)*b*(c + d*x))/d])/(b^4*E^((2*I)*(a - (b*c)/
d))))/(448*d*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{(dx+c)^{\frac{7}{2}}}{7}+\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad}{4b}\right.\right.\right.}{d}$
default	$\frac{(dx+c)^{\frac{7}{2}}}{7}+\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad}{4b}\right.\right.\right.}{d}$

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/14*(d*x+c)^(7/2)+1/8/b*d*(d*x+c)^(5/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)
/d)-5/8/b*d*(-1/4/b*d*(d*x+c)^(3/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+3/4/b*
d*(1/4/b*d*(d*x+c)^(1/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/
(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.12

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \frac{105\,\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right.$$

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/896*(105*pi*d^4*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) + 105*pi*d^4*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(32*b^4*d^3*x^3 + 96*b^4*c*d
^2*x^2 + 32*b^4*c^3 - 70*b^2*c*d^2 + 140*(b^2*d^3*x + b^2*c*d^2)*cos(b*x +
a)^2 + 7*(16*b^3*d^3*x^2 + 32*b^3*c*d^2*x + 16*b^3*c^2*d - 15*b*d^3)*cos(b*
x + a)*sin(b*x + a) + 2*(48*b^4*c^2*d - 35*b^2*d^3)*x)*sqrt(d*x + c))/(b^4*
d)
```

Sympy [F]

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\cos^2(a+bx)\,dx$$

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(5/2)*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.28

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{512\,\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d}+1120\,\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-105\left(-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\right.\right.$$

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7168*sqrt(2)*(512*sqrt(2)*(d*x + c)^(7/2)*b^4/d + 1120*sqrt(2)*(d*x + c)^
(3/2)*b^2*d*cos(2*((d*x + c)*b - b*c + a*d)/d) - 105*(-(I + 1)*4^(1/4)*sqrt
(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(pi)*d
^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d))
- 105*((I - 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
 (I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)) + 56*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(
2)*sqrt(d*x + c)*b*d^2)*sin(2*((d*x + c)*b - b*c + a*d)/d))/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 1331, normalized size of antiderivative = 5.76

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8960*(2240*(-I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
) + I*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt
(d*x + c))*c^3 - 28*c*d^2*(64*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 1
5*sqrt(d*x + c)*c^2)/d^2 - 15*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)
*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(4*I*(d*x + c)^(
3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - 15*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d
- 3*d^2)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-4*I*
(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*
(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - d^3*(256*(5*(d*x + c)^(7/2)
- 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3
 - 35*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*e
rf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(16*I*(d*x + c)^(5/2
)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*
(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*
e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 35*(-I*sqrt(pi)*(64*b^
3*c^3 + 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(I*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(-16*I*(d*x + c)^(5/2)*b^2*d + 48*I*(d*x +
 c)^(3/2)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2
 + 36*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b
```

```
- I*b*c + I*a*d)/d)/b^3)/d^3) + 560*(3*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 3*I*sqrt(pi)*(4*b*c + I*d)*d*erf
(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2) + 48*
sqrt(d*x + c)*c - 6*I*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)
/d)/b + 6*I*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c^
2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\left(c+d\,x\right)^{5/2}dx$$

```
[In] int(cos(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.49 $\int (c+dx)^{3/2}\cos^2(a+bx)\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 203

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} + \frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{32b^{5/2}} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] 1/5*(d*x+c)^(5/2)/d+1/2*(d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)/b-3/32*d^(3/2)*
cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2
)/b^(5/2)+3/32*d^(3/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*s
in(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)-3/16*d*(d*x+c)^(1/2)/b^2+3/8*d*cos(b*x+a)^
2*(d*x+c)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 32, 3393, 3387, 3386, 3432, 3385, 3433}

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = -\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} + \frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b} - \frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d}$$

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) + (3*d*Sqrt[c + d*x]*
Cos[a + b*x]^2)/(8*b^2) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/2)) + (3*d^(3/2)*S
qrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2
*b*c)/d])/(32*b^(5/2)) + ((c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad + \frac{1}{2}\int (c+dx)^{3/2}\,dx - \frac{(3d^2)\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx}{16b^2} \\
&= \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} \\
&\quad + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} - \frac{(3d^2)\int \left(\frac{1}{2\sqrt{c+dx}} + \frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)\,dx}{16b^2} \\
&= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} \\
&\quad + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} - \frac{(3d^2)\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{32b^2} \\
&= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad - \frac{\left(3d^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{32b^2} + \frac{\left(3d^2\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{32b^2} \\
&= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} \\
&\quad + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b} \\
&\quad - \frac{\left(3d\cos\left(2a-\frac{2bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{16b^2} \\
&\quad + \frac{\left(3d\sin\left(2a-\frac{2bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{16b^2}
\end{aligned}
$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} + \frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{32b^{5/2}} + \frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \frac{\sqrt{c+dx}\left(32(c+dx)^2 + \frac{5\sqrt{2}d^2e^{2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)}{b^2\sqrt{-\frac{ib(c+dx)}{d}}} + \frac{5\sqrt{2}d^2e^{-2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},\frac{2ib(c+dx)}{d}\right)}{b^2\sqrt{\frac{ib(c+dx)}{d}}}\right)}{160d}$$

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*(32*(c + d*x)^2 + (5*Sqrt[2]*d^2*E^((2*I)*(a - (b*c)/d))*Gam
ma[5/2, ((-2*I)*b*(c + d*x))/d])/(b^2*Sqrt[((-I)*b*(c + d*x))/d]) + (5*Sqrt
[2]*d^2*Gamma[5/2, ((2*I)*b*(c + d*x))/d])/(b^2*E^((2*I)*(a - (b*c)/d))*Sqr
t[(I*b*(c + d*x))/d])))/(160*d)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{(dx+c)^{\frac{5}{2}}}{5} + \frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b} - \frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b} + \frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\text{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{}}\right.}{d}$
default	$\frac{(dx+c)^{\frac{5}{2}}}{5} + \frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b} - \frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b} + \frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\text{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{}}\right.}{d}$

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/10*(d*x+c)^(5/2)+1/8/b*d*(d*x+c)^(3/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)
/d)-3/8/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+1/8/b*
d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx =$$

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,(16}{}$$

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/160*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 + 30*b*d^2*cos(b*x + a)^2 - 15*b*d^2 + 40*(b^2*d^2*x + b^2*c
*d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(b^3*d)
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\cos^2\left(a+bx\right)dx$$

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.35

$$\int (c+dx)^{3/2}\cos^2(a$$

$$+bx)\,dx = \frac{\sqrt{2}\left(\frac{128\,\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d}+160\,\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)+120\,\sqrt{2}\sqrt{dx+c}bd\cos\left(\frac{2\,((dx+c)b-}{d}\right.\right.$$

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1280*sqrt(2)*(128*sqrt(2)*(d*x + c)^(5/2)*b^3/d + 160*sqrt(2)*(d*x + c)^(
3/2)*b^2*sin(2*((d*x + c)*b - b*c + a*d)/d) + 120*sqrt(2)*sqrt(d*x + c)*b*d
*cos(2*((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*((I + 1
)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1
/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(-2*I*b/d)))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 821, normalized size of antiderivative = 4.04

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/960*(240*(-I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1))
+ I*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d
*x + c))*c^2 - d^2*(64*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(
d*x + c)*c^2)/d^2 - 15*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(
-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(4*I*(d*x + c)^(3/2)*b*
d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b +
I*b*c - I*a*d)/d)/b^2)/d^2 - 15*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2
)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-4*I*(d*x +
c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x
 + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 40*(3*I*sqrt(pi)*(4*b*c - I*d)*d*er
f(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 3*I*sqrt(pi)*(4*b*c + I*d
)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2
) + 48*sqrt(d*x + c)*c - 6*I*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d
)/b)*c)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\left(c+d\,x\right)^{3/2}\,dx$$

```
[In] int(cos(a + b*x)^2*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.50 $\int \sqrt{c+dx}\cos^2(a+bx)\,dx$

Optimal result . . . 444
Rubi [A] (verified) . . . 444
Mathematica [C] (verified) . . . 446
Maple [A] (verified) . . . 447
Fricas [A] (verification not implemented) . . . 447
Sympy [F] . . . 448
Maxima [C] (verification not implemented) . . . 448
Giac [C] (verification not implemented) . . . 448
Mupad [F(-1)] . . . 449

Optimal result

Integrand size = 18, antiderivative size = 158

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx = \frac{(c+dx)^{3/2}}{3d} - \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}} + \frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}$$

```
[Out] 1/3*(d*x+c)^(3/2)/d-1/8*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(2*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin(2*b
*x+2*a)*(d*x+c)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx = -\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} + \frac{\sqrt{c+dx}\sin(2a+2bx)}{4b} + \frac{(c+dx)^{3/2}}{3d}$$

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(3*d) - (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]
*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d
])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}\sqrt{c+dx} + \frac{1}{2}\sqrt{c+dx}\cos(2a+2bx)\right)\,dx \\
&= \frac{(c+dx)^{3/2}}{3d} + \frac{1}{2}\int \sqrt{c+dx}\cos(2a+2bx)\,dx \\
&= \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{c+dx}\sin(2a+2bx)}{4b} - \frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b} \\
&= \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{c+dx}\sin(2a+2bx)}{4b} - \frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b} \\
&\quad - \frac{\left(d\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b} \\
&= \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{c+dx}\sin(2a+2bx)}{4b} \\
&\quad - \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{4b} \\
&\quad - \frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{4b} \\
&= \frac{(c+dx)^{3/2}}{3d} - \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}} + \frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95

$$\begin{aligned}
&\int \sqrt{c+dx}\cos^2(a+bx)\,dx \\
&= \frac{16(c+dx)^2 + \frac{3\sqrt{2}d^2e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{b^2} + \frac{3\sqrt{2}d^2e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{b^2}}{48d\sqrt{c+dx}}
\end{aligned}$$

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2,x]
```

```
[Out] (16*(c + d*x)^2 + (3*Sqrt[2]*d^2*E^((2*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c +
d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/b^2 + (3*Sqrt[2]*d^2*Sqrt[(I*b
*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d])/(b^2*E^((2*I)*(a - (b*c)/
d))))/(48*d*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$	150
default	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$	150

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/6*(d*x+c)^(3/2)+1/8/b*d*(d*x+c)^(1/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/
d)-1/16/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx = -\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,(2$$
$$\overline{\qquad\qquad 24\,b^2 d}$$

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c)*sqrt(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx = \int \sqrt{c+dx}\cos^2(a+bx)\,dx$$

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2,x)
[Out] Integral(sqrt(c + d*x)*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.45

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx$$
$$= \frac{\sqrt{2}\left(\frac{32\sqrt{2}(dx+c)^{\frac{3}{2}}b^2}{d} + 24\sqrt{2}\sqrt{dx+c}b\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right) - 3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right) - (i\right.\right.}{}$$

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/192*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2/d + 24*sqrt(2)*sqrt(d*x + c)*
b*sin(2*((d*x + c)*b - b*c + a*d)/d) - 3*((I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d
^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4
)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 3*(-(I - 1)*4^(
1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqr
t(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*
b/d)))/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx =$$
$$-\frac{12\left(-\frac{i\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)} + \frac{i\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)} - 4\sqrt{dx+}\right.}{}$$

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/48*(12*(-I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) +
I*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d*x
 + c))*c + 3*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) - 3*I*sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2) + 48*sqrt(d*x + c)*c - 6*I*sqrt
(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*I*sqrt(d*x + c)*
d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx = \int \cos(a+b\,x)^2\sqrt{c+d\,x}\,dx$$

```
[In] int(cos(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.51 $\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result . . . 450
Rubi [A] (verified) . . . 450
Mathematica [C] (verified) . . . 452
Maple [A] (verified) . . . 452
Fricas [A] (verification not implemented) . . . 453
Sympy [F] . . . 453
Maxima [C] (verification not implemented) . . . 453
Giac [C] (verification not implemented) . . . 454
Mupad [F(-1)] . . . 454

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{c+dx}}{d} + \frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

```
[Out] 1/2*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^
(1/2)/b^(1/2)/d^(1/2)-1/2*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(1/2)/d^(1/2)+(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{\sqrt{c+dx}}{d}$$

```
[In] Int[Cos[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] Sqrt[c + d*x]/d + (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*FresnelS[(2*S
```

```
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[b]*
Sqrt[d])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2\sqrt{c+dx}} + \frac{\cos(2a+2bx)}{2\sqrt{c+dx}} \right) dx \\
&= \frac{\sqrt{c+dx}}{d} + \frac{1}{2} \int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\, dx \\
&= \frac{\sqrt{c+dx}}{d} + \frac{1}{2} \cos\left(2a - \frac{2bc}{d}\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{\sqrt{c+dx}}\, dx - \frac{1}{2} \sin\left(2a - \frac{2bc}{d}\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{\sqrt{c+dx}}\, dx
\end{aligned}$$

$$= \frac{\sqrt{c+dx}}{d} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d} - \frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= \frac{\sqrt{c+dx}}{d} + \frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{8\left(\frac{c}{d}+x\right) - \frac{i\sqrt{2}e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{i\sqrt{2}e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},\frac{2ib(c+dx)}{d}\right)}{b}}{8\sqrt{c+dx}}$$

```
[In] Integrate[Cos[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] (8*(c/d + x) - (I*Sqrt[2]*E^((2*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[1/2, ((-2*I)*b*(c + d*x))/d])/b + (I*Sqrt[2]*Sqrt[(I*b*(c + d*x))/d
]*Gamma[1/2, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))))/(8*Sqrt[c
 + d*x])
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\sqrt{dx+c}+\frac{\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$	108
default	$\frac{\sqrt{dx+c}+\frac{\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$	108

```
[In] int(cos(b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2*(d*x+c)^(1/2)+1/4*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*Fresnel
C(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/P
i^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) + 2\sqrt{dx+c}b}{2\,bd}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 2*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx$$

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/sqrt(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.44

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx =$$
$$-\frac{\sqrt{2}\left(\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right) + (i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{2}{}}\right.\right.}{}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*(((I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)
/d) + (I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*co
s(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - 8*sqrt(2)*sqrt(d*x + c)*b/d)
/b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.28

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{-\frac{i\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)} + \frac{i\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)} - 4\sqrt{dx+c}}{4d}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqr
t(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d*x + c)
)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos(a+bx)^2}{\sqrt{c+dx}}\,dx$$

```
[In] int(cos(a + b*x)^2/(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x)^(1/2), x)
```

3.52 $\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 135

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{2\sqrt{b}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^{3/2}}$$

```
[Out] -2*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*b^(1
/2)*Pi^(1/2)/d^(3/2)-2*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*s
in(2*a-2*b*c/d)*b^(1/2)*Pi^(1/2)/d^(3/2)-2*cos(b*x+a)^2/d/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3394, 12, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\sqrt{\pi}\sqrt{b}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}} - \frac{2\sqrt{\pi}\sqrt{b}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}} - \frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^2)/(d*Sqrt[c + d*x]) - (2*Sqrt[b]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/d^(3/2) - (2*
```

```
Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin
[2*a - (2*b*c)/d])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} + \frac{(4b)\int -\frac{\sin(2a+2bx)}{2\sqrt{c+dx}}\,dx}{d} \\
&= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{(2b)\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{d} \\
&= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{\left(2b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{d} - \frac{\left(2b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{d} \\
&= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{\left(4b\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2} \\
&\quad - \frac{\left(4b\sin\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2} \\
&= -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}} - \frac{2\sqrt{b}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}} \\
&\quad - \frac{2\sqrt{b}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.30

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-\frac{2i(ad+b(c+dx))}{d}}\left(\sqrt{2}e^{2i(2a+bx)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(-\left(1+e^{2i(a+bx)}\right)^2+\sqrt{}\right.\right.}{2d\sqrt{c+dx}}$$

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^((2*I)*(2*a + b*x))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-2*I
)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*(-(1 + E^((2*I)*(a + b*x)))^2 + Sqrt[
2]*E^(((2*I)*b*(c + d*x))/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, ((2*I)*b*(c
 + d*x))/d]))/(2*d*E^(((2*I)*(a*d + b*(c + d*x)))/d)*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{-\frac{1}{\sqrt{dx+c}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	146
default	$\frac{-\frac{1}{\sqrt{dx+c}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	146

```
[In] int(cos(b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/2/(d*x+c)^(1/2)-1/2/(d*x+c)^(1/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-
b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(
d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx =$$

$$-\frac{2\left((\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-a}{d}\right.\right.}{d^2x+cd}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt
(d*x + c)*sqrt(b/(pi*d))) + (pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(2*sq
rt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + sqrt(d*x + c)*cos(b*x +
 a)^2)/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos^2(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(3/2),x)
[Out] Integral(cos(a + b*x)**2/(c + d*x)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{2}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{2i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)\right.}{8}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 1/8*(sqrt(2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) + (I - 1)*sq
rt(2)*gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + ((I - 1)*sqr
t(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*(d*x
 + c)*b/d))*sin(-2*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) - 8)/(sqrt(d*x + c)*
d)
```

Giac [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^2}{(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(c+d\,x\right)^{3/2}}\,dx$$

```
[In] int(cos(a + b*x)^2/(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^2/(c + d*x)^(3/2), x)
```

3.53 $\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result . . . 461
Rubi [A] (verified) . . . 461
Mathematica [C] (verified) . . . 464
Maple [A] (verified) . . . 464
Fricas [A] (verification not implemented) . . . 465
Sympy [F] . . . 465
Maxima [C] (verification not implemented) . . . 465
Giac [F] . . . 466
Mupad [F(-1)] . . . 466

Optimal result

Integrand size = 18, antiderivative size = 170

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}} + \frac{8b^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^{5/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}}$$

```
[Out] -2/3*cos(b*x+a)^2/d/(d*x+c)^(3/2)-8/3*b^(3/2)*cos(2*a-2*b*c/d)*FresnelC(2*b
^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/d^(5/2)+8/3*b^(3/2)*Fresnel
S(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(5/
2)+8/3*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 32, 3393, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{8\sqrt{\pi}b^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}} + \frac{8\sqrt{\pi}b^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}} + \frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}$$

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x]^2)/(3*d*(c + d*x)^(3/2)) - (8*b^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(3*d^(5/2
)) + (8*b^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[P
i])]*Sin[2*a - (2*b*c)/d])/(3*d^(5/2)) + (8*b*Cos[a + b*x]*Sin[a + b*x])/(3
*d^2*Sqrt[c + d*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} \\
&\quad + \frac{(8b^2)\int \frac{1}{\sqrt{c+dx}}\,dx}{3d^2} - \frac{(16b^2)\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2} \\
&= \frac{16b^2\sqrt{c+dx}}{3d^3} - \frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} \\
&\quad - \frac{(16b^2)\int \left(\frac{1}{2\sqrt{c+dx}} + \frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)\,dx}{3d^2} \\
&= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} - \frac{(8b^2)\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{3d^2} \\
&= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} \\
&\quad - \frac{\left(8b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(8b^2\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{3d^2} \\
&= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}} \\
&\quad - \frac{\left(16b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{3d^3} \\
&\quad + \frac{\left(16b^2\sin\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{3d^3} \\
&= -\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}} \\
&\quad + \frac{8b^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^{5/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{e^{-\frac{2i(bc+ad)}{d}}\left(-2\sqrt{2}de^{4ia}\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)-2\sqrt{2}de^{\frac{4ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{2ib}{}\right.\right.}{3d^2(c+dx)^{3/2}}$$

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*d*E^((4*I)*a)*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*b
*(c + d*x))/d] - 2*Sqrt[2]*d*E^(((4*I)*b*c)/d)*((I*b*(c + d*x))/d)^(3/2)*Ga
mma[1/2, ((2*I)*b*(c + d*x))/d] + 2*E^(((2*I)*(b*c + a*d))/d)*(-(d*Cos[a +
b*x]^2) + 2*b*(c + d*x)*Sin[2*(a + b*x)]))/(3*d^2*E^(((2*I)*(b*c + a*d))/d)
*(c + d*x)^(3/2))
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11

method	result
derivativedivides	$-\frac{1}{3(dx+c)^{\frac{3}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2bc}{d}\right.\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$
default	$-\frac{1}{3(dx+c)^{\frac{3}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2bc}{d}\right.\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}}{d}$

```
[In] int(cos(b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/6/(d*x+c)^(3/2)-1/6/(d*x+c)^(3/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-
2/3*b/d*(-1/(d*x+c)^(1/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(
b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.21

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{2\left(4\left(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4\left(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2\right)\sqrt{}\right.}{3\left(d^4x^2+2\right.}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-2*(b*c
 - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 4*(pi*b*d^2*x^2 +
2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-2*(b*c - a*d)/d) + (d*cos(b*x + a)^2 - 4*(b*d*x + b*c)*cos(b*
x + a)*sin(b*x + a))*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/(c + d*x)**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{3\sqrt{2}\left(\left(-(i-1)\,\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{2i\,(dx+c)b}{d}\right)+(i+1)\,\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{2i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right.\right.}{}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I + 1)
*sqrt(2)*gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I + 1)
*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-3/2, -2*I*
(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) - 4)/((d*x + c
)^(3/2)*d)
```

Giac [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos\left(bx+a\right)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{(c+d\,x)^{5/2}}\,dx$$

```
[In] int(cos(a + b*x)^2/(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)^2/(c + d*x)^(5/2), x)
```

3.54 $\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result . . . 467
Rubi [A] (verified) . . . 467
Mathematica [C] (verified) . . . 470
Maple [A] (verified) . . . 470
Fricas [A] (verification not implemented) . . . 471
Sympy [F] . . . 472
Maxima [C] (verification not implemented) . . . 472
Giac [F] . . . 472
Mupad [F(-1)] . . . 473

Optimal result

Integrand size = 18, antiderivative size = 216

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{32b^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}} + \frac{32b^{5/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{15d^{7/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}}$$

```
[Out] -2/5*cos(b*x+a)^2/d/(d*x+c)^(5/2)+8/15*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^
(3/2)+32/15*b^(5/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))*Pi^(1/2)/d^(7/2)+32/15*b^(5/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(7/2)-16/15*b^2/d^3/(d*x+c)
^(1/2)+32/15*b^2*cos(b*x+a)^2/d^3/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3395, 32, 3394, 12, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{32\sqrt{\pi}b^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}} + \frac{32\sqrt{\pi}b^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{16b^2}{15d^3\sqrt{c+dx}}$$

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (2*Cos[a + b*x]^2)/(5*d*(c + d*x)^(5/2))
 + (32*b^2*Cos[a + b*x]^2)/(15*d^3*Sqrt[c + d*x]) + (32*b^(5/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(15*d^(7/2)) + (32*b^(5/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(15*d^(7/2)) + (8*b*Cos[a + b*x]*Sin[
a + b*x])/(15*d^2*(c + d*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} \\
&\quad + \frac{(8b^2)\int \frac{1}{(c+dx)^{3/2}}\,dx}{15d^2} - \frac{(16b^2)\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2} \\
&= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} \\
&\quad + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{(64b^3)\int -\frac{\sin(2a+2bx)}{2\sqrt{c+dx}}\,dx}{15d^3} \\
&= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} \\
&\quad + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(32b^3)\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,dx}{15d^3} \\
&= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}} \\
&\quad + \frac{\left(32b^3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{15d^3} + \frac{\left(32b^3\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{15d^3}
\end{aligned}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}}$$
$$+ \frac{\left(64b^3\cos\left(2a-\frac{2bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{15d^4}$$
$$+ \frac{\left(64b^3\sin\left(2a-\frac{2bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{15d^4}$$
$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}}$$
$$+ \frac{32b^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}}$$
$$+ \frac{32b^{5/2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{15d^{7/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.10

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{-6d^2 + e^{2ia}\left(-3d^2e^{2ibx} + 4be^{-\frac{2ibc}{d}}(c+dx)\left(e^{\frac{2ib(c+dx)}{d}}(-id+4b(c+dx)) - 4i\sqrt{2}d\left(-\frac{ib(c}{}\right.\right.\right.}{}$$

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-6*d^2 + E^((2*I)*a)*(-3*d^2*E^((2*I)*b*x) + (4*b*(c + d*x)*(E^(((2*I)*b*(
c + d*x))/d)*((-I)*d + 4*b*(c + d*x)) - (4*I)*Sqrt[2]*d*(((-I)*b*(c + d*x))
/d)^(3/2)*Gamma[1/2, ((-2*I)*b*(c + d*x))/d]))/E^(((2*I)*b*c)/d)) + (-3*d^2
 - (2*I)*b*(c + d*x)*(-2*d + (8*I)*b*(c + d*x) - 8*Sqrt[2]*d*E^(((2*I)*b*(c
 + d*x))/d)*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((2*I)*b*(c + d*x))/d]))/E
^((2*I)*(a + b*x)))/(30*d^3*(c + d*x)^(5/2))
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{1}{5(dx+c)^{\frac{5}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad}{d}\right.\right.\right.}{5d}$
default	$-\frac{1}{5(dx+c)^{\frac{5}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad}{d}\right.\right.\right.}{5d}$

```
[In] int(cos(b*x+a)^2/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/10/(d*x+c)^(5/2)-1/10/(d*x+c)^(5/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d
)-2/5*b/d*(-1/3/(d*x+c)^(3/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+4/3*b/d*(-1/
(d*x+c)^(1/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-2*b/d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.50

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{2\left(16\left(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\right.\right.}}{}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c
^3)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b
/(pi*d))) + 16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi
*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2
*(b*c - a*d)/d) - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - (16*b^2*d^2*x
^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 3*d^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d
)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*
d^4*x + c^3*d^3)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos^2(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(7/2),x)
[Out] Integral(cos(a + b*x)**2/(c + d*x)**(7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.63

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{5\sqrt{2}\left(\left((i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{2i(dx+c)b}{d}\right)-(i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\right.}{10}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
[Out] 1/10*(5*sqrt(2)*(((I + 1)*sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) - (I - 1)*
sqrt(2)*gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I - 1)*
sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) + (I + 1)*sqrt(2)*gamma(-5/2, -2*I*(
d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) - 2)/((d*x + c)
^(5/2)*d)
```

Giac [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos\left(bx+a\right)^2}{(dx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(c+d\,x\right)^{7/2}}\,dx$$

```
[In] int(cos(a + b*x)^2/(c + d*x)^(7/2),x)
[Out] int(cos(a + b*x)^2/(c + d*x)^(7/2), x)
```

3.55 $\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx$

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Mupad [F(-1)] . . . 480

Optimal result

Integrand size = 18, antiderivative size = 247

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{128b^{7/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}} - \frac{128b^{7/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{105d^{9/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}}$$

```
[Out] -16/105*b^2/d^3/(d*x+c)^(3/2)-2/7*cos(b*x+a)^2/d/(d*x+c)^(7/2)+32/105*b^2*c
os(b*x+a)^2/d^3/(d*x+c)^(3/2)+8/35*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(5/2
)+128/105*b^(7/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/Pi^(1/2))*Pi^(1/2)/d^(9/2)-128/105*b^(7/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(9/2)-128/105*b^3*cos(b*x+a
)*sin(b*x+a)/d^4/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 32, 3393, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{128\sqrt{\pi}b^{7/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}}$$
$$-\frac{128\sqrt{\pi}b^{7/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}} - \frac{128b^3\sin(a+bx)\cos(a+bx)}{105d^4\sqrt{c+dx}}$$
$$+\frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{16b^2}{105d^3(c+dx)^{3/2}}$$

```
[In] Int[Cos[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (2*Cos[a + b*x]^2)/(7*d*(c + d*x)^(7/
2)) + (32*b^2*Cos[a + b*x]^2)/(105*d^3*(c + d*x)^(3/2)) + (128*b^(7/2)*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])])/(105*d^(9/2)) - (128*b^(7/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d
*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(105*d^(9/2)) + (8*b*Cos[a +
 b*x]*Sin[a + b*x])/(35*d^2*(c + d*x)^(5/2)) - (128*b^3*Cos[a + b*x]*Sin[a
+ b*x])/(105*d^4*Sqrt[c + d*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} \\
&\quad + \frac{(8b^2)\int \frac{1}{(c+dx)^{5/2}}\,dx}{35d^2} - \frac{(16b^2)\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx}{35d^2} \\
&= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} \\
&\quad - \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} - \frac{(128b^4)\int \frac{1}{\sqrt{c+dx}}\,dx}{105d^4} + \frac{(256b^4)\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx}{105d^4} \\
&= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{256b^4\sqrt{c+dx}}{105d^5} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} \\
&\quad + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} \\
&\quad - \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} + \frac{(256b^4)\int \left(\frac{1}{2\sqrt{c+dx}} + \frac{\cos(2a+2bx)}{2\sqrt{c+dx}}\right)\,dx}{105d^4}
\end{aligned}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} + \frac{(128b^4)\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{105d^4}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} + \frac{\left(128b^4\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{105d^4} - \frac{\left(128b^4\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{105d^4}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}} + \frac{\left(256b^4\cos\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{105d^5} - \frac{\left(256b^4\sin\left(2a-\frac{2bc}{d}\right)\right)\text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{105d^5}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{128b^{7/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}} - \frac{128b^{7/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{105d^{9/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{2\left(-8b^2d(c+dx)^2 - 15d^3\cos^2(a+bx) + 16b^2d(c+dx)^2\cos^2(a+bx) + 16\sqrt{2}b^2de^{2i\left(a-\right.}\right.}{}$$

```
[In] Integrate[Cos[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (2*(-8*b^2*d*(c + d*x)^2 - 15*d^3*Cos[a + b*x]^2 + 16*b^2*d*(c + d*x)^2*Cos
[a + b*x]^2 + 16*Sqrt[2]*b^2*d*E^((2*I)*(a - (b*c)/d))*(c + d*x)^2*(((-I)*b
*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*b*(c + d*x))/d] + (16*Sqrt[2]*b^2*d
*(c + d*x)^2*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((2*I)*b*(c + d*x))/d])/E
^((2*I)*(a - (b*c)/d)) + 6*b*d^2*(c + d*x)*Sin[2*(a + b*x)] - 32*b^3*(c + d
*x)^3*Sin[2*(a + b*x)]))/(105*d^4*(c + d*x)^(7/2))
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{7(dx+c)^{\frac{7}{2}}}-\left(4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\left(4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\left(4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\cdots\right)\right)\right)\right)\right)}{d}$
default	$\frac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{7(dx+c)^{\frac{7}{2}}}-\left(4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\left(4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\left(4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\cdots\right)\right)\right)\right)\right)}{d}$

```
[In] int(cos(b*x+a)^2/(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] 2/d*(-1/14/(d*x+c)^(7/2)-1/14/(d*x+c)^(7/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d
)-2/7*b/d*(-1/5/(d*x+c)^(5/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+4/5*b/d*(-1/
```

```
3/(d*x+c)^(3/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-4/3*b/d*(-1/(d*x+c)^(1/2)*
sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c
)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*
FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(195) = 390$.

Time $= 0.33$ (sec) , antiderivative size $= 417$, normalized size of antiderivative $= 1.69$

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{2\left(64\left(\pi b^3 d^4 x^4 + 4\,\pi b^3 c d^3 x^3 + 6\,\pi b^3 c^2 d^2 x^2 + 4\,\pi b^3 c^3 d x + \pi b^3 c^4\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\right.}{}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3 + 6*pi*b^3*c^2*d^2*x^2 + 4*p
i*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_co
s(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3
 + 6*pi*b^3*c^2*d^2*x^2 + 4*pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*fre
snel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - (8*b^2*d^3
*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - (16*b^2*d^3*x^2 + 32*b^2*c*d^2*x + 16
*b^2*c^2*d - 15*d^3)*cos(b*x + a)^2 + 4*(16*b^3*d^3*x^3 + 48*b^3*c*d^2*x^2
+ 16*b^3*c^3 - 3*b*c*d^2 + 3*(16*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)*sin(b*x
 + a))*sqrt(d*x + c))/(d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c^3*d^5*x
+ c^4*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**2/(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.55

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx =$$

$$-\frac{7\sqrt{2}\left(\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{7}{2},\frac{2i\,(dx+c)b}{d}\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{7}{2},-\frac{2i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+\left(-(i+1)\sqrt{2}\Gamma\left(\right.\right.\right.}{7\,(dx+c)^{\frac{7}{2}}d}$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*(7*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I + 1)
*sqrt(2)*gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I + 1)
*sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-7/2, -2*I*
(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(7/2) + 1)/((d*x + c
)^(7/2)*d)
```

Giac [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\cos\left(bx+a\right)^2}{(dx+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{(c+d\,x)^{9/2}}\,dx$$

```
[In] int(cos(a + b*x)^2/(c + d*x)^(9/2),x)
```

```
[Out] int(cos(a + b*x)^2/(c + d*x)^(9/2), x)
```

3.56 $\int (c+dx)^{5/2}\cos^3(a+bx)\,dx$

Optimal result . . . 481
Rubi [A] (verified) . . . 482
Mathematica [C] (verified) . . . 486
Maple [A] (verified) . . . 486
Fricas [A] (verification not implemented) . . . 487
Sympy [F(-1)] . . . 487
Maxima [C] (verification not implemented) . . . 488
Giac [C] (verification not implemented) . . . 488
Mupad [F(-1)] . . . 490

Optimal result

Integrand size = 18, antiderivative size = 410

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2}$$
$$+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}$$
$$+\frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}$$
$$+\frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}}$$
$$+\frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}}$$
$$-\frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3}+\frac{2(c+dx)^{5/2}\sin(a+bx)}{3b}$$
$$+\frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b}-\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}$$

```
[Out] 5/3*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2+5/18*d*(d*x+c)^(3/2)*cos(b*x+a)^3/b^2+2/
3*(d*x+c)^(5/2)*sin(b*x+a)/b+1/3*(d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)/b+5/
864*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+4
5/32*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+45/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^
```

```
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-45/16*d^
2*sin(b*x+a)*(d*x+c)^(1/2)/b^3-5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 3377, 3387, 3386, 3432, 3385, 3433, 3393}

$$\begin{aligned}
\int (c+dx)^{5/2}\cos^3(a+bx)\,dx &= \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} \\
&+ \frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} \\
&+ \frac{45\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} \\
&+ \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} \\
&- \frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} - \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3} \\
&+ \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} \\
&+ \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}
\end{aligned}$$

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(3*b^2) + (5*d*(c + d*x)^(3/2)*Cos[a + b
*x]^3)/(18*b^2) + (45*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*C
os[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi/2]
*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16
*b^(7/2)) - (45*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + (2*(c + d*x)^(5/
2)*Sin[a + b*x])/(3*b) + ((c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b
) - (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad + \frac{2}{3}\int (c+dx)^{5/2}\cos(a+bx)\,dx - \frac{(5d^2)\int \sqrt{c+dx}\cos^3(a+bx)\,dx}{12b^2} \\
&= \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} \\
&\quad + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{(5d)\int (c+dx)^{3/2}\sin(a+bx)\,dx}{3b} \\
&\quad - \frac{(5d^2)\int \left(\frac{3}{4}\sqrt{c+dx}\cos(a+bx) + \frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)\,dx}{12b^2} \\
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} \\
&\quad + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{(5d^2)\int \sqrt{c+dx}\cos(3a+3bx)\,dx}{48b^2} \\
&\quad - \frac{(5d^2)\int \sqrt{c+dx}\cos(a+bx)\,dx}{16b^2} - \frac{(5d^2)\int \sqrt{c+dx}\cos(a+bx)\,dx}{2b^2} \\
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} \\
&\quad - \frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} \\
&\quad + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3} \\
&\quad + \frac{(5d^3)\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx}{288b^3} + \frac{(5d^3)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{32b^3} + \frac{(5d^3)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{4b^3} \\
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} \\
&\quad - \frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} \\
&\quad + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3} \\
&\quad + \frac{\left(5d^3\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{288b^3} + \frac{\left(5d^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{32b^3} \\
&\quad + \frac{\left(5d^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{4b^3} + \frac{\left(5d^3\sin\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{288b^3} \\
&\quad + \frac{\left(5d^3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{32b^3} + \frac{\left(5d^3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{4b^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} \\
&\quad - \frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} \\
&\quad + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3} \\
&\quad + \frac{\left(5d^2\cos\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{144b^3} \\
&\quad + \frac{\left(5d^2\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{16b^3} \\
&\quad + \frac{\left(5d^2\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{2b^3} \\
&\quad + \frac{\left(5d^2\sin\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{144b^3} \\
&\quad + \frac{\left(5d^2\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{16b^3} \\
&\quad + \frac{\left(5d^2\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{2b^3} \\
&= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} + \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} \\
&\quad + \frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} \\
&\quad + \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} \\
&\quad + \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}} \\
&\quad + \frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}} \\
&\quad - \frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} \\
&\quad + \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.58

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx =$$

$$-\frac{d^3 e^{-\frac{3i(bc+ad)}{d}}\left(243e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)+243e^{2ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e^{6ia}\sqrt{-}\right.\right.}{648b^4\sqrt{c+dx}}$$

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3,x]
```

```
[Out] -1/648*(d^3*(243*E^((2*I)*(2*a + (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma
[7/2, ((-I)*b*(c + d*x))/d] + 243*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[7/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqrt[((-I)*
b*(c + d*x))/d]*Gamma[7/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*Sqrt
[(I*b*(c + d*x))/d]*Gamma[7/2, ((3*I)*b*(c + d*x))/d])))/(b^4*E^(((3*I)*(b*
c + a*d))/d)*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{3d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{15d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\right.\right.}{4b}$
default	$\frac{3d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{15d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\right.\right.}{4b}$

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/8/b*d*(d*x+c)^(5/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-15/8/b*d*(-1/2/b*d*
(d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2)*s
```

```
in(b*(d*x+c)/d+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-
b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b
*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/24/b*d
*(d*x+c)^(5/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-5/24/b*d*(-1/6/b*d*(d*x+c)^
(3/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)*sin(3
*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(
cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.90

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+1215\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+}\right.}$$

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 1215*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 1215*
sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
 + b^2*c*d)*cos(b*x + a)^3 + 60*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) + (24*b^
3*d^2*x^2 + 48*b^3*c*d*x + 24*b^3*c^2 - 100*b*d^2 + (12*b^3*d^2*x^2 + 24*b^
3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c)
)/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.33

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)+6480\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-5\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{}\right.\right.$$

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) + 6480*(
d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) - 5*(-(I + 1)*9^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/
4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(3*I*b/d)) - 1215*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 1215*((I - 1)*sqrt(2)*s
qrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi
)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)
) - 5*((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c -
 a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12*(d*x + c)^(5/2)*b^
4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c + a*d)/d) + 648*(4*(d
*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x + c)*b - b*c + a*d)
/d))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 2480, normalized size of antiderivative = 6.05

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/1728*(72*(9*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sq
```

```
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(27*(I*sqrt(2)
*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x
 + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b
 + I*b*c - I*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*
c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2
) + 2*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*
d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(
12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b^2) + 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*
c*d - sqrt(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2)
 + d^3*(81*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 -
15*I*d^3)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
 + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) +
 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d
*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 1
5*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (I
*sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*er
f(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(12*I*(d*
x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*
c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x
+ c)*d^3)*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 81*(I*sqrt(2
)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*
I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(4*I*(d*x + c)^(5/
2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10
*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)
*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + (-I*sqrt(6)*sqrt(pi)*(72*
b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(1/2*I*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x + c)^(5/2)*b^2*d
+ 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c
)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^(-3*(I*
(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3) + 36*(-27*I*sqrt(2)*sqrt(pi)*(2*b
*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
```

```
 + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I
*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b) + 27*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c +
 I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 5
4*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(d*x
+ c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*I*sqrt(d*x + c)*d*e^
((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b)*c^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \int \cos(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

```
[In] int(cos(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.57 $\int (c+dx)^{3/2}\cos^3(a+bx)\,dx$

Optimal result . . . 491
Rubi [A] (verified) . . . 492
Mathematica [C] (verified) . . . 496
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Fricas [A] (verification not implemented) . . . 497
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Maxima [C] (verification not implemented) . . . 497
Giac [C] (verification not implemented) . . . 498
Mupad [F(-1)] . . . 499

Optimal result

Integrand size = 18, antiderivative size = 354

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} - \frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} + \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}} + \frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}} + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b}$$

```
[Out] 2/3*(d*x+c)^(3/2)*sin(b*x+a)/b+1/3*(d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)/b-
1/144*d^(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)+1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)
-9/16*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+9/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+d*cos(b*
x+a)*(d*x+c)^(1/2)/b^2+1/6*d*cos(b*x+a)^3*(d*x+c)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 3377, 3387, 3386, 3432, 3385, 3433, 3393}

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = -\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}$$
$$-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}$$
$$+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}$$
$$+\frac{9\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}$$
$$+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{d\sqrt{c+dx}\cos(a+bx)}{b^2}$$
$$+\frac{2(c+dx)^{3/2}\sin(a+bx)}{3b}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3,x]
```

```
[Out] (d*Sqrt[c + d*x]*Cos[a + b*x])/b^2 + (d*Sqrt[c + d*x]*Cos[a + b*x]^3)/(6*b^
2) - (9*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/
d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) + (d^
(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(24*b^(5/2)) + (9*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (2*(c + d*x
)^(3/2)*Sin[a + b*x])/(3*b) + ((c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x])
/(3*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b} \\ &\quad + \frac{2}{3}\int (c+dx)^{3/2}\cos(a+bx)\,dx - \frac{d^2\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx}{12b^2}\end{aligned}$$

$$
\begin{aligned}
&= \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} \\
&\quad + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad - \frac{d\int \sqrt{c+dx}\sin(a+bx)\,dx}{b} - \frac{d^2\int \left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}} + \frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)\,dx}{12b^2} \\
&= \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} \\
&\quad + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad - \frac{d^2\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx}{48b^2} - \frac{d^2\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{16b^2} - \frac{d^2\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{2b^2} \\
&= \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} \\
&\quad + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad - \frac{\left(d^2\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{48b^2} - \frac{\left(d^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{16b^2} \\
&\quad - \frac{\left(d^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b^2} + \frac{\left(d^2\sin\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{48b^2} \\
&\quad + \frac{\left(d^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{16b^2} + \frac{\left(d^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{2b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} \\
&\quad + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b} \\
&\quad - \frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{24b^2} \\
&\quad - \frac{\left(d\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{8b^2} \\
&\quad - \frac{\left(d\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{b^2} \\
&\quad + \frac{\left(d\sin\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{24b^2} \\
&\quad + \frac{\left(d\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{8b^2} \\
&\quad + \frac{\left(d\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{b^2} \\
&= \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} + \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} \\
&\quad - \frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} \\
&\quad - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} \\
&\quad + \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}} \\
&\quad + \frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}} \\
&\quad + \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.71

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \frac{e^{-\frac{3i(bc+ad)}{d}}(c+dx)^{5/2}\left(81e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)+81e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},\frac{ib(c+dx)}{d}\right)\right.}{216d\left(\frac{b^2(c+dx)^2}{d^2}\right)^{3/2}}$$

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3,x]
```

```
[Out] ((c + d*x)^(5/2)*(81*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamm
a[5/2, ((-I)*b*(c + d*x))/d] + 81*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((-I)*b*
(c + d*x))/d]*Gamma[5/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqrt[(I*
b*(c + d*x))/d]*Gamma[5/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*Sqrt
[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((3*I)*b*(c + d*x))/d])))/(216*d*E^(((3*I
)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{3d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{9d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$
default	$\frac{3d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{9d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/8/b*d*(d*x+c)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-9/8/b*d*(-1/2/b*d*(
d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1
/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)
/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)))+1/24/b*d*(d*x+c)^(3/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/8/b*d*(-1/6/
b*d*(d*x+c)^(1/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2
)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)
```

```
/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2
)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx =$$

$$-\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+81\,\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-8$$

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 81*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 +
 6*b*d*cos(b*x + a) + 2*(2*b^2*d*x + 2*b^2*c + (b^2*d*x + b^2*c)*cos(b*x +
a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\cos^3\left(a+bx\right)dx$$

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.40

$$\int (c+dx)^{3/2}\cos^3(a$$

$$+bx)\,dx = \left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{432\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+24\,\sqrt{dx+c}b^2\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)\right.$$

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d + 432*(d
*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*b^2
*cos(3*((d*x + c)*b - b*c + a*d)/d) + 648*sqrt(d*x + c)*b^2*cos(((d*x + c)*
b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)
*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 81*(-(I - 1)*sqr
t(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqr
t(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d
)) - 81*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) +
 (I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^
(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2
)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 1549, normalized size of antiderivative = 4.38

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/288*(12*(9*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(27*(I*sqrt(2)*sqrt(
pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*
b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^
2 + (-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(2*I*(d*x + c)^(3/2)*b*d
 - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b + I*b
*c - I*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d -
3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
```

```
1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*
(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*b^2
*c^2 + 4*I*b*c*d - d^2)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)*b^2) + 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d -
sqrt(d*x + c)*d^2)*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 4*(
-27*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*I*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(2)*sqrt(pi)*(2*
b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) -
I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b) + 54*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c +
I*a*d)/d)/b - 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/
b - 54*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqr
t(d*x + c)*d*e^(-3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\left(c+d\,x\right)^{3/2}\,dx$$

```
[In] int(cos(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.58 $\int \sqrt{c+dx}\cos^3(a+bx)\,dx$

Optimal result . . . 500
Rubi [A] (verified) . . . 501
Mathematica [C] (verified) . . . 503
Maple [A] (verified) . . . 504
Fricas [A] (verification not implemented) . . . 504
Sympy [F] . . . 505
Maxima [C] (verification not implemented) . . . 505
Giac [C] (verification not implemented) . . . 505
Mupad [F(-1)] . . . 506

Optimal result

Integrand size = 18, antiderivative size = 304

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx = -\frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}} - \frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}} + \frac{3\sqrt{c+dx}\sin(a+bx)}{4b} + \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

```
[Out] -1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-
3/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d
^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-3/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+3/4*sin(b*x
+a)*(d*x+c)^(1/2)/b+1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$\begin{aligned}\int \sqrt{c+dx}\cos^3(a+bx)\,dx = &-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}\\ &-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}\\ &-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}\\ &-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}\\ &+\frac{3\sqrt{c+dx}\sin(a+bx)}{4b}+\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}\end{aligned}$$

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt[d
]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a -
 (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (3*Sqrt[c + d*x
]*Sin[a + b*x])/(4*b) + (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}\sqrt{c+dx}\cos(a+bx) + \frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)\,dx \\
&= \frac{1}{4}\int \sqrt{c+dx}\cos(3a+3bx)\,dx + \frac{3}{4}\int \sqrt{c+dx}\cos(a+bx)\,dx \\
&= \frac{3\sqrt{c+dx}\sin(a+bx)}{4b} + \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} - \frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx}{24b} - \frac{(3d)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{8b} \\
&= \frac{3\sqrt{c+dx}\sin(a+bx)}{4b} + \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} \\
&\quad - \frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{24b} - \frac{\left(3d\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{8b} \\
&\quad - \frac{\left(d\sin\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{24b} - \frac{\left(3d\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{8b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3\sqrt{c+dx}\sin(a+bx)}{4b} + \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} \\
&\quad - \frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{12b} \\
&\quad - \frac{\left(3\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{4b} \\
&\quad - \frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{12b} \\
&\quad - \frac{\left(3\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{4b} \\
&= -\frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}} \\
&\quad - \frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}} \\
&\quad + \frac{3\sqrt{c+dx}\sin(a+bx)}{4b} + \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.77

$$
\begin{aligned}
&\int \sqrt{c+dx}\cos^3(a+bx)\,dx \\
&= \frac{de^{-\frac{3i(bc+ad)}{d}}\left(27e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right) + 27e^{2ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right) + \sqrt{3}\left(e^{6ia}\sqrt{-\frac{ib}{d}}\right.\right.}{72b^2\sqrt{c+dx}}
\end{aligned}
$$

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3,x]

[Out] (d*(27*E^((2*I)*(2*a + (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I
)*b*(c + d*x))/d] + 27*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*
Gamma[3/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqrt[((-I)*b*(c + d*x)
```

```
)/d]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, ((3*I)*b*(c + d*x))/d]))/(72*b^2*E^(((3*I)*(b*c + a*d)
)/d)*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{3d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{3d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{3b(dx+c)}{d}+\frac{3(ad-bc)}{d}\right)}{12b}}{d}$
default	$\frac{3d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{3d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{3b(dx+c)}{d}+\frac{3(ad-bc)}{d}\right)}{12b}}{d}$

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/8/b*d*(d*x+c)^(1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-3/16/b*d*2^(1/2)*Pi
^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d))+1/24/b*d*(d*x+c)^(1/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-
1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS
(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)
*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx = -\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27$$

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b
*c - a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c
)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 + 2*b)*sqrt(
d*x + c)*sin(b*x + a))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx = \int \sqrt{c+dx}\cos^3(a+bx)\,dx$$

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3,x)
[Out] Integral(sqrt(c + d*x)*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx$$
$$= \frac{\left(\frac{24\sqrt{dx+c}b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d} + \frac{216\sqrt{dx+c}b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d} + \left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)\right.\right.}{}$$

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="maxima")
[Out] 1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d + 216*sqrt
(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) - 27*((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)
/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(I*b/d)) - 27*(-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)
*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c
 - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt
(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt
(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b
/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.79

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/144*(-27*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1
/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(2)*sqrt
(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*(9*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*
I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d
*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt
(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 5
4*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 6*I*sqrt(d*x
+ c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*I*sqrt(d*x + c)*d*e^
((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx = \int \cos(a+b\,x)^3\sqrt{c+d\,x}\,dx$$

```
[In] int(cos(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.59 $\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result . . . 507
Rubi [A] (verified) . . . 508
Mathematica [C] (verified) . . . 510
Maple [A] (verified) . . . 510
Fricas [A] (verification not implemented) . . . 511
Sympy [F] . . . 511
Maxima [C] (verification not implemented) . . . 511
Giac [C] (verification not implemented) . . . 512
Mupad [F(-1)] . . . 513

Optimal result

Integrand size = 18, antiderivative size = 257

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{2\sqrt{b}\sqrt{d}} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

```
[Out] 1/12*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*6^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-1/12*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)+3
/4*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^
(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-3/4*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

```
[In] Int[Cos[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(
Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(2*Sqrt[b]*Sqrt[d]) - (3*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}} + \frac{\cos(3a+3bx)}{4\sqrt{c+dx}} \right) dx \\
&= \frac{1}{4}\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\, dx + \frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\, dx \\
&= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\, dx + \frac{1}{4}\left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\, dx \\
&\quad -\frac{1}{4}\sin\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\, dx - \frac{1}{4}\left(3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\, dx \\
&= \frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)\, dx, x, \sqrt{c+dx}\right)}{2d} \\
&\quad + \frac{\left(3\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\, dx, x, \sqrt{c+dx}\right)}{2d} \\
&\quad - \frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)\, dx, x, \sqrt{c+dx}\right)}{2d} \\
&\quad - \frac{\left(3\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\, dx, x, \sqrt{c+dx}\right)}{2d}
\end{aligned}
$$

$$= \frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$
$$- \frac{\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{2\sqrt{b}\sqrt{d}} - \frac{3\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.92

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{ie^{-\frac{3i(bc+ad)}{d}}\left(-9e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+9e^{2ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(-e^{6ia}\sqrt{-\frac{ib(}{}\right.\right.}{24b\sqrt{c+dx}}$$

```
[In] Integrate[Cos[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] ((I/24)*(-9*E^((2*I)*(2*a + (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2,
 ((-I)*b*(c + d*x))/d] + 9*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))
/d]*Gamma[1/2, (I*b*(c + d*x))/d] + Sqrt[3]*(-(E^((6*I)*a)*Sqrt[((-I)*b*(c
 + d*x))/d]*Gamma[1/2, ((-3*I)*b*(c + d*x))/d]) + E^(((6*I)*b*c)/d)*Sqrt[(I*
b*(c + d*x))/d]*Gamma[1/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(((3*I)*(b*c + a*
d))/d)*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{3\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\text{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\text{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{4\sqrt{\frac{b}{d}}} + \frac{\sqrt{2}\sqrt{\pi}\sqrt{3}\left(\cos\left(\frac{3ad-3bc}{d}\right)\text{C}\left(\frac{\sqrt{2}\sqrt{3}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{3a}{}\right.\right.}{12\sqrt{\frac{b}{d}}}$ $\Big)\Big)\Big)$ $\Big)\Big)\Big)$ $\Big)\Big)\Big)

```
(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))+1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi}{12\,b}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(6)*pi*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*s
qrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)
/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sqrt(b
/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)
/d) - sqrt(6)*pi*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-3*(b*c - a*d)/d))/b
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx$$

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**3/sqrt(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.47

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx =$$
$$-\frac{\left(\left(\frac{(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)}{d}+\frac{(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{d}\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{3i\,b}{d}}\right)-9\left(-\frac{(i-1}{}\right.\right.}{}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*
d)/d)/d + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c -
a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 9*(-(I - 1)*sqrt(2)*sqrt(pi)*
b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I + 1
)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c
 - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b
*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.29

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{9i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)} - \frac{i\sqrt{6}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)} - \frac{9i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}}{}$$
$$- \frac{}{24\,d}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(9*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\sqrt{c+d\,x}}\,dx$$

```
[In] int(cos(a + b*x)^3/(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^3/(c + d*x)^(1/2), x)
```

3.60 $\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result . . . 514
Rubi [A] (verified) . . . 515
Mathematica [C] (verified) . . . 517
Maple [A] (verified) . . . 518
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Optimal result

Integrand size = 18, antiderivative size = 271

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{d^{3/2}} - \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}}$$

```
[Out] -3/2*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-3/2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-1/2*cos(3*
a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)
*6^(1/2)*Pi^(1/2)/d^(3/2)-1/2*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*sin(3*a-3*b*c/d)*b^(1/2)*6^(1/2)*Pi^(1/2)/d^(3/2)-2*cos(b*x+a)^
3/d/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3394, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\cos^3(a+bx)}{d\sqrt{c+dx}}$$

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(d*Sqrt[c + d*x]) - (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a - (b*c)
/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b
]*Sqrt[(3*Pi)/2]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[b]*Sqrt[(3*Pi)/2]*FresnelC[(Sqrt[b]*Sqrt[6
/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/d^(3/2) - (3*Sqrt[b]*Sqr
t[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/
d])/d^(3/2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} + \frac{(6b)\int \left(-\frac{\sin(a+bx)}{4\sqrt{c+dx}} - \frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right) dx}{d} \\
&= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\, dx}{2d} - \frac{(3b)\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\, dx}{2d} \\
&= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\, dx}{2d} \\
&\quad - \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\, dx}{2d} - \frac{\left(3b\sin\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\, dx}{2d} \\
&\quad - \frac{\left(3b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\, dx}{2d}
\end{aligned}$$

$$= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$- \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$- \frac{\left(3b\sin\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$- \frac{\left(3b\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}$$
$$- \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}$$
$$- \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{d^{3/2}}$$
$$- \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-3ia}\left(-e^{-3ibx} - 3e^{2ia-ibx} - e^{3i(2a+bx)} - 3e^{i(4a+bx)} + 3e^{4ia-\frac{ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2}, -\frac{ib(c+dx)}{d}\right)\right.}{}$$

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-E^((-3*I)*b*x) - 3*E^((2*I)*a - I*b*x) - E^((3*I)*(2*a + b*x)) - 3*E^(I*(
4*a + b*x)) + 3*E^((4*I)*a - (I*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/
2, ((-I)*b*(c + d*x))/d] + 3*E^(I*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*
Gamma[1/2, (I*b*(c + d*x))/d] + Sqrt[3]*E^((6*I)*a - ((3*I)*b*c)/d)*Sqrt[((
-I)*b*(c + d*x))/d]*Gamma[1/2, ((-3*I)*b*(c + d*x))/d] + Sqrt[3]*E^(((3*I)*
b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, ((3*I)*b*(c + d*x))/d])/(4*d*E^(
(3*I)*a)*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2\sqrt{dx+c}}-\frac{3b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2d\sqrt{\frac{b}{d}}}-\frac{\cos\left(\frac{3b(dx+c)}{d}+\frac{3ad-3bc}{d}\right.}{2\sqrt{dx+c}}$ / d
default	$-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2\sqrt{dx+c}}-\frac{3b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2d\sqrt{\frac{b}{d}}}-\frac{\cos\left(\frac{3b(dx+c)}{d}+\frac{3ad-3bc}{d}\right.}{2\sqrt{dx+c}}$ / d

```
[In] int(cos(b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/4/(d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-3/4*b/d*2^(1/2)*Pi^(1/
2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d))-1/4/(d*x+c)^(1/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/4*b/d*2
^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi
^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2
^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx =$$

$$-\frac{\sqrt{6}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+}\right.}{}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(6)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_
sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(
b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*
d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(6)*(pi*d*x + pi*c)*sqrt(b
/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) + 4*sqrt(d*x + c)*cos(b*x + a)^3)/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos^3(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(3/2),x)
[Out] Integral(cos(a + b*x)**3/(c + d*x)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.93

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{3}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{3i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)\right.}{}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 1/16*(sqrt(3)*((-(I + 1)*sqrt(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) + (I - 1)*s
qrt(2)*gamma(-1/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I - 1)*sq
rt(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -3*I*(d*
x + c)*b/d))*sin(-3*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) - 3*(((I + 1)*sqrt(
2)*gamma(-1/2, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*
b/d))*cos(-(b*c - a*d)/d) + (-(I - 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)*b/d)
+ (I + 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*sqrt(
(d*x + c)*b/d))/(sqrt(d*x + c)*d)
```

Giac [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos(bx+a)^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^3/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(c+d\,x\right)^{3/2}}\,dx$$

```
[In] int(cos(a + b*x)^3/(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3/(c + d*x)^(3/2), x)
```

3.61 $\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result . . . 521
Rubi [A] (verified) . . . 522
Mathematica [C] (verified) . . . 525
Maple [A] (verified) . . . 525
Fricas [A] (verification not implemented) . . . 526
Sympy [F] . . . 527
Maxima [C] (verification not implemented) . . . 527
Giac [F] . . . 527
Mupad [F(-1)] . . . 528

Optimal result

Integrand size = 18, antiderivative size = 292

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{b^{3/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{b^{3/2}\sqrt{6\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{d^{5/2}} + \frac{b^{3/2}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}}$$

```
[Out] -2/3*cos(b*x+a)^3/d/(d*x+c)^(3/2)-b^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)+b^(3/2)*Fresn
elS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi
^(1/2)/d^(5/2)-b^(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/d^(5/2)+b^(3/2)*FresnelS(b^(1/2)*6^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/d^(5
/2)+4*b*cos(b*x+a)^2*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3395, 3387, 3386, 3432, 3385, 3433, 3393}

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{\sqrt{2\pi}b^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{\sqrt{6\pi}b^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{\sqrt{6\pi}b^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{\sqrt{2\pi}b^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(3*d*(c + d*x)^(3/2)) - (b^(3/2)*Sqrt[2*Pi]*Cos[a - (b*
c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/d^(5/2) - (b^(3
/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/d^(5/2) + (b^(3/2)*Sqrt[6*Pi]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/d^(5/2) + (b^(3/2)*Sqrt[2*Pi]*
FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/d^(5
/2) + (4*b*Cos[a + b*x]^2*Sin[a + b*x])/(d^2*Sqrt[c + d*x])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

```
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} \\
&\quad + \frac{(8b^2)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(12b^2)\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} \\
&= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} \\
&\quad - \frac{(12b^2)\int \left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}} + \frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)\,dx}{d^2} \\
&\quad + \frac{\left(8b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{\left(8b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} - \frac{(3b^2)\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx}{d^2} \\
&\quad - \frac{(9b^2)\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} + \frac{\left(16b^2\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^3} \\
&\quad - \frac{\left(16b^2\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^3} \\
&= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} \\
&\quad - \frac{8b^{3/2}\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} \\
&\quad - \frac{\left(3b^2\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{d^2} - \frac{\left(9b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d^2} \\
&\quad + \frac{\left(3b^2\sin\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{d^2} + \frac{\left(9b^2\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{d^2} \\
&= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{8b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} \\
&\quad - \frac{8b^{3/2}\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}} \\
&\quad - \frac{\left(6b^2\cos\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^3} \\
&\quad - \frac{\left(18b^2\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^3} \\
&\quad + \frac{\left(6b^2\sin\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^3} \\
&\quad + \frac{\left(18b^2\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{d^3}
\end{aligned}
$$

$$= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}$$
$$- \frac{b^{3/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}}$$
$$+ \frac{b^{3/2}\sqrt{6\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{d^{5/2}}$$
$$+ \frac{b^{3/2}\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.92

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{-4d\cos^3(a+bx) - 3de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right) - 3de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{3}$$

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^(5/2),x]

[Out] (-4*d*Cos[a + b*x]^3 - 3*d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(3/2)
*Gamma[1/2, ((-I)*b*(c + d*x))/d] - (3*d*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/
2, (I*b*(c + d*x))/d])/E^(I*(a - (b*c)/d)) - 3*Sqrt[3]*d*E^((3*I)*(a - (b*c
)/d))*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-3*I)*b*(c + d*x))/d] - (3*
Sqrt[3]*d*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((3*I)*b*(c + d*x))/d])/E^((
3*I)*(a - (b*c)/d)) + 24*b*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(6*d^2*(c
 + d*x)^(3/2))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2(dx+c)^{\frac{3}{2}}}-\frac{b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)\right)}{d\sqrt{\frac{b}{d}}}\right)}{d}$
default	$-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2(dx+c)^{\frac{3}{2}}}-\frac{b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)\right)}{d\sqrt{\frac{b}{d}}}\right)}{d}$

```
[In] int(cos(b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/4/(d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^(1
/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d
-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-
b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/12/(d*
x+c)^(3/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^(1/2)*sin(3
*b*(d*x+c)/d+3*(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3
*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)
/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.26

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{3\sqrt{6}(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(6)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos
(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sq
rt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-(b*c - a
*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(pi*b*
d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 3*sqrt(6)*(pi*b*d^2*x^2 + 2*
pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqr
t(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 2*(d*cos(b*x + a)^3 - 6*(b*d*x + b*c)*
cos(b*x + a)^2*sin(b*x + a))*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(5/2),x)
[Out] Integral(cos(a + b*x)**3/(c + d*x)**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{3\left(\sqrt{3}\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{3i\,(dx+c)b}{d}\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{3i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+\left((i+1)\sqrt{2}\Gamma\left(\right.\right.\right.}{}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
[Out] -3/16*(sqrt(3)*(((I - 1)*sqrt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) - (I + 1)*s
qrt(2)*gamma(-3/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I + 1)*sq
rt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-3/2, -3*I*(d*
x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) - ((-(I - 1)*sqrt
(2)*gamma(-3/2, I*(d*x + c)*b/d) + (I + 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)
*b/d))*cos(-(b*c - a*d)/d) + (-(I + 1)*sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d)
 + (I - 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*((d*
x + c)*b/d)^(3/2))/((d*x + c)^(3/2)*d)
```

Giac [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos\left(bx+a\right)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^3/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(c+d\,x\right)^{5/2}}\,dx$$

```
[In] int(cos(a + b*x)^3/(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)^3/(c + d*x)^(5/2), x)
```

3.62 $\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result . . . 529
Rubi [A] (verified) . . . 530
Mathematica [C] (verified) . . . 534
Maple [A] (verified) . . . 534
Fricas [A] (verification not implemented) . . . 535
Sympy [F] . . . 536
Maxima [C] (verification not implemented) . . . 536
Giac [F] . . . 536
Mupad [F(-1)] . . . 537

Optimal result

Integrand size = 18, antiderivative size = 356

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{2b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{6b^{5/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{6b^{5/2}\sqrt{6\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{5d^{7/2}} + \frac{2b^{5/2}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{5d^{7/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}}$$

```
[Out] -2/5*cos(b*x+a)^3/d/(d*x+c)^(5/2)+4/5*b*cos(b*x+a)^2*sin(b*x+a)/d^2/(d*x+c)
^(3/2)+2/5*b^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)+2/5*b^(5/2)*FresnelC(b^(1/2)*2^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/d^(7/2)+6/5*
b^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*6^(1/2)*Pi^(1/2)/d^(7/2)+6/5*b^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/d^(7/2)-16/5*b^
2*cos(b*x+a)/d^3/(d*x+c)^(1/2)+24/5*b^2*cos(b*x+a)^3/d^3/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 3378, 3387, 3386, 3432, 3385, 3433, 3394}

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{6\sqrt{6\pi}b^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$
$$+\frac{2\sqrt{2\pi}b^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$
$$+\frac{2\sqrt{2\pi}b^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$
$$+\frac{6\sqrt{6\pi}b^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}}$$
$$-\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

```
[In] Int[Cos[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (-16*b^2*Cos[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (2*Cos[a + b*x]^3)/(5*d*(c +
 d*x)^(5/2)) + (24*b^2*Cos[a + b*x]^3)/(5*d^3*Sqrt[c + d*x]) + (2*b^(5/2)*S
qrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (6*b^(5/2)*Sqrt[6*P
i]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(5*d^(7/2)) + (2*b^(5/2)*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
 + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(5*d^(7/2)) + (4*b*Cos[a + b*x]^2*Sin[a
 + b*x])/(5*d^2*(c + d*x)^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} = &-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} \\ &+ \frac{(8b^2)\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2} - \frac{(12b^2)\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} \\
&\quad + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{(16b^3)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{5d^3} \\
&\quad - \frac{(72b^3)\int \left(-\frac{\sin(a+bx)}{4\sqrt{c+dx}} - \frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)\,dx}{5d^3} \\
&= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} \\
&\quad + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{(18b^3)\int \frac{\sin(a+bx)}{\sqrt{c+dx}}\,dx}{5d^3} + \frac{(18b^3)\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}\,dx}{5d^3} \\
&\quad - \frac{\left(16b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{5d^3} - \frac{\left(16b^3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{5d^3} \\
&= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} \\
&\quad + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} + \frac{\left(18b^3\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{5d^3} \\
&\quad - \frac{\left(32b^3\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{5d^4} \\
&\quad + \frac{\left(18b^3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{5d^3} + \frac{\left(18b^3\sin\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{\sqrt{c+dx}}\,dx}{5d^3} \\
&\quad - \frac{\left(32b^3\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{5d^4} \\
&\quad + \frac{\left(18b^3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}\,dx}{5d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} \\
&\quad - \frac{16b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\
&\quad - \frac{16b^{5/2}\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{5d^{7/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}} \\
&\quad + \frac{\left(36b^3\cos\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{5d^4} \\
&\quad + \frac{\left(36b^3\cos\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \sin\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{5d^4} \\
&\quad + \frac{\left(36b^3\sin\left(3a-\frac{3bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{3bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{5d^4} \\
&\quad + \frac{\left(36b^3\sin\left(a-\frac{bc}{d}\right)\right)\operatorname{Subst}\left(\int \cos\left(\frac{bx^2}{d}\right)\,dx, x, \sqrt{c+dx}\right)}{5d^4} \\
&= -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}} \\
&\quad + \frac{2b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\
&\quad + \frac{6b^{5/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\
&\quad + \frac{6b^{5/2}\sqrt{6\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{5d^{7/2}} \\
&\quad + \frac{2b^{5/2}\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{5d^{7/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.26

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{e^{-3ia}\left(2e^{4ia}\left(-3d^2e^{ibx}+2be^{-\frac{ibc}{d}}(c+dx)\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right)^{3}\right.\right.\right.$$

```
[In] Integrate[Cos[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (2*E^((4*I)*a)*(-3*d^2*E^(I*b*x) + (2*b*(c + d*x)*(E^((I*b*(c + d*x))/d)*((
-I)*d + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((
-I)*b*(c + d*x))/d]))/E^((I*b*c)/d)) + E^((2*I)*a - I*b*x)*(-6*d^2 + (4*I)*
b*d*(c + d*x) + 8*b^2*(c + d*x)^2 + 8*d^2*E^((I*b*(c + d*x))/d)*((I*b*(c +
d*x))/d)^(5/2)*Gamma[1/2, (I*b*(c + d*x))/d]) + 2*E^((6*I)*a)*(-(d^2*E^((3*
I)*b*x)) + (2*b*(c + d*x)*(E^(((3*I)*b*(c + d*x))/d)*((-I)*d + 6*b*(c + d*x
)) - (6*I)*Sqrt[3]*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-3*I)*b*(c +
 d*x))/d]))/E^(((3*I)*b*c)/d)) + (2*(-d^2 - I*b*(c + d*x)*(-2*d + (12*I)*b*
(c + d*x) - 12*Sqrt[3]*d*E^(((3*I)*b*(c + d*x))/d)*((I*b*(c + d*x))/d)^(3/2
)*Gamma[1/2, ((3*I)*b*(c + d*x))/d]))/E^((3*I)*b*x))/(40*d^3*E^((3*I)*a)*(
c + d*x)^(5/2))
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{10(dx+c)^{\frac{5}{2}}}-\frac{3b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}\right.}{5d}$
default	$-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{10(dx+c)^{\frac{5}{2}}}-\frac{3b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{3d}\right.}{5d}$

```
[In] int(cos(b*x+a)^3/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-3/20/(d*x+c)^(5/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x+c
)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^(1/2)*cos(b*(d*x+c
)/d+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*Fresnel
S(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC
(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/20/(d*x+c)^(5/2)*cos(
3*b*(d*x+c)/d+3*(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x+c)^(3/2)*sin(3*b*(d*x+c)/d
+3*(a*d-b*c)/d)+2*b/d*(-1/(d*x+c)^(1/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-b/
d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)
/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*Fresnel
C(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{2\left(3\sqrt{6}(\pi b^2 d^3 x^3 + 3\,\pi b^2 c d^2 x^2 + 3\,\pi b^2 c^2 dx + \pi b^2 c^3)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+\right.\right.}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(6)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi
*b^2*c^3)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi
*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(s
qrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c
*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 3*sqrt(6)*(pi*b^2*d^3
*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*f
resnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + ((
12*b^2*d^2*x^2 + 24*b^2*c*d*x + 12*b^2*c^2 - d^2)*cos(b*x + a)^3 + 2*(b*d^2
*x + b*c*d)*cos(b*x + a)^2*sin(b*x + a) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*cos(b*x + a))*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
c^3*d^3)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos^3(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)**3/(d*x+c)**(7/2),x)
[Out] Integral(cos(a + b*x)**3/(c + d*x)**(7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.71

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx =$$

$$-\frac{3\left(3\sqrt{3}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{3i\,(dx+c)b}{d}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{3i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+\left((i-1)\sqrt{2}\Gamma\right.\right.}{}$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
[Out] -3/16*(3*sqrt(3)*((-(I + 1)*sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) + (I - 1
)*sqrt(2)*gamma(-5/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I - 1)
*sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-5/2, -3*I*
(d*x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) - (((I + 1)*sq
rt(2)*gamma(-5/2, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-5/2, -I*(d*x +
c)*b/d))*cos(-(b*c - a*d)/d) + (-(I - 1)*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/
d) + (I + 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d))*((
d*x + c)*b/d)^(5/2))/((d*x + c)^(5/2)*d)
```

Giac [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos\left(bx+a\right)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^3/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(c+d\,x\right)^{7/2}}\,dx$$

```
[In] int(cos(a + b*x)^3/(c + d*x)^(7/2),x)
[Out] int(cos(a + b*x)^3/(c + d*x)^(7/2), x)
```

3.63 $\int x^{3/2}\cos(x)\,dx$

Optimal result . . . 538
Rubi [A] (verified) . . . 538
Mathematica [C] (verified) . . . 539
Maple [A] (verified) . . . 539
Fricas [A] (verification not implemented) . . . 540
Sympy [A] (verification not implemented) . . . 540
Maxima [C] (verification not implemented) . . . 541
Giac [C] (verification not implemented) . . . 541
Mupad [F(-1)] . . . 542

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int x^{3/2}\cos(x)\,dx = \frac{3}{2}\sqrt{x}\cos(x) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right) + x^{3/2}\sin(x)$$

```
[Out] x^(3/2)*sin(x)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+3/2*
x^(1/2)*cos(x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3385, 3433}

$$\int x^{3/2}\cos(x)\,dx = -\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right) + x^{3/2}\sin(x) + \frac{3}{2}\sqrt{x}\cos(x)$$

```
[In] Int[x^(3/2)*Cos[x],x]
```

```
[Out] (3*Sqrt[x]*Cos[x])/2 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[x]])/2 + x^(3
/2)*Sin[x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x^{3/2}\sin(x) - \frac{3}{2}\int \sqrt{x}\sin(x)\,dx \\
&= \frac{3}{2}\sqrt{x}\cos(x) + x^{3/2}\sin(x) - \frac{3}{4}\int \frac{\cos(x)}{\sqrt{x}}\,dx \\
&= \frac{3}{2}\sqrt{x}\cos(x) + x^{3/2}\sin(x) - \frac{3}{2}\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{x}\right) \\
&= \frac{3}{2}\sqrt{x}\cos(x) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right) + x^{3/2}\sin(x)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int x^{3/2}\cos(x)\,dx = \frac{\sqrt{x}\Gamma\left(\frac{5}{2}, -ix\right)}{2\sqrt{-ix}} + \frac{\sqrt{x}\Gamma\left(\frac{5}{2}, ix\right)}{2\sqrt{ix}}$$

```
[In] Integrate[x^(3/2)*Cos[x],x]
```

```
[Out] (Sqrt[x]*Gamma[5/2, (-I)*x])/(2*Sqrt[(-I)*x]) + (Sqrt[x]*Gamma[5/2, I*x])/(
2*Sqrt[I*x])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$x^{\frac{3}{2}} \sin(x) - \frac{3\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi}}{4} + \frac{3\sqrt{x}\cos(x)}{2}$	34
default	$x^{\frac{3}{2}} \sin(x) - \frac{3\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi}}{4} + \frac{3\sqrt{x}\cos(x)}{2}$	34
meijerg	$2\sqrt{2}\sqrt{\pi}\left(\frac{3\sqrt{x}\sqrt{2}\cos(x)}{8\sqrt{\pi}} + \frac{x^{\frac{3}{2}}\sqrt{2}\sin(x)}{4\sqrt{\pi}} - \frac{3\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)}{8}\right)$	49

```
[In] int(x^(3/2)*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] x^(3/2)*sin(x)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+3/2*
x^(1/2)*cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int x^{3/2}\cos(x)\,dx = -\frac{3}{4}\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right) + \frac{1}{2}\left(2\,x\sin(x) + 3\,\cos(x)\right)\sqrt{x}$$

```
[In] integrate(x^(3/2)*cos(x),x, algorithm="fricas")
```

```
[Out] -3/4*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x)/sqrt(pi)) + 1/2*(2*x*sin(
x) + 3*cos(x))*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int x^{3/2}\cos(x)\,dx = \frac{5x^{\frac{3}{2}}\sin(x)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{15\sqrt{x}\cos(x)\Gamma\left(\frac{5}{4}\right)}{8\Gamma\left(\frac{9}{4}\right)} - \frac{15\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{16\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x**(3/2)*cos(x),x)
```

```
[Out] 5*x**(3/2)*sin(x)*gamma(5/4)/(4*gamma(9/4)) + 15*sqrt(x)*cos(x)*gamma(5/4)/
(8*gamma(9/4)) - 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*sqrt(x)/sqrt(pi))*gam
ma(5/4)/(16*gamma(9/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int x^{3/2}\cos(x)\,dx = x^{\frac{3}{2}}\sin(x)$$
$$-\frac{3}{32}\sqrt{\pi}\Big(-(i-1)\sqrt{2}\operatorname{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)-(i+1)\sqrt{2}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)+(i+1)\sqrt{2}\operatorname{erf}\left(\sqrt{}\right.$$
$$+\frac{3}{2}\sqrt{x}\cos(x)$$

```
[In] integrate(x^(3/2)*cos(x),x, algorithm="maxima")
```

```
[Out] x^(3/2)*sin(x) - 3/32*sqrt(pi)*(-(I - 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*
sqrt(x)) - (I + 1)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (I + 1)*sqr
t(2)*erf(sqrt(-I)*sqrt(x)) - (I - 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x))) + 3/2
*sqrt(x)*cos(x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\begin{aligned}\int x^{3/2}\cos(x)\,dx &= \left(\frac{3}{16}i+\frac{3}{16}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)\\ &-\left(\frac{3}{16}i-\frac{3}{16}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)\\ &-\frac{1}{4}\left(2i\,x^{\frac{3}{2}}-3\sqrt{x}\right)e^{(i\,x)}-\frac{1}{4}\left(-2i\,x^{\frac{3}{2}}-3\sqrt{x}\right)e^{(-i\,x)}\end{aligned}$$

```
[In] integrate(x^(3/2)*cos(x),x, algorithm="giac")
```

```
[Out] (3/16*I + 3/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (3/16
*I - 3/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x)) - 1/4*(2*I*
x^(3/2) - 3*sqrt(x))*e^(I*x) - 1/4*(-2*I*x^(3/2) - 3*sqrt(x))*e^(-I*x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\cos(x)\,dx = \int x^{3/2}\,\cos\left(x\right)\,dx$$

```
[In] int(x^(3/2)*cos(x),x)
[Out] int(x^(3/2)*cos(x), x)
```

3.64 $\int \sqrt{x}\cos(x)\,dx$

Optimal result . . . 543
Rubi [A] (verified) . . . 543
Mathematica [C] (verified) . . . 544
Maple [A] (verified) . . . 544
Fricas [A] (verification not implemented) . . . 545
Sympy [A] (verification not implemented) . . . 545
Maxima [C] (verification not implemented) . . . 545
Giac [C] (verification not implemented) . . . 546
Mupad [B] (verification not implemented) . . . 546

Optimal result

Integrand size = 8, antiderivative size = 36

$$\int \sqrt{x}\cos(x)\,dx = -\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right) + \sqrt{x}\sin(x)$$

```
[Out] -1/2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+sin(x)*x^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3377, 3386, 3432}

$$\int \sqrt{x}\cos(x)\,dx = \sqrt{x}\sin(x) - \sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

```
[In] Int[Sqrt[x]*Cos[x],x]
```

```
[Out] -(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]) + Sqrt[x]*Sin[x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{x}\sin(x) - \frac{1}{2}\int \frac{\sin(x)}{\sqrt{x}}\,dx \\ &= \sqrt{x}\sin(x) - \text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{x}\right) \\ &= -\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right) + \sqrt{x}\sin(x)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \sqrt{x}\cos(x)\,dx = \frac{\sqrt{-ix}\Gamma\left(\frac{3}{2},-ix\right) + \sqrt{ix}\Gamma\left(\frac{3}{2},ix\right)}{2\sqrt{x}}$$

```
[In] Integrate[Sqrt[x]*Cos[x],x]
[Out] (Sqrt[(-I)*x]*Gamma[3/2, (-I)*x] + Sqrt[I*x]*Gamma[3/2, I*x])/(2*Sqrt[x])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{\text{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi}}{2} + \sin\left(x\right)\sqrt{x}$	27
default	$-\frac{\text{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi}}{2} + \sin\left(x\right)\sqrt{x}$	27
meijerg	$\sqrt{2}\sqrt{\pi}\left(\frac{\sqrt{x}\sqrt{2}\sin(x)}{2\sqrt{\pi}} - \frac{\text{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)}{2}\right)$	35

```
[In] int(x^(1/2)*cos(x),x,method=_RETURNVERBOSE)
[Out] -1/2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+sin(x)*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\cos(x)\,dx = -\frac{1}{2}\sqrt{2}\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right) + \sqrt{x}\sin(x)$$

```
[In] integrate(x^(1/2)*cos(x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(pi)*fresnel_sin(sqrt(2)*sqrt(x)/sqrt(pi)) + sqrt(x)*sin(x
)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \sqrt{x}\cos(x)\,dx = \frac{3\sqrt{x}\sin(x)\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{3\sqrt{2}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x**(1/2)*cos(x),x)
```

```
[Out] 3*sqrt(x)*sin(x)*gamma(3/4)/(4*gamma(7/4)) - 3*sqrt(2)*sqrt(pi)*fresnels(sq
rt(2)*sqrt(x)/sqrt(pi))*gamma(3/4)/(8*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

$$\int \sqrt{x}\cos(x)\,dx =$$
$$-\frac{1}{16}\sqrt{\pi}\Bigg((i+1)\,\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{x}\right) + (i-1)\,\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{x}\right) - (i-1)\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{}\right.$$
$$+\sqrt{x}\sin(x)$$

```
[In] integrate(x^(1/2)*cos(x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*((I + 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*sqrt(x)) + (I - 1
)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (I - 1)*sqrt(2)*erf(sqrt(-I)
*sqrt(x)) + (I + 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x))) + sqrt(x)*sin(x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \sqrt{x}\cos(x)\,dx = -\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) + \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) - \frac{1}{2}i\sqrt{x}e^{(i\,x)} + \frac{1}{2}i\sqrt{x}e^{(-i\,x)}$$

```
[In] integrate(x^(1/2)*cos(x),x, algorithm="giac")
```

```
[Out] -(1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (1/8*I
 + 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x)) - 1/2*I*sqrt(x
)*e^(I*x) + 1/2*I*sqrt(x)*e^(-I*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\cos(x)\,dx = \sqrt{x}\,\sin(x) - \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)}{2}$$

```
[In] int(x^(1/2)*cos(x),x)
```

```
[Out] x^(1/2)*sin(x) - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*x^(1/2))/pi^(1/2)))/2
```

3.65 $\int \frac{\cos(x)}{\sqrt{x}}\,dx$

Optimal result . . . 547
Rubi [A] (verified) . . . 547
Mathematica [C] (verified) . . . 548
Maple [A] (verified) . . . 548
Fricas [A] (verification not implemented) . . . 549
Sympy [A] (verification not implemented) . . . 549
Maxima [C] (verification not implemented) . . . 549
Giac [C] (verification not implemented) . . . 550
Mupad [B] (verification not implemented) . . . 550

Optimal result

Integrand size = 8, antiderivative size = 24

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = \sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

```
[Out] FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3385, 3433}

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = \sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

```
[In] Int[Cos[x]/Sqrt[x],x]
```

```
[Out] Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[x]]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \cos\left(x^2\right)\, dx, x, \sqrt{x}\right)$$

$$= \sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{\cos(x)}{\sqrt{x}}\, dx = -\frac{i\left(\sqrt{-ix}\,\Gamma\left(\frac{1}{2}, -ix\right) - \sqrt{ix}\,\Gamma\left(\frac{1}{2}, ix\right)\right)}{2\sqrt{x}}$$

```
[In] Integrate[Cos[x]/Sqrt[x],x]
```

```
[Out] ((-1/2*I)*(Sqrt[(-I)*x]*Gamma[1/2, (-I)*x] - Sqrt[I*x]*Gamma[1/2, I*x]))/Sq
rt[x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\text{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi}$	19
default	$\text{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi}$	19
meijerg	$\text{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi}$	19

```
[In] int(cos(x)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = \sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)$$

```
[In] integrate(cos(x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x)/sqrt(pi))
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = \frac{\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(cos(x)/x**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*sqrt(x)/sqrt(pi))*gamma(1/4)/(4*gamma(5/4
))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx =$$
$$-\frac{1}{8}\sqrt{\pi}\left((i-1)\,\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{x}\right)+(i+1)\,\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{x}\right)-(i+1)\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{}\right.\right.$$

```
[In] integrate(cos(x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*((I - 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*sqrt(x)) + (I + 1)
*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (I + 1)*sqrt(2)*erf(sqrt(-I)*
sqrt(x)) + (I - 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x)))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = -\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) + \left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)$$

```
[In] integrate(cos(x)/x^(1/2),x, algorithm="giac")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (1/4*I
 - 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = \sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)$$

```
[In] int(cos(x)/x^(1/2),x)
```

```
[Out] 2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*x^(1/2))/pi^(1/2))
```

3.66 $\int \frac{\cos(x)}{x^{3/2}}\,dx$

Optimal result . . . 551
Rubi [A] (verified) . . . 551
Mathematica [C] (verified) . . . 552
Maple [A] (verified) . . . 552
Fricas [A] (verification not implemented) . . . 553
Sympy [A] (verification not implemented) . . . 553
Maxima [C] (verification not implemented) . . . 553
Giac [F] . . . 554
Mupad [F(-1)] . . . 554

Optimal result

Integrand size = 8, antiderivative size = 35

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = -\frac{2\cos(x)}{\sqrt{x}} - 2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

```
[Out] -2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)-2*cos(x)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3378, 3386, 3432}

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = -2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right) - \frac{2\cos(x)}{\sqrt{x}}$$

```
[In] Int[Cos[x]/x^(3/2),x]
```

```
[Out] (-2*Cos[x])/Sqrt[x] - 2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(x)}{\sqrt{x}} - 2\int \frac{\sin(x)}{\sqrt{x}}\,dx \\ &= -\frac{2\cos(x)}{\sqrt{x}} - 4\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{x}\right) \\ &= -\frac{2\cos(x)}{\sqrt{x}} - 2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = \frac{-e^{-ix}(1+e^{2ix}) + \sqrt{-ix}\Gamma\left(\frac{1}{2}, -ix\right) + \sqrt{ix}\Gamma\left(\frac{1}{2}, ix\right)}{\sqrt{x}}$$

```
[In] Integrate[Cos[x]/x^(3/2),x]
```

```
[Out] (-((1 + E^((2*I)*x))/E^(I*x)) + Sqrt[(-I)*x]*Gamma[1/2, (-I)*x] + Sqrt[I*x]
*Gamma[1/2, I*x])/Sqrt[x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-2\ \text{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi} - \frac{2\cos(x)}{\sqrt{x}}$	28
default	$-2\ \text{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi} - \frac{2\cos(x)}{\sqrt{x}}$	28
meijerg	$\frac{\sqrt{2}\sqrt{\pi}\left(-\frac{4\sqrt{2}\cos(x)}{\sqrt{\pi}\sqrt{x}} - 8\ \text{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\right)}{4}$	36

```
[In] int(cos(x)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)-2*cos(x)/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = -\frac{2\left(\sqrt{2}\sqrt{\pi}x\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)+\sqrt{x}\cos\left(x\right)\right)}{x}$$

```
[In] integrate(cos(x)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*sqrt(pi)*x*fresnel_sin(sqrt(2)*sqrt(x)/sqrt(pi)) + sqrt(x)*cos(
x))/x
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{2\Gamma\left(\frac{3}{4}\right)}+\frac{\cos\left(x\right)\Gamma\left(-\frac{1}{4}\right)}{2\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

```
[In] integrate(cos(x)/x**(3/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*fresnels(sqrt(2)*sqrt(x)/sqrt(pi))*gamma(-1/4)/(2*gamma(3/
4)) + cos(x)*gamma(-1/4)/(2*sqrt(x)*gamma(3/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = -\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\Gamma\left(-\frac{1}{2},i\,x\right)+\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\Gamma\left(-\frac{1}{2},-i\,x\right)$$

```
[In] integrate(cos(x)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*gamma(-1/2, I*x) + (1/4*I - 1/4)*sqrt(2)*gamma(-1/2,
 -I*x)
```

Giac [F]

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = \int \frac{\cos(x)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(x)/x^(3/2),x, algorithm="giac")
[Out] integrate(cos(x)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = \int \frac{\cos(x)}{x^{3/2}}\,dx$$

```
[In] int(cos(x)/x^(3/2),x)
[Out] int(cos(x)/x^(3/2), x)
```

3.67 $\int (c+dx)^{4/3}\cos(a+bx)\,dx$

Optimal result . . . 555
Rubi [A] (verified) . . . 555
Mathematica [A] (verified) . . . 557
Maple [F] . . . 557
Fricas [A] (verification not implemented) . . . 557
Sympy [F] . . . 558
Maxima [A] (verification not implemented) . . . 558
Giac [F] . . . 558
Mupad [F(-1)] . . . 559

Optimal result

Integrand size = 16, antiderivative size = 183

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2} + \frac{2id^2e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}} - \frac{2id^2e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}} + \frac{(c+dx)^{4/3}\sin(a+bx)}{b}$$

```
[Out] 4/3*d*(d*x+c)^(1/3)*cos(b*x+a)/b^2+2/9*I*d^2*exp(I*(a-b*c/d))*(-I*b*(d*x+c)
/d)^(2/3)*GAMMA(1/3,-I*b*(d*x+c)/d)/b^3/(d*x+c)^(2/3)-2/9*I*d^2*(I*b*(d*x+c
)/d)^(2/3)*GAMMA(1/3,I*b*(d*x+c)/d)/b^3/exp(I*(a-b*c/d))/(d*x+c)^(2/3)+(d*x
+c)^(4/3)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3377, 3388, 2212}

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \frac{2id^2e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}} - \frac{2id^2e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}} + \frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2} + \frac{(c+dx)^{4/3}\sin(a+bx)}{b}$$

```
[In] Int[(c + d*x)^(4/3)*Cos[a + b*x],x]
```

```
[Out] (4*d*(c + d*x)^(1/3)*Cos[a + b*x])/(3*b^2) + (((2*I)/9)*d^2*E^(I*(a - (b*c)
/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(b^3*(c
 + d*x)^(2/3)) - (((2*I)/9)*d^2*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(
c + d*x))/d])/(b^3*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3)) + ((c + d*x)^(4/3)*
Sin[a + b*x])/b
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c+dx)^{4/3}\sin(a+bx)}{b} - \frac{(4d)\int \sqrt[3]{c+dx}\sin(a+bx)\,dx}{3b} \\
&= \frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2} + \frac{(c+dx)^{4/3}\sin(a+bx)}{b} - \frac{(4d^2)\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx}{9b^2} \\
&= \frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2} + \frac{(c+dx)^{4/3}\sin(a+bx)}{b} - \frac{(2d^2)\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx}{9b^2} - \frac{(2d^2)\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}\,dx}{9b^2} \\
&= \frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2} + \frac{2id^2 e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}} \\
&\quad - \frac{2id^2 e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}} + \frac{(c+dx)^{4/3}\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \frac{de^{-\frac{i(bc+ad)}{d}}\sqrt[3]{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{7}{3},-\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{-\frac{ib(c+dx)}{d}}}+\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{7}{3},\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

```
[In] Integrate[(c + d*x)^(4/3)*Cos[a + b*x],x]
```

```
[Out] (d*(c + d*x)^(1/3)*((E^((2*I)*a)*Gamma[7/3, ((-I)*b*(c + d*x))/d])/(((-I)*b
*(c + d*x))/d)^(1/3) + (E^(((2*I)*b*c)/d)*Gamma[7/3, (I*b*(c + d*x))/d])/((
I*b*(c + d*x))/d)^(1/3)))/(2*b^2*E^((I*(b*c + a*d))/d))
```

Maple [F]

$$\int (dx+c)^{\frac{4}{3}}\cos(bx+a)\,dx$$

```
[In] int((d*x+c)^(4/3)*cos(b*x+a),x)
```

```
[Out] int((d*x+c)^(4/3)*cos(b*x+a),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.95

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = -\frac{2\left(i\,d^2\cos\left(-\frac{bc-ad}{d}\right)+d^2\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{i\,b}{d}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},\frac{i\,bdx+i\,bc}{d}\right)+2\left(-i\,d^2\cos\left(-\frac{bc-ad}{d}\right)+d^2\sin\left(-\frac{bc-ad}{d}\right)\right)\left(-\frac{i\,b}{d}\right.}{9\,b^3}$$

```
[In] integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/9*(2*(I*d^2*cos(-(b*c - a*d)/d) + d^2*sin(-(b*c - a*d)/d))*(I*b/d)^(2/3)
*gamma(1/3, (I*b*d*x + I*b*c)/d) + 2*(-I*d^2*cos(-(b*c - a*d)/d) + d^2*sin(
-(b*c - a*d)/d))*(-I*b/d)^(2/3)*gamma(1/3, (-I*b*d*x - I*b*c)/d) - 3*(4*b*d
*cos(b*x + a) + 3*(b^2*d*x + b^2*c)*sin(b*x + a))*(d*x + c)^(1/3))/b^3
```

Sympy [F]

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \int (c+dx)^{\frac{4}{3}}\cos\left(a+bx\right)dx$$

```
[In] integrate((d*x+c)**(4/3)*cos(b*x+a),x)
[Out] Integral((c + d*x)**(4/3)*cos(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.28

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \frac{9\,(dx+c)^{\frac{4}{3}}b\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+12\,(dx+c)^{\frac{1}{3}}\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left(\left(($$

```
[In] integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="maxima")
[Out] 1/9*(9*(d*x + c)^(4/3)*b*((d*x + c)*b/d)^(1/3)*d*sin(((d*x + c)*b - b*c + a
*d)/d) + 12*(d*x + c)^(1/3)*((d*x + c)*b/d)^(1/3)*d^2*cos(((d*x + c)*b - b*
c + a*d)/d) + (((sqrt(3) - I)*gamma(1/3, I*(d*x + c)*b/d) + (sqrt(3) + I)*g
amma(1/3, -I*(d*x + c)*b/d))*d^2*cos(-(b*c - a*d)/d) + ((-I*sqrt(3) - 1)*ga
mma(1/3, I*(d*x + c)*b/d) + (I*sqrt(3) - 1)*gamma(1/3, -I*(d*x + c)*b/d))*d
^2*sin(-(b*c - a*d)/d))*(d*x + c)^(1/3))/(b^2*((d*x + c)*b/d)^(1/3)*d)
```

Giac [F]

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \int (dx+c)^{\frac{4}{3}}\cos\left(bx+a\right)\,dx$$

```
[In] integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^(4/3)*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\left(c+d\,x\right)^{4/3}\,dx$$

```
[In] int(cos(a + b*x)*(c + d*x)^(4/3),x)
[Out] int(cos(a + b*x)*(c + d*x)^(4/3), x)
```

3.68 $\int (c+dx)^{2/3}\cos(a+bx)\,dx$

Optimal result . . . 560
Rubi [A] (verified) . . . 560
Mathematica [A] (verified) . . . 562
Maple [F] . . . 562
Fricas [A] (verification not implemented) . . . 562
Sympy [F] . . . 563
Maxima [A] (verification not implemented) . . . 563
Giac [F] . . . 563
Mupad [F(-1)] . . . 564

Optimal result

Integrand size = 16, antiderivative size = 152

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{(c+dx)^{2/3}\sin(a+bx)}{b}$$

```
[Out] 1/3*d*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,-I*b*(d*x+c)/d)/b^2
/(d*x+c)^(1/3)+1/3*d*(I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,I*b*(d*x+c)/d)/b^2/exp
(I*(a-b*c/d))/(d*x+c)^(1/3)+(d*x+c)^(2/3)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3377, 3389, 2212}

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{(c+dx)^{2/3}\sin(a+bx)}{b}$$

```
[In] Int[(c + d*x)^(2/3)*Cos[a + b*x],x]
```

```
[Out] (d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c +
 d*x))/d])/(3*b^2*(c + d*x)^(1/3)) + (d*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3
, (I*b*(c + d*x))/d])/(3*b^2*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3)) + ((c + d
*x)^(2/3)*Sin[a + b*x])/b
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c+dx)^{2/3}\sin(a+bx)}{b} - \frac{(2d)\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}\,dx}{3b} \\
&= \frac{(c+dx)^{2/3}\sin(a+bx)}{b} - \frac{(id)\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{3b} + \frac{(id)\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{3b} \\
&= \frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} \\
&\quad + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{(c+dx)^{2/3}\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b^2\sqrt[3]{c+dx}}$$

```
[In] Integrate[(c + d*x)^(2/3)*Cos[a + b*x],x]
```

```
[Out] (d*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[5/3, ((-I)*b*(c + d*x))/
d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[5/3, (I*b*(c + d*x))
/d]))/(2*b^2*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [F]

$$\int (dx+c)^{\frac{2}{3}}\cos(bx+a)\,dx$$

```
[In] int((d*x+c)^(2/3)*cos(b*x+a),x)
```

```
[Out] int((d*x+c)^(2/3)*cos(b*x+a),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \frac{3\,(dx+c)^{\frac{2}{3}}b\sin(bx+a)+\left(d\cos\left(-\frac{bc-ad}{d}\right)-i\,d\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{ib}{d}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},\frac{ibdx+ibc}{d}\right)+\left(d\cos\left(-\frac{bc-ad}{d}\right)\right.}{3\,b^2}$$

```
[In] integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(d*x + c)^(2/3)*b*sin(b*x + a) + (d*cos(-(b*c - a*d)/d) - I*d*sin(-(
b*c - a*d)/d))*(I*b/d)^(1/3)*gamma(2/3, (I*b*d*x + I*b*c)/d) + (d*cos(-(b*c
 - a*d)/d) + I*d*sin(-(b*c - a*d)/d))*(-I*b/d)^(1/3)*gamma(2/3, (-I*b*d*x -
 I*b*c)/d))/b^2
```

Sympy [F]

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \int (c+dx)^{\frac{2}{3}}\cos\left(a+bx\right)dx$$

```
[In] integrate((d*x+c)**(2/3)*cos(b*x+a),x)
[Out] Integral((c + d*x)**(2/3)*cos(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \frac{6\,(dx+c)^{\frac{2}{3}}\left(\frac{(dx+c)b}{d}\right)^{\frac{2}{3}} d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right) + \left(\left((\sqrt{3}+i)\Gamma\left(\frac{2}{3},\frac{i\,(dx+c)b}{d}\right) + (\sqrt{3}-i)\Gamma\left(\frac{2}{3},-\frac{i\,(dx+c)}{d}\right.\right.\right.}{6\,b}$$

```
[In] integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="maxima")
[Out] 1/6*(6*(d*x + c)^(2/3)*((d*x + c)*b/d)^(2/3)*d*sin(((d*x + c)*b - b*c + a*d
)/d) + (((sqrt(3) + I)*gamma(2/3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(2/
3, -I*(d*x + c)*b/d))*d*cos(-(b*c - a*d)/d) + ((-I*sqrt(3) + 1)*gamma(2/3,
I*(d*x + c)*b/d) + (I*sqrt(3) + 1)*gamma(2/3, -I*(d*x + c)*b/d))*d*sin(-(b*
c - a*d)/d))*(d*x + c)^(2/3))/(b*((d*x + c)*b/d)^(2/3)*d)
```

Giac [F]

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \int (dx+c)^{\frac{2}{3}}\cos\left(bx+a\right)\,dx$$

```
[In] integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^(2/3)*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \int \cos(a+b\,x)\,(c+d\,x)^{2/3}\,dx$$

```
[In] int(cos(a + b*x)*(c + d*x)^(2/3),x)
[Out] int(cos(a + b*x)*(c + d*x)^(2/3), x)
```

3.69 $\int \sqrt[3]{c+dx}\cos(a+bx)\,dx$

Optimal result . . . 565
Rubi [A] (verified) . . . 565
Mathematica [A] (verified) . . . 567
Maple [F] . . . 567
Fricas [A] (verification not implemented) . . . 567
Sympy [F] . . . 568
Maxima [A] (verification not implemented) . . . 568
Giac [F] . . . 568
Mupad [F(-1)] . . . 569

Optimal result

Integrand size = 16, antiderivative size = 152

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} + \frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}$$

```
[Out] 1/6*d*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,-I*b*(d*x+c)/d)/b^2
/(d*x+c)^(2/3)+1/6*d*(I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,I*b*(d*x+c)/d)/b^2/exp
(I*(a-b*c/d))/(d*x+c)^(2/3)+(d*x+c)^(1/3)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3377, 3389, 2212}

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} + \frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}$$

```
[In] Int[(c + d*x)^(1/3)*Cos[a + b*x],x]
```

```
[Out] (d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c +
 d*x))/d])/(6*b^2*(c + d*x)^(2/3)) + (d*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3
```

```
, (I*b*(c + d*x))/d])/(6*b^2*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3)) + ((c + d
*x)^(1/3)*Sin[a + b*x])/b
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{c+dx}\sin(a+bx)}{b} - \frac{d\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}\,dx}{3b} \\
&= \frac{\sqrt[3]{c+dx}\sin(a+bx)}{b} - \frac{(id)\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx}{6b} + \frac{(id)\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}\,dx}{6b} \\
&= \frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} \\
&\quad + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} + \frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx$$
$$= \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{4}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{4}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b^2(c+dx)^{2/3}}$$

```
[In] Integrate[(c + d*x)^(1/3)*Cos[a + b*x],x]
```

```
[Out] (d*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[4/3, ((-I)*b*(c + d*x))/
d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[4/3, (I*b*(c + d*x))
/d]))/(2*b^2*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

Maple [F]

$$\int (dx+c)^{\frac{1}{3}}\cos(bx+a)\,dx$$

```
[In] int((d*x+c)^(1/3)*cos(b*x+a),x)
```

```
[Out] int((d*x+c)^(1/3)*cos(b*x+a),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx$$
$$= \frac{\left(d\cos\left(-\frac{bc-ad}{d}\right)-i\,d\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{ib}{d}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},\frac{ibdx+ibc}{d}\right)+\left(d\cos\left(-\frac{bc-ad}{d}\right)+i\,d\sin\left(-\frac{bc-ad}{d}\right)\right)\left(-\frac{ib}{d}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},\right.}{6\,b^2}$$

```
[In] integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*((d*cos(-(b*c - a*d)/d) - I*d*sin(-(b*c - a*d)/d))*(I*b/d)^(2/3)*gamma(
1/3, (I*b*d*x + I*b*c)/d) + (d*cos(-(b*c - a*d)/d) + I*d*sin(-(b*c - a*d)/d
))*(-I*b/d)^(2/3)*gamma(1/3, (-I*b*d*x - I*b*c)/d) + 6*(d*x + c)^(1/3)*b*si
n(b*x + a))/b^2
```

Sympy [F]

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \int \sqrt[3]{c+dx}\cos\left(a+bx\right)\,dx$$

```
[In] integrate((d*x+c)**(1/3)*cos(b*x+a),x)
[Out] Integral((c + d*x)**(1/3)*cos(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx$$
$$= \frac{12\,(dx+c)^{\frac{1}{3}}\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}} d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right) + \left(\left((i\sqrt{3}+1)\Gamma\left(\frac{1}{3},\frac{i\,(dx+c)b}{d}\right) + (-i\sqrt{3}+1)\Gamma\left(\frac{1}{3},-\frac{i\,(dx+c)b}{d}\right)\right)\right.}{12\,b\left(\frac{(dx+c)b}{d}\right)}$$

```
[In] integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="maxima")
[Out] 1/12*(12*(d*x + c)^(1/3)*((d*x + c)*b/d)^(1/3)*d*sin(((d*x + c)*b - b*c + a
*d)/d) + (((I*sqrt(3) + 1)*gamma(1/3, I*(d*x + c)*b/d) + (-I*sqrt(3) + 1)*g
amma(1/3, -I*(d*x + c)*b/d))*d*cos(-(b*c - a*d)/d) + ((sqrt(3) - I)*gamma(1
/3, I*(d*x + c)*b/d) + (sqrt(3) + I)*gamma(1/3, -I*(d*x + c)*b/d))*d*sin(-(
b*c - a*d)/d))*(d*x + c)^(1/3))/(b*((d*x + c)*b/d)^(1/3)*d)
```

Giac [F]

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \int (dx+c)^{\frac{1}{3}}\cos\left(bx+a\right)\,dx$$

```
[In] integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/3)*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\left(c+d\,x\right)^{1/3}\,dx$$

```
[In] int(cos(a + b*x)*(c + d*x)^(1/3),x)
[Out] int(cos(a + b*x)*(c + d*x)^(1/3), x)
```

3.70 $\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx$

Optimal result . . . 570
Rubi [A] (verified) . . . 570
Mathematica [A] (verified) . . . 571
Maple [F] . . . 572
Fricas [A] (verification not implemented) . . . 572
Sympy [F] . . . 572
Maxima [A] (verification not implemented) . . . 572
Giac [F] . . . 573
Mupad [F(-1)] . . . 573

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}$$

```
[Out] -1/2*I*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,-I*b*(d*x+c)/d)/b/
(d*x+c)^(1/3)+1/2*I*(I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,I*b*(d*x+c)/d)/b/exp(I*
(a-b*c/d))/(d*x+c)^(1/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3388, 2212}

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}} - \frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] ((-1/2*I)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)
*b*(c + d*x))/d])/(b*(c + d*x)^(1/3)) + ((I/2)*((I*b*(c + d*x))/d)^(1/3)*Ga
mma[2/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx + \frac{1}{2}\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx \\ &= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\begin{aligned}&\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx \\ &= \frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right) + e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt[3]{c+dx}}\end{aligned}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] ((I/2)*(-(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c +
d*x))/d]) + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c
+ d*x))/d]))/(b*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [F]

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{1}{3}}}dx$$

```
[In] int(cos(b*x+a)/(d*x+c)^(1/3),x)
[Out] int(cos(b*x+a)/(d*x+c)^(1/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx$$
$$= \frac{\left(\frac{ib}{d}\right)^{\frac{1}{3}}\left(i\cos\left(-\frac{bc-ad}{d}\right)+\sin\left(-\frac{bc-ad}{d}\right)\right)\Gamma\left(\frac{2}{3},\frac{ibdx+ibc}{d}\right)+\left(-\frac{ib}{d}\right)^{\frac{1}{3}}\left(-i\cos\left(-\frac{bc-ad}{d}\right)+\sin\left(-\frac{bc-ad}{d}\right)\right)\Gamma\left(\frac{2}{3},\frac{-ibdx-}{d}\right.}{2b}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] 1/2*((I*b/d)^(1/3)*(I*cos(-(b*c - a*d)/d) + sin(-(b*c - a*d)/d))*gamma(2/3,
 (I*b*d*x + I*b*c)/d) + (-I*b/d)^(1/3)*(-I*cos(-(b*c - a*d)/d) + sin(-(b*c
- a*d)/d))*gamma(2/3, (-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**(1/3),x)
[Out] Integral(cos(a + b*x)/(c + d*x)**(1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx$$
$$= \frac{(dx+c)^{\frac{2}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},\frac{i(dx+c)b}{d}\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},\frac{i(dx+}{d}\right.\right.}{4\left(\frac{(dx+c)b}{d}\right)^{\frac{2}{3}}d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(d*x + c)^(2/3)*(((I*sqrt(3) - 1)*gamma(2/3, I*(d*x + c)*b/d) + (-I*sqr
t(3) - 1)*gamma(2/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((sqrt(3) + I
)*gamma(2/3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)*b/d))
*sin(-(b*c - a*d)/d))/(((d*x + c)*b/d)^(2/3)*d)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \int \frac{\cos(bx+a)}{(dx+c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^{1/3}}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^(1/3),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^(1/3), x)
```

3.71 $\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx$

Optimal result . . . 574
Rubi [A] (verified) . . . 574
Mathematica [A] (verified) . . . 575
Maple [F] . . . 576
Fricas [A] (verification not implemented) . . . 576
Sympy [F] . . . 576
Maxima [A] (verification not implemented) . . . 576
Giac [F] . . . 577
Mupad [F(-1)] . . . 577

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}$$

```
[Out] -1/2*I*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,-I*b*(d*x+c)/d)/b/
(d*x+c)^(2/3)+1/2*I*(I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,I*b*(d*x+c)/d)/b/exp(I*
(a-b*c/d))/(d*x+c)^(2/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3388, 2212}

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}} - \frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] ((-1/2*I)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)
*b*(c + d*x))/d])/(b*(c + d*x)^(2/3)) + ((I/2)*((I*b*(c + d*x))/d)^(2/3)*Ga
mma[1/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx + \frac{1}{2}\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}\,dx \\ &= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right) + e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b(c+dx)^{2/3}}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] ((I/2)*(-(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c +
d*x))/d]) + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(c
+ d*x))/d]))/(b*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

Maple [F]

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{2}{3}}}dx$$

```
[In] int(cos(b*x+a)/(d*x+c)^(2/3),x)
[Out] int(cos(b*x+a)/(d*x+c)^(2/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \frac{\left(\frac{ib}{d}\right)^{\frac{2}{3}}\left(i\cos\left(-\frac{bc-ad}{d}\right)+\sin\left(-\frac{bc-ad}{d}\right)\right)\Gamma\left(\frac{1}{3},\frac{ibdx+ibc}{d}\right)+\left(-\frac{ib}{d}\right)^{\frac{2}{3}}\left(-i\cos\left(-\frac{bc-ad}{d}\right)+\sin\left(-\right.\right.}{2b}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] 1/2*((I*b/d)^(2/3)*(I*cos(-(b*c - a*d)/d) + sin(-(b*c - a*d)/d))*gamma(1/3,
 (I*b*d*x + I*b*c)/d) + (-I*b/d)^(2/3)*(-I*cos(-(b*c - a*d)/d) + sin(-(b*c
- a*d)/d))*gamma(1/3, (-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \int \frac{\cos(a+bx)}{(c+dx)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**(2/3),x)
[Out] Integral(cos(a + b*x)/(c + d*x)**(2/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx =$$
$$-\frac{(dx+c)^{\frac{1}{3}}\left(\left((\sqrt{3}-i)\Gamma\left(\frac{1}{3},\frac{i(dx+c)b}{d}\right)+(\sqrt{3}+i)\Gamma\left(\frac{1}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left((i\sqrt{3}+1)\Gamma\left(\frac{1}{3},\frac{i(dx+c)b}{d}\right)\right.\right.}{4\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(d*x + c)^(1/3)*(((sqrt(3) - I)*gamma(1/3, I*(d*x + c)*b/d) + (sqrt(3)
 + I)*gamma(1/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) - ((I*sqrt(3) + 1)*
gamma(1/3, I*(d*x + c)*b/d) + (-I*sqrt(3) + 1)*gamma(1/3, -I*(d*x + c)*b/d)
)*sin(-(b*c - a*d)/d))/(((d*x + c)*b/d)^(1/3)*d)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \int \frac{\cos\left(bx+a\right)}{\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x + c)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^{2/3}}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^(2/3),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x)^(2/3), x)
```

3.72 $\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx$

Optimal result . . . 578
Rubi [A] (verified) . . . 578
Mathematica [A] (verified) . . . 580
Maple [F] . . . 580
Fricas [A] (verification not implemented) . . . 580
Sympy [F] . . . 581
Maxima [A] (verification not implemented) . . . 581
Giac [F] . . . 581
Mupad [F(-1)] . . . 582

Optimal result

Integrand size = 16, antiderivative size = 151

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}$$

```
[Out] -3*cos(b*x+a)/d/(d*x+c)^(1/3)+3/2*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(1/3)*G
AMMA(2/3,-I*b*(d*x+c)/d)/d/(d*x+c)^(1/3)+3/2*(I*b*(d*x+c)/d)^(1/3)*GAMMA(2/
3,I*b*(d*x+c)/d)/d/exp(I*(a-b*c/d))/(d*x+c)^(1/3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3378, 3389, 2212}

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^(4/3),x]
```

```
[Out] (-3*Cos[a + b*x])/(d*(c + d*x)^(1/3)) + (3*E^(I*(a - (b*c)/d))*(((-I)*b*(c
+ d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(2*d*(c + d*x)^(1/3)) +
(3*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c + d*x))/d])/(2*d*E^(I*(a -
(b*c)/d))*(c + d*x)^(1/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} - \frac{(3b)\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}\,dx}{d} \\
&= -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} - \frac{(3ib)\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{2d} + \frac{(3ib)\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{2d} \\
&= -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}} \\
&\quad + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = -\frac{e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\Gamma\left(-\frac{1}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\Gamma\left(-\frac{1}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d\sqrt[3]{c+dx}}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(4/3),x]
```

```
[Out] -1/2*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[-1/3, ((-I)*b*(c + d*x
))/d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[-1/3, (I*b*(c + d
*x))/d])/(d*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [F]

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{4}{3}}}dx$$

```
[In] int(cos(b*x+a)/(d*x+c)^(4/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(4/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.12

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = \frac{3\left(\left((dx+c)\cos\left(-\frac{bc-ad}{d}\right)-(i\,dx+i\,c)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{ib}{d}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},\frac{ibdx+ibc}{d}\right)+\left((dx+c)\cos\right.\right.}{2\,(d^2}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/2*(((d*x + c)*cos(-(b*c - a*d)/d) - (I*d*x + I*c)*sin(-(b*c - a*d)/d))*(I
*b/d)^(1/3)*gamma(2/3, (I*b*d*x + I*b*c)/d) + ((d*x + c)*cos(-(b*c - a*d)/d
) - (-I*d*x - I*c)*sin(-(b*c - a*d)/d))*(-I*b/d)^(1/3)*gamma(2/3, (-I*b*d*x
 - I*b*c)/d) - 2*(d*x + c)^(2/3)*cos(b*x + a))/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = \int \frac{\cos\left(a+bx\right)}{\left(c+dx\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**(4/3),x)
[Out] Integral(cos(a + b*x)/(c + d*x)**(4/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx =$$

$$-\frac{\left(\left((\sqrt{3}+i)\Gamma\left(-\frac{1}{3},\frac{i\,(dx+c)b}{d}\right)+(\sqrt{3}-i)\Gamma\left(-\frac{1}{3},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left((i\sqrt{3}-1)\Gamma\left(-\frac{1}{3},\frac{i\,(dx+c)b}{d}\right)+\right.}{4\left(dx+c\right)^{\frac{1}{3}}d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] -1/4*(((sqrt(3) + I)*gamma(-1/3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(-1/
3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) - ((I*sqrt(3) - 1)*gamma(-1/3, I*
(d*x + c)*b/d) + (-I*sqrt(3) - 1)*gamma(-1/3, -I*(d*x + c)*b/d))*sin(-(b*c
- a*d)/d))*((d*x + c)*b/d)^(1/3)/((d*x + c)^(1/3)*d)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = \int \frac{\cos\left(bx+a\right)}{\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/(d*x + c)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^{4/3}}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^(4/3),x)
[Out] int(cos(a + b*x)/(c + d*x)^(4/3), x)
```

3.73 $\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx$

Optimal result . . . 583
Rubi [A] (verified) . . . 583
Mathematica [A] (verified) . . . 585
Maple [F] . . . 585
Fricas [A] (verification not implemented) . . . 585
Sympy [F] . . . 586
Maxima [A] (verification not implemented) . . . 586
Giac [F] . . . 586
Mupad [F(-1)] . . . 587

Optimal result

Integrand size = 16, antiderivative size = 153

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}$$

```
[Out] -3/2*cos(b*x+a)/d/(d*x+c)^(2/3)+3/4*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(2/3)
*GAMMA(1/3,-I*b*(d*x+c)/d)/d/(d*x+c)^(2/3)+3/4*(I*b*(d*x+c)/d)^(2/3)*GAMMA(
1/3,I*b*(d*x+c)/d)/d/exp(I*(a-b*c/d))/(d*x+c)^(2/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3378, 3389, 2212}

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^(5/3),x]
```

```
[Out] (-3*Cos[a + b*x])/(2*d*(c + d*x)^(2/3)) + (3*E^(I*(a - (b*c)/d))*(((-I)*b*(
c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(4*d*(c + d*x)^(2/3))
```

```
+ (3*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(c + d*x))/d])/(4*d*E^(I*(a
- (b*c)/d))*(c + d*x)^(2/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} - \frac{(3b)\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}\,dx}{2d} \\
&= -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} - \frac{(3ib)\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}\,dx}{4d} + \frac{(3ib)\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}\,dx}{4d} \\
&= -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}} \\
&\quad + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = -\frac{e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(-\frac{2}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d(c+dx)^{2/3}}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(5/3),x]
```

```
[Out] -1/2*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[-2/3, ((-I)*b*(c + d*x
))/d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[-2/3, (I*b*(c + d
*x))/d])/(d*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

Maple [F]

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{5}{3}}}dx$$

```
[In] int(cos(b*x+a)/(d*x+c)^(5/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(5/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \frac{3\left(\left((dx+c)\cos\left(-\frac{bc-ad}{d}\right)-(i\,dx+i\,c)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{ib}{d}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},\frac{i\,bdx+i\,bc}{d}\right)+\left((dx+c)\,c\right.\right.}{4\,(d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] 3/4*(((d*x + c)*cos(-(b*c - a*d)/d) - (I*d*x + I*c)*sin(-(b*c - a*d)/d))*(I
*b/d)^(2/3)*gamma(1/3, (I*b*d*x + I*b*c)/d) + ((d*x + c)*cos(-(b*c - a*d)/d
) - (-I*d*x - I*c)*sin(-(b*c - a*d)/d))*(-I*b/d)^(2/3)*gamma(1/3, (-I*b*d*x
 - I*b*c)/d) - 2*(d*x + c)^(1/3)*cos(b*x + a))/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \int \frac{\cos\left(a+bx\right)}{(c+dx)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**(5/3),x)
[Out] Integral(cos(a + b*x)/(c + d*x)**(5/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \frac{\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},\frac{i\,(dx+c)b}{d}\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\left(\sqrt{3}-\right.\right.\right.}{4\,(dx+c)^{\frac{2}{3}}d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="maxima")
[Out] 1/4*(((-I*sqrt(3) - 1)*gamma(-2/3, I*(d*x + c)*b/d) + (I*sqrt(3) - 1)*gamma
(-2/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) - ((sqrt(3) - I)*gamma(-2/3,
I*(d*x + c)*b/d) + (sqrt(3) + I)*gamma(-2/3, -I*(d*x + c)*b/d))*sin(-(b*c -
 a*d)/d))*((d*x + c)*b/d)^(2/3)/((d*x + c)^(2/3)*d)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \int \frac{\cos\left(bx+a\right)}{(dx+c)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/(d*x + c)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^{5/3}}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^(5/3),x)
[Out] int(cos(a + b*x)/(c + d*x)^(5/3), x)
```

3.74 $\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx$

Optimal result . . . 588
Rubi [A] (verified) . . . 588
Mathematica [A] (verified) . . . 590
Maple [F] . . . 590
Fricas [A] (verification not implemented) . . . 590
Sympy [F] . . . 591
Maxima [A] (verification not implemented) . . . 591
Giac [F] . . . 591
Mupad [F(-1)] . . . 592

Optimal result

Integrand size = 16, antiderivative size = 182

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} - \frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} + \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}}$$

```
[Out] -3/4*cos(b*x+a)/d/(d*x+c)^(4/3)+9/8*I*b*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(
1/3)*GAMMA(2/3,-I*b*(d*x+c)/d)/d^2/(d*x+c)^(1/3)-9/8*I*b*(I*b*(d*x+c)/d)^(1
/3)*GAMMA(2/3,I*b*(d*x+c)/d)/d^2/exp(I*(a-b*c/d))/(d*x+c)^(1/3)+9/4*b*sin(b
*x+a)/d^2/(d*x+c)^(1/3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3378, 3388, 2212}

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}} + \frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} - \frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} - \frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}$$

```
[In] Int[Cos[a + b*x]/(c + d*x)^(7/3),x]
```

```
[Out] (-3*Cos[a + b*x])/(4*d*(c + d*x)^(4/3)) + (((9*I)/8)*b*E^(I*(a - (b*c)/d))*
(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(d^2*(c + d*
x)^(1/3)) - (((9*I)/8)*b*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c + d*x
))/d])/(d^2*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3)) + (9*b*Sin[a + b*x])/(4*d^
2*(c + d*x)^(1/3))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} - \frac{(3b)\int \frac{\sin(a+bx)}{(c+dx)^{4/3}}\,dx}{4d} \\
&= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}} - \frac{(9b^2)\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx}{4d^2} \\
&= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}} - \frac{(9b^2)\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{8d^2} - \frac{(9b^2)\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}\,dx}{8d^2} \\
&= -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} \\
&\quad - \frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} + \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = \frac{ibe^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\Gamma\left(-\frac{4}{3},-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\Gamma\left(-\frac{4}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d^2\sqrt[3]{c+dx}}$$

```
[In] Integrate[Cos[a + b*x]/(c + d*x)^(7/3),x]
```

```
[Out] ((I/2)*b*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[-4/3, ((-I)*b*(c +
 d*x))/d] - E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[-4/3, (I*b*(c
 + d*x))/d]))/(d^2*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

Maple [F]

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{7}{3}}}dx$$

```
[In] int(cos(b*x+a)/(d*x+c)^(7/3),x)
```

```
[Out] int(cos(b*x+a)/(d*x+c)^(7/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.42

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx =$$

$$-\frac{3\left(3\left(\left(i\,bd^2x^2+2i\,bcdx+i\,bc^2\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(bd^2x^2+2\,bcdx+bc^2\right)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{i\,b}{d}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},\frac{i\,bdx+i\,bc}{d}\right)+3\right.}{}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/8*(3*((I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)*cos(-(b*c - a*d)/d) + (b*d^2
*x^2 + 2*b*c*d*x + b*c^2)*sin(-(b*c - a*d)/d))*(I*b/d)^(1/3)*gamma(2/3, (I*
b*d*x + I*b*c)/d) + 3*((-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)*cos(-(b*c - a
*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(-(b*c - a*d)/d))*(-I*b/d)^(1/3
)*gamma(2/3, (-I*b*d*x - I*b*c)/d) + 2*(d*x + c)^(2/3)*(d*cos(b*x + a) - 3*
(b*d*x + b*c)*sin(b*x + a)))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = \int \frac{\cos\left(a+bx\right)}{(c+dx)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)**(7/3),x)
[Out] Integral(cos(a + b*x)/(c + d*x)**(7/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.75

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx =$$

$$-\frac{\left(\left((i\sqrt{3}-1)\Gamma\left(-\frac{4}{3},\frac{i\,(dx+c)b}{d}\right)+(-i\sqrt{3}-1)\Gamma\left(-\frac{4}{3},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((\sqrt{3}+i)\Gamma\left(-\frac{4}{3},\frac{i\,(dx+c)b}{d}\right.\right.}{4\,(dx+c)^{\frac{4}{3}}d}$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="maxima")
[Out] -1/4*(((I*sqrt(3) - 1)*gamma(-4/3, I*(d*x + c)*b/d) + (-I*sqrt(3) - 1)*gamm
a(-4/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((sqrt(3) + I)*gamma(-4/3,
 I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(-4/3, -I*(d*x + c)*b/d))*sin(-(b*c
- a*d)/d))*((d*x + c)*b/d)^(4/3)/((d*x + c)^(4/3)*d)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = \int \frac{\cos\left(bx+a\right)}{(dx+c)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/(d*x + c)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^{7/3}}\,dx$$

```
[In] int(cos(a + b*x)/(c + d*x)^(7/3),x)
[Out] int(cos(a + b*x)/(c + d*x)^(7/3), x)
```

3.75 $\int x\sqrt{\cos(a+bx)}\,dx$

Optimal result . . . 593
Rubi [N/A] . . . 593
Mathematica [N/A] . . . 594
Maple [N/A] (verified) . . . 594
Fricas [F(-2)] . . . 594
Sympy [N/A] . . . 594
Maxima [N/A] . . . 595
Giac [N/A] . . . 595
Mupad [N/A] . . . 595

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x\sqrt{\cos(a+bx)}\,dx = \text{Int}\left(x\sqrt{\cos(a+bx)}, x\right)$$

```
[Out] Unintegrable(x*cos(b*x+a)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos(a+bx)}\,dx$$

```
[In] Int[x*Sqrt[Cos[a + b*x]],x]
```

```
[Out] Defer[Int][x*Sqrt[Cos[a + b*x]], x]
```

Rubi steps

$$\text{integral} = \int x\sqrt{\cos(a+bx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 26.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos(a+bx)}\,dx$$

```
[In] Integrate[x*Sqrt[Cos[a + b*x]],x]
[Out] Integrate[x*Sqrt[Cos[a + b*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x\left(\sqrt{\cos}(bx+a)\right)\,dx$$

```
[In] int(x*cos(b*x+a)^(1/2),x)
[Out] int(x*cos(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\cos(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cos(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 6.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos\left(a+bx\right)}\,dx$$

```
[In] integrate(x*cos(b*x+a)**(1/2),x)
[Out] Integral(x*sqrt(cos(a + b*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos\left(bx+a\right)}\,dx$$

```
[In] integrate(x*cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(cos(b*x + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos\left(bx+a\right)}\,dx$$

```
[In] integrate(x*cos(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*sqrt(cos(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 13.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\,\sqrt{\cos\left(a+b\,x\right)}\,dx$$

```
[In] int(x*cos(a + b*x)^(1/2),x)
[Out] int(x*cos(a + b*x)^(1/2), x)
```

3.76 $\int \sqrt{\cos(a+bx)}\,dx$

Optimal result . . . 596
Rubi [A] (verified) . . . 596
Mathematica [A] (verified) . . . 597
Maple [B] (verified) . . . 597
Fricas [C] (verification not implemented) . . . 597
Sympy [F] . . . 598
Maxima [F] . . . 598
Giac [F] . . . 598
Mupad [B] (verification not implemented) . . . 598

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2719}

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[In] Int[Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{b}$$

```
[In] Integrate[Sqrt[Cos[a + b*x]],x]
[Out] (2*EllipticE[(a + b*x)/2, 2])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(42) = 84.

Time = 1.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 8.31

method	result
default	$\frac{2\sqrt{\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,b}$
risch	$-\frac{i\sqrt{2}\sqrt{\left(e^{2i(bx+a)}+1\right)e^{-i(bx+a)}}}{b} - \frac{i\left(-\frac{2\left(e^{2i(bx+a)}+1\right)}{\sqrt{\left(e^{2i(bx+a)}+1\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(bx+a)}-i\right)}\sqrt{ie^{i(bx+a)}}\left(-2iE\left(\sqrt{-i\left(e\right.\right.}\right.}{\sqrt{e^{3i(bx+a)}+e^{i(bx+}}}}{b(e}$

```
[In] int(cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*((-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2
*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a)
,2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+
1/2*a)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \sqrt{\cos(a+bx)}\,dx$$
$$= \frac{i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))) - i\sqrt{2}\text{weierstrassZe}}{b}$$

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) +
 I*sin(b*x + a))) - I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int \sqrt{\cos(a+bx)}\,dx = \int \sqrt{\cos\left(a+bx\right)}\,dx$$

```
[In] integrate(cos(b*x+a)**(1/2),x)
[Out] Integral(sqrt(cos(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\cos(a+bx)}\,dx = \int \sqrt{\cos\left(bx+a\right)}\,dx$$

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cos(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\cos(a+bx)}\,dx = \int \sqrt{\cos\left(bx+a\right)}\,dx$$

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cos(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2\,\mathrm{E}\left(\frac{a}{2}+\frac{bx}{2}\middle|2\right)}{b}$$

```
[In] int(cos(a + b*x)^(1/2),x)
[Out] (2*ellipticE(a/2 + (b*x)/2, 2))/b
```

3.77 $\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx$

Optimal result . . . 599
Rubi [N/A] . . . 599
Mathematica [N/A] . . . 600
Maple [N/A] (verified) . . . 600
Fricas [F(-2)] . . . 600
Sympy [N/A] . . . 600
Maxima [N/A] . . . 601
Giac [N/A] . . . 601
Mupad [N/A] . . . 601

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \text{Int}\left(\frac{\sqrt{\cos(a+bx)}}{x}, x\right)$$

```
[Out] Unintegrable(cos(b*x+a)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \int \frac{\sqrt{\cos(a+bx)}}{x}\,dx$$

```
[In] Int[Sqrt[Cos[a + b*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[Cos[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{\cos(a+bx)}}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \int \frac{\sqrt{\cos(a+bx)}}{x}\,dx$$

```
[In] Integrate[Sqrt[Cos[a + b*x]]/x,x]
[Out] Integrate[Sqrt[Cos[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\cos}(bx+a)}{x}dx$$

```
[In] int(cos(b*x+a)^(1/2)/x,x)
[Out] int(cos(b*x+a)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cos(b*x+a)^(1/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \int \frac{\sqrt{\cos\left(a+bx\right)}}{x}\,dx$$

```
[In] integrate(cos(b*x+a)**(1/2)/x,x)
[Out] Integral(sqrt(cos(a + b*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \int \frac{\sqrt{\cos\left(bx+a\right)}}{x}\,dx$$

```
[In] integrate(cos(b*x+a)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(cos(b*x + a))/x, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \int \frac{\sqrt{\cos\left(bx+a\right)}}{x}\,dx$$

```
[In] integrate(cos(b*x+a)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(cos(b*x + a))/x, x)
```

Mupad [N/A]

Not integrable

Time = 13.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \int \frac{\sqrt{\cos\left(a+b\,x\right)}}{x}\,dx$$

```
[In] int(cos(a + b*x)^(1/2)/x,x)
[Out] int(cos(a + b*x)^(1/2)/x, x)
```

3.78 $\int x \cos^{\frac{3}{2}}(a+bx)\,dx$

Optimal result . . . 602
Rubi [N/A] . . . 602
Mathematica [N/A] . . . 603
Maple [N/A] (verified) . . . 603
Fricas [F(-2)] . . . 603
Sympy [N/A] . . . 603
Maxima [N/A] . . . 604
Giac [N/A] . . . 604
Mupad [N/A] . . . 604

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2} + \frac{2x\sqrt{\cos(a+bx)}\sin(a+bx)}{3b} + \frac{1}{3}\text{Int}\left(\frac{x}{\sqrt{\cos(a+bx)}}, x\right)$$

```
[Out] 4/9*cos(b*x+a)^(3/2)/b^2+2/3*x*sin(b*x+a)*cos(b*x+a)^(1/2)/b+1/3*Unintegrable(x/cos(b*x+a)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \int x \cos^{\frac{3}{2}}(a+bx)\,dx$$

```
[In] Int[x*Cos[a + b*x]^(3/2),x]
```

```
[Out] (4*Cos[a + b*x]^(3/2))/(9*b^2) + (2*x*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b) + Defer[Int][x/Sqrt[Cos[a + b*x]], x]/3
```

Rubi steps

$$\text{integral} = \frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2} + \frac{2x\sqrt{\cos(a+bx)}\sin(a+bx)}{3b} + \frac{1}{3}\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \int x \cos^{\frac{3}{2}}(a+bx)\,dx$$

```
[In] Integrate[x*Cos[a + b*x]^(3/2),x]
[Out] Integrate[x*Cos[a + b*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x\left(\cos^{\frac{3}{2}}(bx+a)\right)\,dx$$

```
[In] int(x*cos(b*x+a)^(3/2),x)
[Out] int(x*cos(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cos(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 87.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \int x \cos^{\frac{3}{2}}(a+bx)\,dx$$

```
[In] integrate(x*cos(b*x+a)**(3/2),x)
[Out] Integral(x*cos(a + b*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \int x \cos\left(bx+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*cos(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \int x \cos\left(bx+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*cos(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 13.73 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \int x \cos\left(a+b\,x\right)^{3/2}\,dx$$

```
[In] int(x*cos(a + b*x)^(3/2),x)
[Out] int(x*cos(a + b*x)^(3/2), x)
```

3.79 $\int \cos^{\frac{3}{2}}(a+bx)\,dx$

Optimal result . . . 605
Rubi [A] (verified) . . . 605
Mathematica [A] (verified) . . . 606
Maple [B] (verified) . . . 606
Fricas [C] (verification not implemented) . . . 607
Sympy [F] . . . 607
Maxima [F] . . . 607
Giac [F] . . . 607
Mupad [B] (verification not implemented) . . . 608

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b} + \frac{2\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}$$

```
[Out] 2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2
*b*x),2^(1/2))/b+2/3*sin(b*x+a)*cos(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2715, 2720}

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b} + \frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

```
[In] Int[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(
3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{\cos(a+bx)}\sin(a+bx)}{3b} + \frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx \\ &= \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b} + \frac{2\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\left(\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) + \sqrt{\cos(a+bx)}\sin(a+bx)\right)}{3b}$$

```
[In] Integrate[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[a + b*x]))/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(62) = 124.

Time = 2.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 4.26

method	result
default	$-\frac{2\sqrt{\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\right.\right.}}{3\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,b}$

```
[In] int(cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b*
x+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(si
n(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/
2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)
/sin(1/2*b*x+1/2*a)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx$$

$$= \frac{2\sqrt{\cos(bx+a)}\sin(bx+a) - i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) + i\sqrt{2}\text{weiers}}{3\,b}$$

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(cos(b*x + a))*sin(b*x + a) - I*sqrt(2)*weierstrassPInverse(-4,
0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, co
s(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \int \cos^{\frac{3}{2}}(a+bx)\,dx$$

```
[In] integrate(cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{3\,b} + \frac{2\,\sqrt{\cos\left(a+b\,x\right)}\,\sin\left(a+b\,x\right)}{3\,b}$$

```
[In] int(cos(a + b*x)^(3/2),x)
```

```
[Out] (2*ellipticF(a/2 + (b*x)/2, 2))/(3*b) + (2*cos(a + b*x)^(1/2)*sin(a + b*x))
/(3*b)
```

3.80 $\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx$

Optimal result . . . 609
Rubi [N/A] . . . 609
Mathematica [N/A] . . . 610
Maple [N/A] (verified) . . . 610
Fricas [F(-2)] . . . 610
Sympy [N/A] . . . 610
Maxima [N/A] . . . 611
Giac [N/A] . . . 611
Mupad [N/A] . . . 611

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \text{Int}\left(\frac{\cos^{\frac{3}{2}}(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(cos(b*x+a)^(3/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx$$

```
[In] Int[Cos[a + b*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][Cos[a + b*x]^(3/2)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.42 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx$$

```
[In] Integrate[Cos[a + b*x]^(3/2)/x,x]
[Out] Integrate[Cos[a + b*x]^(3/2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cos^{\frac{3}{2}}(bx+a)}{x}dx$$

```
[In] int(cos(b*x+a)^(3/2)/x,x)
[Out] int(cos(b*x+a)^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cos(b*x+a)^(3/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 37.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx$$

```
[In] integrate(cos(b*x+a)**(3/2)/x,x)
[Out] Integral(cos(a + b*x)**(3/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(cos(b*x+a)^(3/2)/x,x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^(3/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(cos(b*x+a)^(3/2)/x,x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(3/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 13.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos\left(a+b\,x\right)^{3/2}}{x}\,dx$$

```
[In] int(cos(a + b*x)^(3/2)/x,x)
[Out] int(cos(a + b*x)^(3/2)/x, x)
```

3.81 $\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}} + x\cos^{\frac{3}{2}}(a+bx)\right)\,dx$

Optimal result . . . 612
Rubi [A] (verified) . . . 612
Mathematica [A] (verified) . . . 613
Maple [F] . . . 613
Fricas [F(-2)] . . . 613
Sympy [F] . . . 614
Maxima [F] . . . 614
Giac [F] . . . 614
Mupad [F(-1)] . . . 614

Optimal result

Integrand size = 28, antiderivative size = 42

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}} + x\cos^{\frac{3}{2}}(a+bx)\right)\,dx = \frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2} + \frac{2x\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}$$

```
[Out] 4/9*cos(b*x+a)^(3/2)/b^2+2/3*x*sin(b*x+a)*cos(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {3391}

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}} + x\cos^{\frac{3}{2}}(a+bx)\right)\,dx = \frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2} + \frac{2x\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

```
[In] Int[-1/3*x/Sqrt[Cos[a + b*x]] + x*Cos[a + b*x]^(3/2),x]
```

```
[Out] (4*Cos[a + b*x]^(3/2))/(9*b^2) + (2*x*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b
)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx\right) + \int x\cos^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2} + \frac{2x\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}} + x\cos^{\frac{3}{2}}(a+bx)\right)\,dx = \frac{\sqrt{\cos(a+bx)}\left(\frac{8\cos(a+bx)}{3b} + 4x\sin(a+bx)\right)}{6b}$$

```
[In] Integrate[-1/3*x/Sqrt[Cos[a + b*x]] + x*Cos[a + b*x]^(3/2),x]
[Out] (Sqrt[Cos[a + b*x]]*((8*Cos[a + b*x])/(3*b) + 4*x*Sin[a + b*x]))/(6*b)
```

Maple [F]

$$\int \left(x\left(\cos^{\frac{3}{2}}(bx+a)\right) - \frac{x}{3\sqrt{\cos(bx+a)}}\right)\,dx$$

```
[In] int(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x)
[Out] int(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}} + x\cos^{\frac{3}{2}}(a+bx)\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\frac{\int \left(-\frac{x}{\sqrt{\cos(a+bx)}}\right)\,dx+\int 3x\cos^{\frac{3}{2}}(a+bx)\,dx}{3}$$

```
[In] integrate(x*cos(b*x+a)**(3/2)-1/3*x/cos(b*x+a)**(1/2),x)
[Out] (Integral(-x/sqrt(cos(a + b*x)), x) + Integral(3*x*cos(a + b*x)**(3/2), x))
/3
```

Maxima [F]

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\int x\cos{(bx+a)}^{\frac{3}{2}}-\frac{x}{3\sqrt{\cos{(bx+a)}}}\,dx$$

```
[In] integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)^(3/2) - 1/3*x/sqrt(cos(b*x + a)), x)
```

Giac [F]

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\int x\cos{(bx+a)}^{\frac{3}{2}}-\frac{x}{3\sqrt{\cos{(bx+a)}}}\,dx$$

```
[In] integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)^(3/2) - 1/3*x/sqrt(cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\int x\cos{(a+b\,x)}^{3/2}-\frac{x}{3\sqrt{\cos{(a+b\,x)}}}\,dx$$

```
[In] int(x*cos(a + b*x)^(3/2) - x/(3*cos(a + b*x)^(1/2)),x)
[Out] int(x*cos(a + b*x)^(3/2) - x/(3*cos(a + b*x)^(1/2)), x)
```

3.82 $\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx$

Optimal result . . . 615
Rubi [N/A] . . . 615
Mathematica [N/A] . . . 616
Maple [N/A] (verified) . . . 616
Fricas [F(-2)] . . . 616
Sympy [N/A] . . . 616
Maxima [N/A] . . . 617
Giac [N/A] . . . 617
Mupad [N/A] . . . 617

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = -\frac{\cos^{\frac{3}{2}}(x)}{2x^2} + \frac{3\sqrt{\cos(x)}\sin(x)}{4x} + \frac{3}{8}\text{Int}\left(\frac{1}{x\sqrt{\cos(x)}}, x\right) - \frac{9}{8}\text{Int}\left(\frac{\cos^{\frac{3}{2}}(x)}{x}, x\right)$$

```
[Out] -1/2*cos(x)^(3/2)/x^2+3/4*sin(x)*cos(x)^(1/2)/x-9/8*Unintegrable(cos(x)^(3/
2)/x,x)+3/8*Unintegrable(1/x/cos(x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx$$

```
[In] Int[Cos[x]^(3/2)/x^3,x]
```

```
[Out] -1/2*Cos[x]^(3/2)/x^2 + (3*Sqrt[Cos[x]]*Sin[x])/(4*x) + (3*Defer[Int][1/(x*
Sqrt[Cos[x]]), x])/8 - (9*Defer[Int][Cos[x]^(3/2)/x, x])/8
```

Rubi steps

$$\text{integral} = -\frac{\cos^{\frac{3}{2}}(x)}{2x^2} + \frac{3\sqrt{\cos(x)}\sin(x)}{4x} + \frac{3}{8}\int \frac{1}{x\sqrt{\cos(x)}}\,dx - \frac{9}{8}\int \frac{\cos^{\frac{3}{2}}(x)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx$$

```
[In] Integrate[Cos[x]^(3/2)/x^3,x]
[Out] Integrate[Cos[x]^(3/2)/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}dx$$

```
[In] int(cos(x)^(3/2)/x^3,x)
[Out] int(cos(x)^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cos(x)^(3/2)/x^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 92.97 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx$$

```
[In] integrate(cos(x)**(3/2)/x**3,x)
[Out] Integral(cos(x)**(3/2)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cos\left(x\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate(cos(x)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(cos(x)^(3/2)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cos\left(x\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate(cos(x)^(3/2)/x^3,x, algorithm="giac")
[Out] integrate(cos(x)^(3/2)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 13.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cos\left(x\right)^{3/2}}{x^3}\,dx$$

```
[In] int(cos(x)^(3/2)/x^3,x)
[Out] int(cos(x)^(3/2)/x^3, x)
```

3.83 $\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$

Optimal result . . . 618
Rubi [N/A] . . . 618
Mathematica [N/A] . . . 619
Maple [N/A] (verified) . . . 619
Fricas [F(-2)] . . . 619
Sympy [N/A] . . . 619
Maxima [N/A] . . . 620
Giac [N/A] . . . 620
Mupad [N/A] . . . 620

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \text{Int}\left(\frac{x}{\sqrt{\cos(a+bx)}}, x\right)$$

```
[Out] Unintegrable(x/cos(b*x+a)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

```
[In] Int[x/Sqrt[Cos[a + b*x]],x]
```

```
[Out] Defer[Int][x/Sqrt[Cos[a + b*x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

```
[In] Integrate[x/Sqrt[Cos[a + b*x]],x]
[Out] Integrate[x/Sqrt[Cos[a + b*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{\cos\left(bx+a\right)}}dx$$

```
[In] int(x/cos(b*x+a)^(1/2),x)
[Out] int(x/cos(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/cos(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos\left(a+bx\right)}}\,dx$$

```
[In] integrate(x/cos(b*x+a)**(1/2),x)
[Out] Integral(x/sqrt(cos(a + b*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos\left(bx+a\right)}}\,dx$$

```
[In] integrate(x/cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(cos(b*x + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos\left(bx+a\right)}}\,dx$$

```
[In] integrate(x/cos(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(cos(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 13.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos\left(a+b\,x\right)}}\,dx$$

```
[In] int(x/cos(a + b*x)^(1/2),x)
[Out] int(x/cos(a + b*x)^(1/2), x)
```

3.84 $\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$

Optimal result . . . 621
Rubi [A] (verified) . . . 621
Mathematica [A] (verified) . . . 622
Maple [C] (verified) . . . 622
Fricas [C] (verification not implemented) . . . 622
Sympy [F] . . . 623
Maxima [F] . . . 623
Giac [F] . . . 623
Mupad [B] (verification not implemented) . . . 623

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b
*x),2^(1/2))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2720}

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

```
[In] Int[1/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

```
[In] Integrate[1/Sqrt[Cos[a + b*x]],x]
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2\,\text{am}^{-1}\left(\frac{bx}{2}+\frac{a}{2}\middle\vert\sqrt{2}\right)}{b}$	18

```
[In] int(1/cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b*InverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$$
$$= \frac{-i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx+}{b}$$

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="fricas")
[Out] (-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + I*s
qrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(a+bx\right)}}\,dx$$

```
[In] integrate(1/cos(b*x+a)**(1/2),x)
[Out] Integral(1/sqrt(cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(bx+a\right)}}\,dx$$

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(bx+a\right)}}\,dx$$

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 13.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{bx}{2}\middle|2\right)}{b}$$

```
[In] int(1/cos(a + b*x)^(1/2),x)
[Out] (2*ellipticF(a/2 + (b*x)/2, 2))/b
```

3.85 $\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$

Optimal result . . . 624
Rubi [N/A] . . . 624
Mathematica [N/A] . . . 625
Maple [N/A] (verified) . . . 625
Fricas [F(-2)] . . . 625
Sympy [N/A] . . . 625
Maxima [N/A] . . . 626
Giac [N/A] . . . 626
Mupad [N/A] . . . 626

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{\cos(a+bx)}}, x\right)$$

```
[Out] Unintegrable(1/x/cos(b*x+a)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

```
[In] Int[1/(x*Sqrt[Cos[a + b*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[Cos[a + b*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

```
[In] Integrate[1/(x*Sqrt[Cos[a + b*x]]),x]
[Out] Integrate[1/(x*Sqrt[Cos[a + b*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{\cos\left(bx+a\right)}}dx$$

```
[In] int(1/x/cos(b*x+a)^(1/2),x)
[Out] int(1/x/cos(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 10.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos\left(a+bx\right)}}\,dx$$

```
[In] integrate(1/x/cos(b*x+a)**(1/2),x)
[Out] Integral(1/(x*sqrt(cos(a + b*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos\left(bx+a\right)}}\,dx$$

```
[In] integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(cos(b*x + a))), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos\left(bx+a\right)}}\,dx$$

```
[In] integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(x*sqrt(cos(b*x + a))), x)
```

Mupad [N/A]

Not integrable

Time = 13.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\,\sqrt{\cos\left(a+b\,x\right)}}\,dx$$

```
[In] int(1/(x*cos(a + b*x)^(1/2)),x)
[Out] int(1/(x*cos(a + b*x)^(1/2)), x)
```

3.86 $\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result . . . 627
Rubi [N/A] . . . 627
Mathematica [N/A] . . . 628
Maple [N/A] (verified) . . . 628
Fricas [F(-2)] . . . 628
Sympy [N/A] . . . 628
Maxima [N/A] . . . 629
Giac [N/A] . . . 629
Mupad [N/A] . . . 629

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{4\sqrt{\cos(a+bx)}}{b^2} + \frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \text{Int}\left(x\sqrt{\cos(a+bx)}, x\right)$$

```
[Out] 2*x*sin(b*x+a)/b/cos(b*x+a)^(1/2)+4*cos(b*x+a)^(1/2)/b^2-Unintegrable(x*cos
(b*x+a)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] Int[x/Cos[a + b*x]^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]])/b^2 + (2*x*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]]) - De
fer[Int][x*Sqrt[Cos[a + b*x]], x]
```

Rubi steps

$$\text{integral} = \frac{4\sqrt{\cos(a+bx)}}{b^2} + \frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \int x\sqrt{\cos(a+bx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] Integrate[x/Cos[a + b*x]^(3/2),x]
[Out] Integrate[x/Cos[a + b*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{\cos\left(bx+a\right)^{\frac{3}{2}}}dx$$

```
[In] int(x/cos(b*x+a)^(3/2),x)
[Out] int(x/cos(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/cos(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 14.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x}{\cos^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

```
[In] integrate(x/cos(b*x+a)**(3/2),x)
[Out] Integral(x/cos(a + b*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x}{\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/cos(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x/cos(b*x + a)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x}{\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/cos(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x/cos(b*x + a)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 13.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x}{\cos\left(a+b\,x\right)^{3/2}}\,dx$$

```
[In] int(x/cos(a + b*x)^(3/2),x)
[Out] int(x/cos(a + b*x)^(3/2), x)
```

3.87 $\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result . . . 630
Rubi [A] (verified) . . . 630
Mathematica [A] (verified) . . . 631
Maple [B] (verified) . . . 631
Fricas [C] (verification not implemented) . . . 632
Sympy [F] . . . 632
Maxima [F] . . . 632
Giac [F] . . . 633
Mupad [B] (verification not implemented) . . . 633

Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b} + \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] -2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*
b*x),2^(1/2))/b+2*sin(b*x+a)/b/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2716, 2719}

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[In] Int[Cos[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \int \sqrt{\cos(a+bx)}\,dx \\ &= -\frac{2E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{b} + \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{b} + \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[Cos[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(62) = 124.

Time = 1.51 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.79

method	result
default	$-\frac{2\left(-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\right.\right.\right.}}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,b}$

```
[In] int(1/cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-2*cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1
/2)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)
^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(
cos(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)
^(1/2)/sin(1/2*b*x+1/2*a)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.45

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{-i\sqrt{2}\cos(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+i\sqrt{2}}$$

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*cos(b*x + a)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*sqrt(co
s(b*x + a))*sin(b*x + a))/(b*cos(b*x + a))
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] integrate(1/cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sin(a+b\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(a+b\,x)^2\right)}{b\sqrt{\cos(a+b\,x)}\sqrt{\sin(a+b\,x)^2}}$$

```
[In] int(1/cos(a + b*x)^(3/2),x)
[Out] (2*sin(a + b*x)*hypergeom([-1/4, 1/2], 3/4, cos(a + b*x)^2))/(b*cos(a + b*x
)^(1/2)*(sin(a + b*x)^2)^(1/2))
```

3.88 $\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result . . . 634
Rubi [N/A] . . . 634
Mathematica [N/A] . . . 635
Maple [N/A] (verified) . . . 635
Fricas [F(-2)] . . . 635
Sympy [N/A] . . . 635
Maxima [N/A] . . . 636
Giac [N/A] . . . 636
Mupad [N/A] . . . 636

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \text{Int}\left(\frac{1}{x \cos^{\frac{3}{2}}(a+bx)}, x\right)$$

```
[Out] Unintegrable(1/x/cos(b*x+a)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] Int[1/(x*Cos[a + b*x]^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*Cos[a + b*x]^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] Integrate[1/(x*Cos[a + b*x]^(3/2)),x]
[Out] Integrate[1/(x*Cos[a + b*x]^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x \cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] int(1/x/cos(b*x+a)^(3/2),x)
[Out] int(1/x/cos(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 36.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x \cos^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

```
[In] integrate(1/x/cos(b*x+a)**(3/2),x)
[Out] Integral(1/(x*cos(a + b*x)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x \cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/(x*cos(b*x + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x \cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/(x*cos(b*x + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 13.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x \cos\left(a+b\,x\right)^{3/2}}\,dx$$

```
[In] int(1/(x*cos(a + b*x)^(3/2)),x)
[Out] int(1/(x*cos(a + b*x)^(3/2)), x)
```

3.89 $\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)} \right) dx$

Optimal result . . . 637
Rubi [A] (verified) . . . 637
Mathematica [A] (verified) . . . 638
Maple [F] . . . 638
Fricas [F(-2)] . . . 638
Sympy [F] . . . 639
Maxima [F] . . . 639
Giac [F] . . . 639
Mupad [B] (verification not implemented) . . . 639

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)} \right) dx = \frac{4\sqrt{\cos(a+bx)}}{b^2} + \frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] 2*x*sin(b*x+a)/b/cos(b*x+a)^(1/2)+4*cos(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3396}

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)} \right) dx = \frac{4\sqrt{\cos(a+bx)}}{b^2} + \frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

```
[In] Int[x/Cos[a + b*x]^(3/2) + x*Sqrt[Cos[a + b*x]],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]])/b^2 + (2*x*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx + \int x\sqrt{\cos(a+bx)}\,dx$$

$$= \frac{4\sqrt{\cos(a+bx)}}{b^2} + \frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)}\right)\,dx = \frac{2(2\cos(a+bx) + bx\sin(a+bx))}{b^2\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[x/Cos[a + b*x]^(3/2) + x*Sqrt[Cos[a + b*x]],x]
[Out] (2*(2*Cos[a + b*x] + b*x*Sin[a + b*x]))/(b^2*Sqrt[Cos[a + b*x]])
```

Maple [F]

$$\int \left(\frac{x}{\cos\left(bx+a\right)^{\frac{3}{2}}} + x\left(\sqrt{\cos}(bx+a)\right)\right)\,dx$$

```
[In] int(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x)
[Out] int(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)} \right) dx = \int \frac{x(\cos^2(a+bx)+1)}{\cos^{\frac{3}{2}}(a+bx)} dx$$

```
[In] integrate(x/cos(b*x+a)**(3/2)+x*cos(b*x+a)**(1/2),x)
[Out] Integral(x*(cos(a + b*x)**2 + 1)/cos(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)} \right) dx = \int x\sqrt{\cos(bx+a)} + \frac{x}{\cos(bx+a)^{\frac{3}{2}}} dx$$

```
[In] integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(cos(b*x + a)) + x/cos(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)} \right) dx = \int x\sqrt{\cos(bx+a)} + \frac{x}{\cos(bx+a)^{\frac{3}{2}}} dx$$

```
[In] integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*sqrt(cos(b*x + a)) + x/cos(b*x + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)} + x\sqrt{\cos(a+bx)} \right) dx$$
$$= \frac{2\sqrt{\cos(a+bx)}\,(2\cos(2a+2bx) + bx\sin(2a+2bx) + 2)}{b^2\,(\cos(2a+2bx)+1)}$$

```
[In] int(x*cos(a + b*x)^(1/2) + x/cos(a + b*x)^(3/2),x)
[Out] (2*cos(a + b*x)^(1/2)*(2*cos(2*a + 2*b*x) + b*x*sin(2*a + 2*b*x) + 2))/(b^2
*(cos(2*a + 2*b*x) + 1))
```

3.90 $\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)} \right) dx$

Optimal result . . . 640
Rubi [A] (verified) . . . 640
Mathematica [A] (verified) . . . 641
Maple [F] . . . 641
Fricas [F(-2)] . . . 641
Sympy [F] . . . 642
Maxima [F] . . . 642
Giac [F] . . . 642
Mupad [B] (verification not implemented) . . . 642

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)} \right) dx = 4\sqrt{\cos(x)} + \frac{2x\sin(x)}{\sqrt{\cos(x)}}$$

```
[Out] 2*x*sin(x)/cos(x)^(1/2)+4*cos(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3396}

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)} \right) dx = 4\sqrt{\cos(x)} + \frac{2x\sin(x)}{\sqrt{\cos(x)}}$$

```
[In] Int[x/Cos[x]^(3/2) + x*Sqrt[Cos[x]],x]
```

```
[Out] 4*Sqrt[Cos[x]] + (2*x*Sin[x])/Sqrt[Cos[x]]
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\cos^{\frac{3}{2}}(x)}\,dx + \int x\sqrt{\cos(x)}\,dx$$

$$= 4\sqrt{\cos(x)} + \frac{2x\sin(x)}{\sqrt{\cos(x)}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)}\right)\,dx = \frac{2(2\cos(x) + x\sin(x))}{\sqrt{\cos(x)}}$$

```
[In] Integrate[x/Cos[x]^(3/2) + x*Sqrt[Cos[x]],x]
[Out] (2*(2*Cos[x] + x*Sin[x]))/Sqrt[Cos[x]]
```

Maple [F]

$$\int \left(\frac{x}{\cos(x)^{\frac{3}{2}}} + x\left(\sqrt{\cos}(x)\right)\right)\,dx$$

```
[In] int(x/cos(x)^(3/2)+x*cos(x)^(1/2),x)
[Out] int(x/cos(x)^(3/2)+x*cos(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)} \right) dx = \int \frac{x(\cos^2(x)+1)}{\cos^{\frac{3}{2}}(x)} dx$$

```
[In] integrate(x/cos(x)**(3/2)+x*cos(x)**(1/2),x)
[Out] Integral(x*(cos(x)**2 + 1)/cos(x)**(3/2), x)
```

Maxima [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)} \right) dx = \int x\sqrt{\cos(x)} + \frac{x}{\cos(x)^{\frac{3}{2}}} dx$$

```
[In] integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(cos(x)) + x/cos(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)} \right) dx = \int x\sqrt{\cos(x)} + \frac{x}{\cos(x)^{\frac{3}{2}}} dx$$

```
[In] integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="giac")
[Out] integrate(x*sqrt(cos(x)) + x/cos(x)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)} \right) dx = \frac{4\cos(x) + 2x\sin(x)}{\sqrt{\cos(x)}}$$

```
[In] int(x*cos(x)^(1/2) + x/cos(x)^(3/2),x)
[Out] (4*cos(x) + 2*x*sin(x))/cos(x)^(1/2)
```

3.91 $\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}} \right) dx$

Optimal result . . . 643
Rubi [A] (verified) . . . 643
Mathematica [A] (verified) . . . 644
Maple [F] . . . 644
Fricas [A] (verification not implemented) . . . 644
Sympy [F] . . . 645
Maxima [F] . . . 645
Giac [F] . . . 645
Mupad [B] (verification not implemented) . . . 645

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}} \right) dx = -\frac{4}{3\sqrt{\cos(x)}} + \frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}$$

```
[Out] 2/3*x*sin(x)/cos(x)^(3/2)-4/3/cos(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3396}

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}} \right) dx = \frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)} - \frac{4}{3\sqrt{\cos(x)}}$$

```
[In] Int[x/Cos[x]^(5/2) - x/(3*Sqrt[Cos[x]]),x]
```

```
[Out] -4/(3*Sqrt[Cos[x]]) + (2*x*Sin[x])/(3*Cos[x]^(3/2))
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\int \frac{x}{\sqrt{\cos(x)}}\,dx\right) + \int \frac{x}{\cos^{\frac{5}{2}}(x)}\,dx \\ &= -\frac{4}{3\sqrt{\cos(x)}} + \frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}}\right)\,dx = -\frac{8-4x\tan(x)}{6\sqrt{\cos(x)}}$$

```
[In] Integrate[x/Cos[x]^(5/2) - x/(3*Sqrt[Cos[x]]),x]
[Out] -1/6*(8 - 4*x*Tan[x])/Sqrt[Cos[x]]
```

Maple [F]

$$\int \left(\frac{x}{\cos(x)^{\frac{5}{2}}} - \frac{x}{3\sqrt{\cos(x)}}\right)\,dx$$

```
[In] int(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x)
[Out] int(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}}\right)\,dx = \frac{2\left(x\sin(x) - 2\cos(x)\right)}{3\cos(x)^{\frac{3}{2}}}$$

```
[In] integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x*sin(x) - 2*cos(x))/cos(x)^(3/2)
```

Sympy [F]

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}} \right) dx = -\frac{\int \left(-\frac{3x}{\cos^{\frac{5}{2}}(x)} \right) dx + \int \frac{x}{\sqrt{\cos(x)}} dx}{3}$$

```
[In] integrate(x/cos(x)**(5/2)-1/3*x/cos(x)**(1/2),x)
[Out] -(Integral(-3*x/cos(x)**(5/2), x) + Integral(x/sqrt(cos(x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}} \right) dx = \int -\frac{x}{3\sqrt{\cos(x)}} + \frac{x}{\cos(x)^{\frac{5}{2}}} dx$$

```
[In] integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-1/3*x/sqrt(cos(x)) + x/cos(x)^(5/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}} \right) dx = \int -\frac{x}{3\sqrt{\cos(x)}} + \frac{x}{\cos(x)^{\frac{5}{2}}} dx$$

```
[In] integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="giac")
[Out] integrate(-1/3*x/sqrt(cos(x)) + x/cos(x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}} \right) dx = -\frac{4\cos(x) - 2x\sin(x)}{3\cos(x)^{3/2}}$$

```
[In] int(x/cos(x)^(5/2) - x/(3*cos(x)^(1/2)),x)
[Out] -(4*cos(x) - 2*x*sin(x))/(3*cos(x)^(3/2))
```

3.92 $\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5} x \sqrt{\cos(x)} \right) dx$

Optimal result . . . 646
Rubi [A] (verified) . . . 646
Mathematica [A] (verified) . . . 647
Maple [F] . . . 647
Fricas [F(-2)] . . . 647
Sympy [F(-1)] . . . 648
Maxima [F] . . . 648
Giac [F] . . . 648
Mupad [B] (verification not implemented) . . . 648

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5} x \sqrt{\cos(x)} \right) dx = -\frac{4}{15 \cos^{\frac{3}{2}}(x)} + \frac{12 \sqrt{\cos(x)}}{5} + \frac{2x \sin(x)}{5 \cos^{\frac{5}{2}}(x)} + \frac{6x \sin(x)}{5 \sqrt{\cos(x)}}$$

```
[Out] -4/15/cos(x)^(3/2)+2/5*x*sin(x)/cos(x)^(5/2)+6/5*x*sin(x)/cos(x)^(1/2)+12/5
*cos(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3396}

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5} x \sqrt{\cos(x)} \right) dx = -\frac{4}{15 \cos^{\frac{3}{2}}(x)} + \frac{12 \sqrt{\cos(x)}}{5} + \frac{2x \sin(x)}{5 \cos^{\frac{5}{2}}(x)} + \frac{6x \sin(x)}{5 \sqrt{\cos(x)}}$$

```
[In] Int[x/Cos[x]^(7/2) + (3*x*Sqrt[Cos[x]])/5,x]
```

```
[Out] -4/(15*Cos[x]^(3/2)) + (12*Sqrt[Cos[x]])/5 + (2*x*Sin[x])/(5*Cos[x]^(5/2))
+ (6*x*Sin[x])/(5*Sqrt[Cos[x]])
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
```

```
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3}{5}\int x\sqrt{\cos(x)}\,dx + \int \frac{x}{\cos^{\frac{7}{2}}(x)}\,dx \\ &= -\frac{4}{15\cos^{\frac{3}{2}}(x)} + \frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)} + \frac{3}{5}\int \frac{x}{\cos^{\frac{3}{2}}(x)}\,dx + \frac{3}{5}\int x\sqrt{\cos(x)}\,dx \\ &= -\frac{4}{15\cos^{\frac{3}{2}}(x)} + \frac{12\sqrt{\cos(x)}}{5} + \frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)} + \frac{6x\sin(x)}{5\sqrt{\cos(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\cos(x)}\right)\,dx = \frac{46\cos(x) + 18\cos(3x) + 21x\sin(x) + 9x\sin(3x)}{30\cos^{\frac{5}{2}}(x)}$$

```
[In] Integrate[x/Cos[x]^(7/2) + (3*x*Sqrt[Cos[x]])/5,x]
[Out] (46*Cos[x] + 18*Cos[3*x] + 21*x*Sin[x] + 9*x*Sin[3*x])/(30*Cos[x]^(5/2))
```

Maple [F]

$$\int \left(\frac{x}{\cos(x)^{\frac{7}{2}}} + \frac{3x\left(\sqrt{\cos}(x)\right)}{5}\right)\,dx$$

```
[In] int(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x)
[Out] int(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\cos(x)}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5} x \sqrt{\cos(x)} \right) dx = \text{Timed out}$$

```
[In] integrate(x/cos(x)**(7/2)+3/5*x*cos(x)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5} x \sqrt{\cos(x)} \right) dx = \int \frac{3}{5} x \sqrt{\cos(x)} + \frac{x}{\cos(x)^{\frac{7}{2}}} dx$$

```
[In] integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="maxima")
[Out] integrate(3/5*x*sqrt(cos(x)) + x/cos(x)^(7/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5} x \sqrt{\cos(x)} \right) dx = \int \frac{3}{5} x \sqrt{\cos(x)} + \frac{x}{\cos(x)^{\frac{7}{2}}} dx$$

```
[In] integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="giac")
[Out] integrate(3/5*x*sqrt(cos(x)) + x/cos(x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)} + \frac{3}{5} x \sqrt{\cos(x)} \right) dx = \frac{36 \cos(x)^3 + 18\, x \sin(x) \cos(x)^2 - 4 \cos(x) + 6\, x \sin(x)}{15 \cos(x)^{5/2}}$$

```
[In] int((3*x*cos(x)^(1/2))/5 + x/cos(x)^(7/2),x)
[Out] (36*cos(x)^3 - 4*cos(x) + 6*x*sin(x) + 18*x*cos(x)^2*sin(x))/(15*cos(x)^(5/
2))
```

3.93 $\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2 \sqrt{\cos(x)} \right) dx$

Optimal result . . . 649
Rubi [A] (verified) . . . 649
Mathematica [A] (verified) . . . 650
Maple [F] . . . 650
Fricas [F(-2)] . . . 651
Sympy [F] . . . 651
Maxima [F] . . . 651
Giac [F] . . . 651
Mupad [F(-1)] . . . 652

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2 \sqrt{\cos(x)} \right) dx = 8x\sqrt{\cos(x)} - 16E\left(\frac{x}{2}\middle| 2\right) + \frac{2x^2 \sin(x)}{\sqrt{\cos(x)}}$$

```
[Out] -16*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),2^(1/2))+2*x^2*sin
(x)/cos(x)^(1/2)+8*x*cos(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3397, 2719}

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2 \sqrt{\cos(x)} \right) dx = \frac{2x^2 \sin(x)}{\sqrt{\cos(x)}} + 8x\sqrt{\cos(x)} - 16E\left(\frac{x}{2}\middle| 2\right)$$

```
[In] Int[x^2/Cos[x]^(3/2) + x^2*Sqrt[Cos[x]],x]
```

```
[Out] 8*x*Sqrt[Cos[x]] - 16*EllipticE[x/2, 2] + (2*x^2*Sin[x])/Sqrt[Cos[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3397

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1)
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[d^2*m*((m - 1)/(b^2*f^2*(n + 1)*(n + 2))), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[d*m*(c + d*x)^(m - 1)*((b*
Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2}{\cos^{\frac{3}{2}}(x)}\,dx + \int x^2\sqrt{\cos(x)}\,dx \\ &= 8x\sqrt{\cos(x)} + \frac{2x^2\sin(x)}{\sqrt{\cos(x)}} - 8\int \sqrt{\cos(x)}\,dx \\ &= 8x\sqrt{\cos(x)} - 16E\left(\frac{x}{2}\middle|\,2\right) + \frac{2x^2\sin(x)}{\sqrt{\cos(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2\sqrt{\cos(x)}\right)\,dx = 2\left(-8E\left(\frac{x}{2}\middle|\,2\right) + \frac{x(4\cos(x) + x\sin(x))}{\sqrt{\cos(x)}}\right)$$

```
[In] Integrate[x^2/Cos[x]^(3/2) + x^2*Sqrt[Cos[x]],x]
```

```
[Out] 2*(-8*EllipticE[x/2, 2] + (x*(4*Cos[x] + x*Sin[x]))/Sqrt[Cos[x]])
```

Maple [F]

$$\int \left(\frac{x^2}{\cos(x)^{\frac{3}{2}}} + x^2\left(\sqrt{\cos}(x)\right)\right)\,dx$$

```
[In] int(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x)
```

```
[Out] int(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2 \sqrt{\cos(x)} \right) dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2 \sqrt{\cos(x)} \right) dx = \int \frac{x^2(\cos^2(x)+1)}{\cos^{\frac{3}{2}}(x)} dx$$

```
[In] integrate(x**2/cos(x)**(3/2)+x**2*cos(x)**(1/2),x)
[Out] Integral(x**2*(cos(x)**2 + 1)/cos(x)**(3/2), x)
```

Maxima [F]

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2 \sqrt{\cos(x)} \right) dx = \int x^2 \sqrt{\cos(x)} + \frac{x^2}{\cos(x)^{\frac{3}{2}}} dx$$

```
[In] integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*sqrt(cos(x)) + x^2/cos(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2 \sqrt{\cos(x)} \right) dx = \int x^2 \sqrt{\cos(x)} + \frac{x^2}{\cos(x)^{\frac{3}{2}}} dx$$

```
[In] integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*sqrt(cos(x)) + x^2/cos(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2 \sqrt{\cos(x)} \right) dx = \int x^2 \sqrt{\cos(x)} + \frac{x^2}{\cos(x)^{3/2}} dx$$

```
[In] int(x^2*cos(x)^(1/2) + x^2/cos(x)^(3/2),x)
[Out] int(x^2*cos(x)^(1/2) + x^2/cos(x)^(3/2), x)
```

3.94 $\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\sec(x)} \right) dx$

Optimal result . . . 653
Rubi [A] (verified) . . . 653
Mathematica [A] (verified) . . . 654
Maple [F] . . . 654
Fricas [F(-2)] . . . 654
Sympy [F] . . . 655
Maxima [F] . . . 655
Giac [F] . . . 655
Mupad [F(-1)] . . . 655

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\sec(x)} \right) dx = \frac{4}{9 \sec^{\frac{3}{2}}(x)} + \frac{2x \sin(x)}{3\sqrt{\sec(x)}}$$

```
[Out] 4/9/sec(x)^(3/2)+2/3*x*sin(x)/sec(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4272, 4274}

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\sec(x)} \right) dx = \frac{4}{9 \sec^{\frac{3}{2}}(x)} + \frac{2x \sin(x)}{3\sqrt{\sec(x)}}$$

```
[In] Int[x/Sec[x]^(3/2) - (x*Sqrt[Sec[x]])/3,x]
```

```
[Out] 4/(9*Sec[x]^(3/2)) + (2*x*Sin[x])/(3*Sqrt[Sec[x]])
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\int x\sqrt{\sec(x)}\,dx\right) + \int \frac{x}{\sec^{\frac{3}{2}}(x)}\,dx \\ &= \frac{4}{9\sec^{\frac{3}{2}}(x)} + \frac{2x\sin(x)}{3\sqrt{\sec(x)}} + \frac{1}{3}\int x\sqrt{\sec(x)}\,dx - \frac{1}{3}\left(\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int \frac{x}{\sqrt{\cos(x)}}\,dx \\ &= \frac{4}{9\sec^{\frac{3}{2}}(x)} + \frac{2x\sin(x)}{3\sqrt{\sec(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x\sqrt{\sec(x)}\right)\,dx = \frac{2(2+3x\tan(x))}{9\sec^{\frac{3}{2}}(x)}$$

```
[In] Integrate[x/Sec[x]^(3/2) - (x*Sqrt[Sec[x]])/3,x]
[Out] (2*(2 + 3*x*Tan[x]))/(9*Sec[x]^(3/2))
```

Maple [F]

$$\int \left(\frac{x}{\sec(x)^{\frac{3}{2}}} - \frac{x\left(\sqrt{\sec}(x)\right)}{3}\right)\,dx$$

```
[In] int(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x)
[Out] int(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x\sqrt{\sec(x)}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\sec(x)} \right) dx = - \frac{\int \left(- \frac{3x}{\sec^{\frac{3}{2}}(x)} \right) dx + \int x \sqrt{\sec(x)}\, dx}{3}$$

```
[In] integrate(x/sec(x)**(3/2)-1/3*x*sec(x)**(1/2),x)
[Out] -(Integral(-3*x/sec(x)**(3/2), x) + Integral(x*sqrt(sec(x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\sec(x)} \right) dx = \int -\frac{1}{3} x \sqrt{\sec(x)} + \frac{x}{\sec(x)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-1/3*x*sqrt(sec(x)) + x/sec(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\sec(x)} \right) dx = \int -\frac{1}{3} x \sqrt{\sec(x)} + \frac{x}{\sec(x)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="giac")
[Out] integrate(-1/3*x*sqrt(sec(x)) + x/sec(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\sec(x)} \right) dx = - \int \frac{x \sqrt{\frac{1}{\cos(x)}}}{3} - \frac{x}{\left(\frac{1}{\cos(x)} \right)^{3/2}}\, dx$$

```
[In] int(x/(1/cos(x))^(3/2) - (x*(1/cos(x))^(1/2))/3,x)
[Out] -int((x*(1/cos(x))^(1/2))/3 - x/(1/cos(x))^(3/2), x)
```

3.95 $\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}} \right) dx$

Optimal result . . . 656
Rubi [A] (verified) . . . 656
Mathematica [A] (verified) . . . 657
Maple [F] . . . 657
Fricas [F(-2)] . . . 657
Sympy [F] . . . 658
Maxima [F] . . . 658
Giac [F] . . . 658
Mupad [F(-1)] . . . 658

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}} \right) dx = \frac{4}{25\sec^{\frac{5}{2}}(x)} + \frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}$$

```
[Out] 4/25/sec(x)^(5/2)+2/5*x*sin(x)/sec(x)^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4272, 4274}

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}} \right) dx = \frac{4}{25\sec^{\frac{5}{2}}(x)} + \frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}$$

```
[In] Int[x/Sec[x]^(5/2) - (3*x)/(5*Sqrt[Sec[x]]),x]
```

```
[Out] 4/(25*Sec[x]^(5/2)) + (2*x*Sin[x])/(5*Sec[x]^(3/2))
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{3}{5}\int \frac{x}{\sqrt{\sec(x)}}\,dx\right)+\int \frac{x}{\sec^{\frac{5}{2}}(x)}\,dx \\ &= \frac{4}{25\sec^{\frac{5}{2}}(x)}+\frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}+\frac{3}{5}\int \frac{x}{\sqrt{\sec(x)}}\,dx-\frac{1}{5}\left(3\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int x\sqrt{\cos(x)}\,dx \\ &= \frac{4}{25\sec^{\frac{5}{2}}(x)}+\frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right)dx = \frac{2(2+5x\tan(x))}{25\sec^{\frac{5}{2}}(x)}$$

```
[In] Integrate[x/Sec[x]^(5/2) - (3*x)/(5*Sqrt[Sec[x]]),x]
[Out] (2*(2 + 5*x*Tan[x]))/(25*Sec[x]^(5/2))
```

Maple [F]

$$\int \left(\frac{x}{\sec(x)^{\frac{5}{2}}}-\frac{3x}{5\sqrt{\sec(x)}}\right)dx$$

```
[In] int(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x)
[Out] int(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right)dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}} \right) dx = -\frac{\int \left(-\frac{5x}{\sec^{\frac{5}{2}}(x)} \right) dx + \int \frac{3x}{\sqrt{\sec(x)}}\, dx}{5}$$

```
[In] integrate(x/sec(x)**(5/2)-3/5*x/sec(x)**(1/2),x)
[Out] -(Integral(-5*x/sec(x)**(5/2), x) + Integral(3*x/sqrt(sec(x)), x))/5
```

Maxima [F]

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}} \right) dx = \int -\frac{3\,x}{5\,\sqrt{\sec(x)}} + \frac{x}{\sec(x)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-3/5*x/sqrt(sec(x)) + x/sec(x)^(5/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}} \right) dx = \int -\frac{3\,x}{5\,\sqrt{\sec(x)}} + \frac{x}{\sec(x)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="giac")
[Out] integrate(-3/5*x/sqrt(sec(x)) + x/sec(x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}} \right) dx = -\int \frac{3\,x}{5\,\sqrt{\frac{1}{\cos(x)}}} - \frac{x}{\left(\frac{1}{\cos(x)}\right)^{5/2}}\, dx$$

```
[In] int(x/(1/cos(x))^(5/2) - (3*x)/(5*(1/cos(x))^(1/2)),x)
[Out] -int((3*x)/(5*(1/cos(x))^(1/2)) - x/(1/cos(x))^(5/2), x)
```

3.96 $\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx$

Optimal result . . . 659
Rubi [A] (verified) . . . 659
Mathematica [A] (verified) . . . 660
Maple [F] . . . 661
Fricas [F(-2)] . . . 661
Sympy [F] . . . 661
Maxima [F] . . . 661
Giac [F] . . . 662
Mupad [F(-1)] . . . 662

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx = \frac{4}{49 \sec^{\frac{7}{2}}(x)} + \frac{20}{63 \sec^{\frac{3}{2}}(x)} + \frac{2x \sin(x)}{7 \sec^{\frac{5}{2}}(x)} + \frac{10x \sin(x)}{21 \sqrt{\sec(x)}}$$

```
[Out] 4/49/sec(x)^(7/2)+20/63/sec(x)^(3/2)+2/7*x*sin(x)/sec(x)^(5/2)+10/21*x*sin(
x)/sec(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4272, 4274}

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx = \frac{20}{63 \sec^{\frac{3}{2}}(x)} + \frac{4}{49 \sec^{\frac{7}{2}}(x)} + \frac{2x \sin(x)}{7 \sec^{\frac{5}{2}}(x)} + \frac{10x \sin(x)}{21 \sqrt{\sec(x)}}$$

```
[In] Int[x/Sec[x]^(7/2) - (5*x*Sqrt[Sec[x]])/21,x]
```

```
[Out] 4/(49*Sec[x]^(7/2)) + 20/(63*Sec[x]^(3/2)) + (2*x*Sin[x])/(7*Sec[x]^(5/2))
+ (10*x*Sin[x])/(21*Sqrt[Sec[x]])
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
```

```
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{5}{21}\int x\sqrt{\sec(x)}\,dx\right) + \int \frac{x}{\sec^{\frac{7}{2}}(x)}\,dx \\
&= \frac{4}{49\sec^{\frac{7}{2}}(x)} + \frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)} + \frac{5}{7}\int \frac{x}{\sec^{\frac{3}{2}}(x)}\,dx - \frac{1}{21}\left(5\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int \frac{x}{\sqrt{\cos(x)}}\,dx \\
&= \frac{4}{49\sec^{\frac{7}{2}}(x)} + \frac{20}{63\sec^{\frac{3}{2}}(x)} + \frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)} + \frac{10x\sin(x)}{21\sqrt{\sec(x)}} \\
&\quad + \frac{5}{21}\int x\sqrt{\sec(x)}\,dx - \frac{1}{21}\left(5\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int \frac{x}{\sqrt{\cos(x)}}\,dx \\
&= \frac{4}{49\sec^{\frac{7}{2}}(x)} + \frac{20}{63\sec^{\frac{3}{2}}(x)} + \frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)} + \frac{10x\sin(x)}{21\sqrt{\sec(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.13$ (sec) , antiderivative size $= 45$, normalized size of antiderivative $= 0.96$

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\sec(x)}\right)\,dx = \sqrt{\sec(x)}\left(\frac{167}{882} + \frac{88}{441}\cos(2x) + \frac{1}{98}\cos(4x) + \frac{13}{42}x\sin(2x) + \frac{1}{28}x\sin(4x)\right)$$

```
[In] Integrate[x/Sec[x]^(7/2) - (5*x*Sqrt[Sec[x]])/21,x]
```

```
[Out] Sqrt[Sec[x]]*(167/882 + (88*Cos[2*x])/441 + Cos[4*x]/98 + (13*x*Sin[2*x])/4
2 + (x*Sin[4*x])/28)
```

Maple [F]

$$\int \left(\frac{x}{\sec(x)^{\frac{7}{2}}} - \frac{5x\left(\sqrt{\sec}(x)\right)}{21} \right) dx$$

```
[In] int(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x)
[Out] int(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx = -\frac{\int \left(-\frac{21x}{\sec^{\frac{7}{2}}(x)} \right) dx + \int 5x\sqrt{\sec(x)}\, dx}{21}$$

```
[In] integrate(x/sec(x)**(7/2)-5/21*x*sec(x)**(1/2),x)
[Out] -(Integral(-21*x/sec(x)**(7/2), x) + Integral(5*x*sqrt(sec(x)), x))/21
```

Maxima [F]

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx = \int -\frac{5}{21}\, x\sqrt{\sec(x)} + \frac{x}{\sec(x)^{\frac{7}{2}}}\, dx$$

```
[In] integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-5/21*x*sqrt(sec(x)) + x/sec(x)^(7/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx = \int -\frac{5}{21} x \sqrt{\sec(x)} + \frac{x}{\sec(x)^{\frac{7}{2}}} dx$$

```
[In] integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="giac")
[Out] integrate(-5/21*x*sqrt(sec(x)) + x/sec(x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx = -\int \frac{5\, x \sqrt{\frac{1}{\cos(x)}}}{21} - \frac{x}{\left(\frac{1}{\cos(x)}\right)^{7/2}} dx$$

```
[In] int(x/(1/cos(x))^(7/2) - (5*x*(1/cos(x))^(1/2))/21,x)
[Out] -int((5*x*(1/cos(x))^(1/2))/21 - x/(1/cos(x))^(7/2), x)
```

3.97 $\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\sec(x)} \right) dx$

Optimal result . . . 663
Rubi [A] (verified) . . . 663
Mathematica [A] (verified) . . . 665
Maple [F] . . . 665
Fricas [F(-2)] . . . 665
Sympy [F] . . . 666
Maxima [F] . . . 666
Giac [F] . . . 666
Mupad [F(-1)] . . . 666

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\sec(x)} \right) dx = \frac{8x}{9\sec^{\frac{3}{2}}(x)} - \frac{16}{27}\sqrt{\cos(x)}\,\text{EllipticF}\left(\frac{x}{2},2\right)\sqrt{\sec(x)} - \frac{16\sin(x)}{27\sqrt{\sec(x)}} + \frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}$$

```
[Out] 8/9*x/sec(x)^(3/2)-16/27*sin(x)/sec(x)^(1/2)+2/3*x^2*sin(x)/sec(x)^(1/2)-16
/27*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(1/2))*cos(x)^(1
/2)*sec(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4273, 4274, 3854, 3856, 2720}

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\sec(x)} \right) dx = \frac{2x^2\sin(x)}{3\sqrt{\sec(x)}} + \frac{8x}{9\sec^{\frac{3}{2}}(x)} - \frac{16\sin(x)}{27\sqrt{\sec(x)}} - \frac{16}{27}\sqrt{\cos(x)}\sqrt{\sec(x)}\,\text{EllipticF}\left(\frac{x}{2},2\right)$$

```
[In] Int[x^2/Sec[x]^(3/2) - (x^2*Sqrt[Sec[x]])/3,x]
```

```
[Out] (8*x)/(9*Sec[x]^(3/2)) - (16*Sqrt[Cos[x]]*EllipticF[x/2, 2]*Sqrt[Sec[x]])/2
7 - (16*Sin[x])/(27*Sqrt[Sec[x]]) + (2*x^2*Sin[x])/(3*Sqrt[Sec[x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4273

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[(c + d*x)^m*Cos[e + f*x]*((b*Csc[e + f*x])^(n + 1)/(b*f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int x^2\sqrt{\sec(x)}\,dx\right) + \int \frac{x^2}{\sec^{\frac{3}{2}}(x)}\,dx \\
&= \frac{8x}{9\sec^{\frac{3}{2}}(x)} + \frac{2x^2\sin(x)}{3\sqrt{\sec(x)}} + \frac{1}{3}\int x^2\sqrt{\sec(x)}\,dx \\
&\quad - \frac{8}{9}\int \frac{1}{\sec^{\frac{3}{2}}(x)}\,dx - \frac{1}{3}\left(\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int \frac{x^2}{\sqrt{\cos(x)}}\,dx \\
&= \frac{8x}{9\sec^{\frac{3}{2}}(x)} - \frac{16\sin(x)}{27\sqrt{\sec(x)}} + \frac{2x^2\sin(x)}{3\sqrt{\sec(x)}} - \frac{8}{27}\int \sqrt{\sec(x)}\,dx
\end{aligned}$$

$$= \frac{8x}{9\sec^{\frac{3}{2}}(x)} - \frac{16\sin(x)}{27\sqrt{\sec(x)}} + \frac{2x^2\sin(x)}{3\sqrt{\sec(x)}} - \frac{1}{27}\left(8\sqrt{\cos(x)}\sqrt{\sec(x)}\right)\int \frac{1}{\sqrt{\cos(x)}}\,dx$$

$$= \frac{8x}{9\sec^{\frac{3}{2}}(x)} - \frac{16}{27}\sqrt{\cos(x)}\,\text{EllipticF}\left(\frac{x}{2}, 2\right)\sqrt{\sec(x)} - \frac{16\sin(x)}{27\sqrt{\sec(x)}} + \frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\sec(x)}\right)dx = \frac{1}{27}\sqrt{\sec(x)}\Big(12x + 12x\cos(2x) - 16\sqrt{\cos(x)}\,\text{EllipticF}\left(\frac{x}{2}, 2\right) - 8\sin(2x) + 9x^2\sin(2x)\Big)$$

```
[In] Integrate[x^2/Sec[x]^(3/2) - (x^2*Sqrt[Sec[x]])/3,x]
```

```
[Out] (Sqrt[Sec[x]]*(12*x + 12*x*Cos[2*x] - 16*Sqrt[Cos[x]]*EllipticF[x/2, 2] - 8
*Sin[2*x] + 9*x^2*Sin[2*x]))/27
```

Maple [F]

$$\int \left(\frac{x^2}{\sec(x)^{\frac{3}{2}}} - \frac{x^2\left(\sqrt{\sec}(x)\right)}{3}\right)dx$$

```
[In] int(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x)
```

```
[Out] int(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\sec(x)}\right)dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x^2 \sqrt{\sec(x)} \right) dx = - \frac{\int \left(- \frac{3x^2}{\sec^{\frac{3}{2}}(x)} \right) dx + \int x^2 \sqrt{\sec(x)}\, dx}{3}$$

```
[In] integrate(x**2/sec(x)**(3/2)-1/3*x**2*sec(x)**(1/2),x)
[Out] -(Integral(-3*x**2/sec(x)**(3/2), x) + Integral(x**2*sqrt(sec(x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x^2 \sqrt{\sec(x)} \right) dx = \int -\frac{1}{3} x^2 \sqrt{\sec(x)} + \frac{x^2}{\sec(x)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-1/3*x^2*sqrt(sec(x)) + x^2/sec(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x^2 \sqrt{\sec(x)} \right) dx = \int -\frac{1}{3} x^2 \sqrt{\sec(x)} + \frac{x^2}{\sec(x)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="giac")
[Out] integrate(-1/3*x^2*sqrt(sec(x)) + x^2/sec(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3} x^2 \sqrt{\sec(x)} \right) dx = - \int \frac{x^2 \sqrt{\frac{1}{\cos(x)}}}{3} - \frac{x^2}{\left(\frac{1}{\cos(x)} \right)^{3/2}}\, dx$$

```
[In] int(x^2/(1/cos(x))^(3/2) - (x^2*(1/cos(x))^(1/2))/3,x)
[Out] -int((x^2*(1/cos(x))^(1/2))/3 - x^2/(1/cos(x))^(3/2), x)
```

3.98 $\int (c+dx)^m (b\cos(e+fx))^n \, dx$

Optimal result 667
Rubi [N/A] 667
Mathematica [N/A] 668
Maple [N/A] (verified) 668
Fricas [N/A] 668
Sympy [N/A] 668
Maxima [N/A] 669
Giac [N/A] 669
Mupad [N/A] 669

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \text{Int}((c+dx)^m (b\cos(e+fx))^n, x)$$

```
[Out] Unintegrable((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \int (c+dx)^m (b\cos(e+fx))^n \, dx$$

```
[In] Int[(c + d*x)^m*(b*Cos[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(b*Cos[e + f*x])^n, x]
```

Rubi steps

$$\text{integral} = \int (c+dx)^m (b\cos(e+fx))^n \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \int (c+dx)^m (b\cos(e+fx))^n \, dx$$

```
[In] Integrate[(c + d*x)^m*(b*Cos[e + f*x])^n,x]
[Out] Integrate[(c + d*x)^m*(b*Cos[e + f*x])^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \left(b\cos\left(fx+e\right)\right)^n \, dx$$

```
[In] int((d*x+c)^m*(b*cos(f*x+e))^n,x)
[Out] int((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \int \left(dx+c\right)^m \left(b\cos\left(fx+e\right)\right)^n \, dx$$

```
[In] integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

Sympy [N/A]

Not integrable

Time = 8.74 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \int \left(b\cos\left(e+fx\right)\right)^n \left(c+dx\right)^m \, dx$$

```
[In] integrate((d*x+c)**m*(b*cos(f*x+e))**n,x)
[Out] Integral((b*cos(e + f*x))**n*(c + d*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \int (dx+c)^m (b\cos(fx+e))^n \, dx$$

```
[In] integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

Giac [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \int (dx+c)^m (b\cos(fx+e))^n \, dx$$

```
[In] integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

Mupad [N/A]

Not integrable

Time = 14.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \int (b\,\cos(e+f\,x))^n \, (c+d\,x)^m \, dx$$

```
[In] int((b*cos(e + f*x))^n*(c + d*x)^m,x)
[Out] int((b*cos(e + f*x))^n*(c + d*x)^m, x)
```

3.99 $\int (c+dx)^m \cos^3(a+bx)\,dx$

Optimal result . . . 670
Rubi [A] (verified) . . . 671
Mathematica [A] (verified) . . . 672
Maple [F] . . . 673
Fricas [A] (verification not implemented) . . . 673
Sympy [F] . . . 673
Maxima [F] . . . 674
Giac [F] . . . 674
Mupad [F(-1)] . . . 674

Optimal result

Integrand size = 16, antiderivative size = 275

$$\begin{aligned}
&\int (c+dx)^m \cos^3(a+bx)\,dx \\
&= -\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad + \frac{3ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad - \frac{i3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{8b} \\
&\quad + \frac{i3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{8b}
\end{aligned}$$

```
[Out] -3/8*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)+3/8*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(
d*x+c)/d)^m)-1/8*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(
d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/8*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(
d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3388, 2212}

$$\begin{aligned}
&\int (c+dx)^m \cos^3(a+bx)\,dx \\
&= -\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad - \frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b} \\
&\quad + \frac{3ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad + \frac{i3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}
\end{aligned}$$

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3,x]
```

```
[Out] (((-3*I)/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))
/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (((3*I)/8)*(c + d*x)^m*Gamma[1 + m, (I*
b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - ((I/8)*3^(
-1 - m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x
))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1
 + m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d
)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}(c+dx)^m \cos(a+bx) + \frac{1}{4}(c+dx)^m \cos(3a+3bx)\right) dx \\
&= \frac{1}{4}\int (c+dx)^m \cos(3a+3bx)\, dx + \frac{3}{4}\int (c+dx)^m \cos(a+bx)\, dx \\
&= \frac{1}{8}\int e^{-i(3a+3bx)}(c+dx)^m\, dx + \frac{1}{8}\int e^{i(3a+3bx)}(c+dx)^m\, dx \\
&\quad + \frac{3}{8}\int e^{-i(a+bx)}(c+dx)^m\, dx + \frac{3}{8}\int e^{i(a+bx)}(c+dx)^m\, dx \\
&= -\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad + \frac{3ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad - \frac{i3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{8b} \\
&\quad + \frac{i3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int (c+dx)^m \cos^3(a+bx)\, dx \\
&= \frac{i3^{-1-m}e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{2+m}e^{2i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m \Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right) + 3^{2+m}e^{2ia+\frac{4ib}{d}}
\end{aligned}$$

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3,x]
```

```
[Out] ((I/8)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((2*I)*(2*a + (b*c)/d))*((I*b*
(c + d*x))/d)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]) + 3^(2 + m)*E^((2*I)*a
+ ((4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, (I*b*(c + d*x))/d] -
 E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E
^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/
d]))/(b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F]

$$\int (dx+c)^m \left(\cos^3(bx+a)\right) dx$$

```
[In] int((d*x+c)^m*cos(b*x+a)^3,x)
[Out] int((d*x+c)^m*cos(b*x+a)^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

$$\int (c+dx)^m \cos^3(a+bx)\, dx$$

$$= \frac{9i\, e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right) - i\, e^{\left(-\frac{dm\log\left(-\frac{3ib}{d}\right)+3ibc-3iad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(ibdx+ibc)}{d}\right) - 9i\, e^{\left(-\frac{dm\log}{}\right.}}{24\,b}$$

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="fricas")
[Out] 1/24*(9*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I
*b*c)/d) - I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, -3
*(I*b*d*x + I*b*c)/d) - 9*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(
m + 1, (-I*b*d*x - I*b*c)/d) + I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)
/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \cos^3(a+bx)\, dx = \int (c+dx)^m \cos^3(a+bx)\, dx$$

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3,x)
[Out] Integral((c + d*x)**m*cos(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)^m \cos^3(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^3\,dx$$

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)^3, x)
```

Giac [F]

$$\int (c+dx)^m \cos^3(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^3\,dx$$

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^3(a+bx)\,dx = \int \cos(a+b\,x)^3\,(c+d\,x)^m\,dx$$

```
[In] int(cos(a + b*x)^3*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^3*(c + d*x)^m, x)
```

3.100 $\int (c+dx)^m \cos^2(a+bx)\,dx$

Optimal result . . . 675
Rubi [A] (verified) . . . 675
Mathematica [A] (verified) . . . 677
Maple [F] . . . 677
Fricas [A] (verification not implemented) . . . 677
Sympy [F] . . . 678
Maxima [F] . . . 678
Giac [F] . . . 678
Mupad [F(-1)] . . . 678

Optimal result

Integrand size = 16, antiderivative size = 162

$$\int (c+dx)^m \cos^2(a+bx)\,dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} - \frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}$$
$$+ \frac{i2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}$$

```
[Out] 1/2*(d*x+c)^(1+m)/d/(1+m)-I*2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m
,-2*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+I*2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*
I*b*(d*x+c)/d)/b/exp(2*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3388, 2212}

$$\int (c+dx)^m \cos^2(a+bx)\,dx$$
$$= -\frac{i2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}$$
$$+ \frac{i2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b} + \frac{(c+dx)^{m+1}}{2d(m+1)}$$

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c
+ d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)
 + (I*2^(-3 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*
I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}(c+dx)^m + \frac{1}{2}(c+dx)^m \cos(2a+2bx)\right)\,dx \\
&= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{2}\int (c+dx)^m \cos(2a+2bx)\,dx \\
&= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{4}\int e^{-i(2a+2bx)}(c+dx)^m\,dx + \frac{1}{4}\int e^{i(2a+2bx)}(c+dx)^m\,dx \\
&= \frac{(c+dx)^{1+m}}{2d(1+m)} - \frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} \\
&\quad + \frac{i2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \frac{1}{8}(c+dx)^m \left(\frac{4c+4dx}{d+dm} - \frac{i2^{-m} e^{2i\left(a-\frac{bc}{d}\right)} \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2ib(c+dx)}{d}\right)}{b} + \frac{i2^{-m} e^{-2i\left(a-\frac{bc}{d}\right)} \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{2ib(c+dx)}{d}\right)}{b} \right)$$

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2,x]
[Out] ((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) - (I*E^((2*I)*(a - (b*c)/d))*Gamma[1
 + m, ((-2*I)*b*(c + d*x))/d])/(2^m*b*(((-I)*b*(c + d*x))/d)^m) + (I*Gamma[1
 + m, ((2*I)*b*(c + d*x))/d])/(2^m*b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x
))/d)^m)))/8
```

Maple [F]

$$\int (dx+c)^m \left(\cos^2(bx+a)\right) dx$$

```
[In] int((d*x+c)^m*cos(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \frac{(-i\,dm - i\,d)e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(m+1, -\frac{2(ibdx+ibc)}{d}\right) + (i\,dm+i\,d)e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+\right.}{8\,(bdm+bd)}$$

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*((-I*d*m - I*d)*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m
 + 1, -2*(I*b*d*x + I*b*c)/d) + (I*d*m + I*d)*e^(-(d*m*log(2*I*b/d) - 2*I*b*
c + 2*I*a*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) + 4*(b*d*x + b*c)*(d*
x + c)^m)/(b*d*m + b*d)
```

Sympy [F]

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \int (c+dx)^m \cos^2(a+bx)\,dx$$

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2\,dx$$

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*b*x + 2*a), x) + e^(m*log(d*x +
c) + log(d*x + c)))/(d*m + d)
```

Giac [F]

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2\,dx$$

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \int \cos(a+b\,x)^2\,(c+d\,x)^m\,dx$$

```
[In] int(cos(a + b*x)^2*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^2*(c + d*x)^m, x)
```

3.101 $\int (c+dx)^m \cos(a+bx)\,dx$

Optimal result . . . 679
Rubi [A] (verified) . . . 679
Mathematica [A] (verified) . . . 680
Maple [F] . . . 681
Fricas [A] (verification not implemented) . . . 681
Sympy [F] . . . 681
Maxima [F] . . . 681
Giac [F] . . . 682
Mupad [F(-1)] . . . 682

Optimal result

Integrand size = 14, antiderivative size = 131

$$\int (c+dx)^m \cos(a+bx)\,dx = -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b}$$

```
[Out] -1/2*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)+1/2*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(
d*x+c)/d)^m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3388, 2212}

$$\int (c+dx)^m \cos(a+bx)\,dx = \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}$$

```
[In] Int[(c + d*x)^m*Cos[a + b*x],x]
```

```
[Out] ((-1/2*I)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d
])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c +
 d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx + \frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx \\ &= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} \\ &\quad + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

$$\int (c+dx)^m\cos(a+bx)\,dx = -\frac{ie^{-\frac{i(bc+ad)}{d}}(c+dx)^m\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right) - e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)\right)}{2b}$$

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x],x]
```

```
[Out] ((-1/2*I)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((
-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d])
/((I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d))
```

Maple [F]

$$\int (dx+c)^m \cos(bx+a)\,dx$$

```
[In] int((d*x+c)^m*cos(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73

$$\int (c+dx)^m \cos(a+bx)\,dx = \frac{i\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right) - i\,e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+ibc-iad}{d}\right)}\Gamma\left(m+1,\frac{-ibdx-ibc}{d}\right)}{2\,b}$$

```
[In] integrate((d*x+c)^m*cos(b*x+a),x, algorithm="fricas")
[Out] 1/2*(I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*
c)/d) - I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x -
 I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \cos(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\,dx$$

```
[In] integrate((d*x+c)**m*cos(b*x+a),x)
[Out] Integral((c + d*x)**m*cos(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^m \cos(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\ dx$$

```
[In] integrate((d*x+c)^m*cos(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^m \cos(a+bx)\,dx = \int (dx+c)^m \cos\left(bx+a\right)\,dx$$

```
[In] integrate((d*x+c)^m*cos(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\left(c+d\,x\right)^m\,dx$$

```
[In] int(cos(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)*(c + d*x)^m, x)
```

3.102 $\int (c+dx)^m \sec(a+bx)\,dx$

Optimal result 683
Rubi [N/A] 683
Mathematica [N/A] 684
Maple [N/A] (verified) 684
Fricas [N/A] 684
Sympy [N/A] 684
Maxima [N/A] 685
Giac [N/A] 685
Mupad [N/A] 685

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \sec(a+bx)\,dx = \text{Int}((c+dx)^m \sec(a+bx), x)$$

```
[Out] Unintegrable((d*x+c)^m*sec(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\,dx$$

```
[In] Int[(c + d*x)^m*Sec[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x], x]
```

Rubi steps

$$\text{integral} = \int (c+dx)^m \sec(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.54 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\,dx$$

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sec[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sec(bx+a)\,dx$$

```
[In] int((d*x+c)^m*sec(b*x+a),x)
[Out] int((d*x+c)^m*sec(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\,dx$$

```
[In] integrate((d*x+c)^m*sec(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\,dx$$

```
[In] integrate((d*x+c)**m*sec(b*x+a),x)
[Out] Integral((c + d*x)**m*sec(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\,dx$$

```
[In] integrate((d*x+c)^m*sec(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\,dx$$

```
[In] integrate((d*x+c)^m*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 14.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (c+dx)^m \sec(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

```
[In] int((c + d*x)^m/cos(a + b*x),x)
[Out] int((c + d*x)^m/cos(a + b*x), x)
```

3.103 $\int (c+dx)^m \sec^2(a+bx)\,dx$

Optimal result 686
Rubi [N/A] 686
Mathematica [N/A] 687
Maple [N/A] (verified) 687
Fricas [N/A] 687
Sympy [N/A] 687
Maxima [N/A] 688
Giac [N/A] 688
Mupad [N/A] 688

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \text{Int}\big((c+dx)^m \sec^2(a+bx), x\big)$$

```
[Out] Unintegrable((d*x+c)^m*sec(b*x+a)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\,dx$$

```
[In] Int[(c + d*x)^m*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]^2, x]
```

Rubi steps

$$\text{integral} = \int (c+dx)^m \sec^2(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\,dx$$

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \left(\sec^2(bx+a)\right)\,dx$$

```
[In] int((d*x+c)^m*sec(b*x+a)^2,x)
[Out] int((d*x+c)^m*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (dx+c)^m \sec\left(bx+a\right)^2\,dx$$

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 5.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (c+dx)^m \sec^2\left(a+bx\right)\,dx$$

```
[In] integrate((d*x+c)**m*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*sec(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2\,dx$$

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2\,dx$$

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2}\,dx$$

```
[In] int((c + d*x)^m/cos(a + b*x)^2,x)
[Out] int((c + d*x)^m/cos(a + b*x)^2, x)
```

3.104 $\int x^{3+m}\cos(a+bx)\,dx$

Optimal result . . . 689
Rubi [A] (verified) . . . 689
Mathematica [A] (verified) . . . 690
Maple [C] (verified) . . . 690
Fricas [A] (verification not implemented) . . . 691
Sympy [F] . . . 691
Maxima [F] . . . 691
Giac [F] . . . 692
Mupad [F(-1)] . . . 692

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^{3+m}\cos(a+bx)\,dx = -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4} - \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}$$

```
[Out] -1/2*exp(I*a)*x^m*GAMMA(4+m,-I*b*x)/b^4/((-I*b*x)^m)-1/2*x^m*GAMMA(4+m,I*b*
x)/b^4/exp(I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$\int x^{3+m}\cos(a+bx)\,dx = -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(m+4,-ibx)}{2b^4} - \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(m+4,ibx)}{2b^4}$$

```
[In] Int[x^(3 + m)*Cos[a + b*x],x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - (x^m*Gamma[4
 + m, I*b*x])/(2*b^4*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-i(a+bx)}x^{3+m}\,dx + \frac{1}{2}\int e^{i(a+bx)}x^{3+m}\,dx \\ &= -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4} - \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^{3+m}\cos(a+bx)\,dx = -\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4} - \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}$$

```
[In] Integrate[x^(3 + m)*Cos[a + b*x],x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - (x^m*Gamma[4
 + m, I*b*x])/(2*b^4*E^(I*a)*(I*b*x)^m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 455, normalized size of antiderivative = 6.07

method	result
meijerg	$\frac{2^{3+m}\left(b^2\right)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{-4-m}x^{3+m}b^3\left(b^2\right)^{\frac{m}{2}}\left(\frac{8}{3}+\frac{2m}{3}\right)\sin(bx)}{\sqrt{\pi}\,(4+m)} - \frac{2^{-3-m}x^{1+m}b\left(b^2\right)^{\frac{m}{2}}\left(-m^2-7m-12\right)(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,(4+m)} + \frac{2^{-3-m}x^{2+m}b^2\left(\right.}{}\right.$

```
[In] int(x^(3+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(3+m)/b^4*(b^2)^(-1/2*m)*Pi^(1/2)*(3*2^(-4-m)/Pi^(1/2)/(4+m)*x^(3+m)*b^3*
(b^2)^(1/2*m)*(8/3+2/3*m)*sin(b*x)-2^(-3-m)/Pi^(1/2)/(4+m)*x^(1+m)*b*(b^2)^
(1/2*m)*(-m^2-7*m-12)*(cos(b*x)*x*b-sin(b*x))+2^(-3-m)/Pi^(1/2)/(4+m)*x^(2+
m)*b^2*(b^2)^(1/2*m)*(-m^3-8*m^2-19*m-12)*(b*x)^(-3/2-m)*LommelS1(3/2+m,3/2
,b*x)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2+m)*(1+m)*(3+m
)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(1/2+m,1/2,b*x))*cos(a)-2^
```

```
(3+m)*b^(-4-m)*Pi^(1/2)*(2^(-3-m)/Pi^(1/2)/(5+m)*x^(2+m)*b^(2+m)*(m^2+7*m+1
0)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(cos(b*x)*x*b-sin(b*x))-2^(-3
-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*m*(3+m)*(2+m)*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/
2,b*x)*sin(b*x)+2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(3+m)*(2+m)*(b*x)^(-5/2-m
)*(cos(b*x)*x*b-sin(b*x))*LommelS1(3/2+m,1/2,b*x))*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int x^{3+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m+3)\log(i\,b)-i\,a)}\Gamma(m+4,i\,bx) - i\,e^{(-(m+3)\log(-i\,b)+i\,a)}\Gamma(m+4,-i\,bx)}{2\,b}$$

```
[In] integrate(x^(3+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m + 3)*log(I*b) - I*a)*gamma(m + 4, I*b*x) - I*e^(-(m + 3)*log(
-I*b) + I*a)*gamma(m + 4, -I*b*x))/b
```

Sympy [F]

$$\int x^{3+m}\cos(a+bx)\,dx = \int x^{m+3}\cos\left(a+bx\right)dx$$

```
[In] integrate(x**(3+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m + 3)*cos(a + b*x), x)
```

Maxima [F]

$$\int x^{3+m}\cos(a+bx)\,dx = \int x^{m+3}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(3+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 3)*cos(b*x + a), x)
```

Giac [F]

$$\int x^{3+m}\cos(a+bx)\,dx = \int x^{m+3}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(3+m)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m + 3)*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\cos(a+bx)\,dx = \int x^{m+3}\cos\left(a+b\,x\right)\,dx$$

```
[In] int(x^(m + 3)*cos(a + b*x),x)
[Out] int(x^(m + 3)*cos(a + b*x), x)
```

3.105 $\int x^{2+m}\cos(a+bx)\,dx$

Optimal result . . . 693
Rubi [A] (verified) . . . 693
Mathematica [A] (verified) . . . 694
Maple [C] (verified) . . . 694
Fricas [A] (verification not implemented) . . . 695
Sympy [F] . . . 695
Maxima [F] . . . 695
Giac [F] . . . 696
Mupad [F(-1)] . . . 696

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int x^{2+m}\cos(a+bx)\,dx = \frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3} - \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}$$

```
[Out] 1/2*I*exp(I*a)*x^m*GAMMA(3+m,-I*b*x)/b^3/((-I*b*x)^m)-1/2*I*x^m*GAMMA(3+m,I
*b*x)/b^3/exp(I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$\int x^{2+m}\cos(a+bx)\,dx = \frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(m+3,-ibx)}{2b^3} - \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(m+3,ibx)}{2b^3}$$

```
[In] Int[x^(2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(b^3*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[3 + m, I*b*x])/(b^3*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-i(a+bx)}x^{2+m}\,dx + \frac{1}{2}\int e^{i(a+bx)}x^{2+m}\,dx \\ &= \frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3} - \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int x^{2+m}\cos(a+bx)\,dx = \frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3} - \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}$$

```
[In] Integrate[x^(2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(b^3*((-I)*b*x)^m) - ((I/2)*x^m*
Gamma[3 + m, I*b*x])/(b^3*E^(I*a)*(I*b*x)^m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.48

method	result
meijerg	$\frac{2^{2+m}\left(b^2\right)^{-\frac{1}{2}-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{-3-m}x^{2+m}\left(b^2\right)^{\frac{3}{2}+\frac{m}{2}}\left(2+\frac{2m}{3}\right)\sin(bx)}{\sqrt{\pi}\,(3+m)b} - \frac{2^{-2-m}x^{2+m}\left(b^2\right)^{\frac{3}{2}+\frac{m}{2}}(2+m)m(bx)^{-\frac{3}{2}-m}s^{(+)}_{\frac{1}{2}+m,\frac{3}{2}}(bx)\sin(bx)}{\sqrt{\pi}\,b} + \frac{2^{-2-m}$ … b^2}

```
[In] int(x^(2+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(2+m)/b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-3-m)/Pi^(1/2)/(3+m)*x^(2+m)*
(b^2)^(3/2+1/2*m)*(2+2/3*m)/b*sin(b*x)-2^(-2-m)/Pi^(1/2)*x^(2+m)*(b^2)^(3/2
+1/2*m)/b*(2+m)*m*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2,b*x)*sin(b*x)+2^(-2-m)/
Pi^(1/2)*x^(2+m)*(b^2)^(3/2+1/2*m)/b*(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin
(b*x))*LommelS1(3/2+m,1/2,b*x))*cos(a)-2^(2+m)*b^(-3-m)*Pi^(1/2)*(-2^(-2-m)
/Pi^(1/2)*x^(1+m)*b^(1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-2-m)/Pi^(1/2)/(4+m)*x
```

```
^(2+m)*b^(2+m)*(m^2+5*m+4)*(b*x)^(-3/2-m)*LommelS1(3/2+m,3/2,b*x)*sin(b*x)+
2^(-2-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(2+m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-
sin(b*x))*LommelS1(1/2+m,1/2,b*x))*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int x^{2+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m+2)\log(i\,b)-i\,a)}\Gamma(m+3,i\,bx) - i\,e^{(-(m+2)\log(-i\,b)+i\,a)}\Gamma(m+3,-i\,bx)}{2\,b}$$

```
[In] integrate(x^(2+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m + 2)*log(I*b) - I*a)*gamma(m + 3, I*b*x) - I*e^(-(m + 2)*log(
-I*b) + I*a)*gamma(m + 3, -I*b*x))/b
```

Sympy [F]

$$\int x^{2+m}\cos(a+bx)\,dx = \int x^{m+2}\cos\left(a+bx\right)\,dx$$

```
[In] integrate(x**(2+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m + 2)*cos(a + b*x), x)
```

Maxima [F]

$$\int x^{2+m}\cos(a+bx)\,dx = \int x^{m+2}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(2+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2)*cos(b*x + a), x)
```

Giac [F]

$$\int x^{2+m}\cos(a+bx)\,dx = \int x^{m+2}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(2+m)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m + 2)*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2+m}\cos(a+bx)\,dx = \int x^{m+2}\cos\left(a+b\,x\right)\,dx$$

```
[In] int(x^(m + 2)*cos(a + b*x),x)
[Out] int(x^(m + 2)*cos(a + b*x), x)
```

3.106 $\int x^{1+m}\cos(a+bx)\,dx$

Optimal result . . . 697
Rubi [A] (verified) . . . 697
Mathematica [A] (verified) . . . 698
Maple [C] (verified) . . . 698
Fricas [A] (verification not implemented) . . . 699
Sympy [F] . . . 699
Maxima [F] . . . 699
Giac [F] . . . 700
Mupad [F(-1)] . . . 700

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^{1+m}\cos(a+bx)\,dx = \frac{e^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2} + \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}$$

```
[Out] 1/2*exp(I*a)*x^m*GAMMA(2+m,-I*b*x)/b^2/((-I*b*x)^m)+1/2*x^m*GAMMA(2+m,I*b*x
)/b^2/exp(I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$\int x^{1+m}\cos(a+bx)\,dx = \frac{e^{ia}x^m(-ibx)^{-m}\Gamma(m+2,-ibx)}{2b^2} + \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(m+2,ibx)}{2b^2}$$

```
[In] Int[x^(1 + m)*Cos[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(2*b^2*((-I)*b*x)^m) + (x^m*Gamma[2 +
m, I*b*x])/(2*b^2*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-i(a+bx)}x^{1+m}\,dx + \frac{1}{2}\int e^{i(a+bx)}x^{1+m}\,dx \\ &= \frac{e^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2} + \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^{1+m}\cos(a+bx)\,dx = \frac{e^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2} + \frac{e^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}$$

```
[In] Integrate[x^(1 + m)*Cos[a + b*x],x]
```

```
[Out] (E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(2*b^2*((-I)*b*x)^m) + (x^m*Gamma[2 +
m, I*b*x])/(2*b^2*E^(I*a)*(I*b*x)^m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.88

method	result
meijerg	$\frac{2^{1+m}\left(b^2\right)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{2^{-1-m}x^{1+m}b\left(b^2\right)^{\frac{m}{2}}\sin(bx)}{\sqrt{\pi}\,(2+m)}+\frac{3\,2^{-2-m}x^{2+m}b^2\left(b^2\right)^{\frac{m}{2}}\left(\frac{2}{3}+\frac{2m}{3}\right)(bx)^{-\frac{3}{2}-m}s_{\frac{3}{2}+m,\frac{3}{2}}^{(+)}(bx)\sin(bx)}{\sqrt{\pi}\,(2+m)}+\frac{2^{-1-m}x^{2+m}b^2\left(b^2\right)}{b^2}$

```
[In] int(x^(1+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1+m)/b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)/(2+m)*x^(1+m)*b*(b^2
)^(1/2*m)*sin(b*x)+3*2^(-2-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2/3
+2/3*m)*(b*x)^(-3/2-m)*LommelS1(3/2+m,3/2,b*x)*sin(b*x)+2^(-1-m)/Pi^(1/2)*x
^(2+m)*b^2*(b^2)^(1/2*m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lomme
lS1(1/2+m,1/2,b*x))*cos(a)-2^(1+m)*b^(-2-m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)*x^(
2+m)*b^(2+m)*m*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2,b*x)*sin(b*x)-2^(-1-m)/Pi^
```

```
(1/2)*x^(2+m)*b^(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(3/2+m
,1/2,b*x))*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int x^{1+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m+1)\log(i\,b)-i\,a)}\Gamma(m+2,i\,bx) - i\,e^{(-(m+1)\log(-i\,b)+i\,a)}\Gamma(m+2,-i\,bx)}{2\,b}$$

```
[In] integrate(x^(1+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m + 1)*log(I*b) - I*a)*gamma(m + 2, I*b*x) - I*e^(-(m + 1)*log(
-I*b) + I*a)*gamma(m + 2, -I*b*x))/b
```

Sympy [F]

$$\int x^{1+m}\cos(a+bx)\,dx = \int x^{m+1}\cos\left(a+bx\right)\,dx$$

```
[In] integrate(x**(1+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m + 1)*cos(a + b*x), x)
```

Maxima [F]

$$\int x^{1+m}\cos(a+bx)\,dx = \int x^{m+1}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(1+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 1)*cos(b*x + a), x)
```

Giac [F]

$$\int x^{1+m}\cos(a+bx)\,dx = \int x^{m+1}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(1+m)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m + 1)*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\cos(a+bx)\,dx = \int x^{m+1}\cos\left(a+b\,x\right)\,dx$$

```
[In] int(x^(m + 1)*cos(a + b*x),x)
[Out] int(x^(m + 1)*cos(a + b*x), x)
```

3.107 $\int x^m \cos(a+bx)\,dx$

Optimal result . . . 701
Rubi [A] (verified) . . . 701
Mathematica [A] (verified) . . . 702
Maple [C] (verified) . . . 702
Fricas [A] (verification not implemented) . . . 703
Sympy [F] . . . 703
Maxima [F] . . . 703
Giac [F] . . . 704
Mupad [F(-1)] . . . 704

Optimal result

Integrand size = 10, antiderivative size = 79

$$\int x^m \cos(a+bx)\,dx = -\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b} + \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}$$

```
[Out] -1/2*I*exp(I*a)*x^m*GAMMA(1+m,-I*b*x)/b/((-I*b*x)^m)+1/2*I*x^m*GAMMA(1+m,I*
b*x)/b/exp(I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3388, 2212}

$$\int x^m \cos(a+bx)\,dx = \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(m+1,ibx)}{2b} - \frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(m+1,-ibx)}{2b}$$

```
[In] Int[x^m*Cos[a + b*x],x]
```

```
[Out] ((-1/2*I)*E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) + ((I/2)*x^m
*Gamma[1 + m, I*b*x])/(b*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-i(a+bx)}x^m\,dx + \frac{1}{2}\int e^{i(a+bx)}x^m\,dx \\ &= -\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b} + \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int x^m\cos(a+bx)\,dx = -\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b} + \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}$$

```
[In] Integrate[x^m*Cos[a + b*x],x]
```

```
[Out] ((-1/2*I)*E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) + ((I/2)*x^m
*Gamma[1 + m, I*b*x])/(b*E^(I*a)*(I*b*x)^m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.80

method	result
meijerg	$2^m(b^2)^{-\frac{1}{2}-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{-1-m}(b^2)^{\frac{1}{2}+\frac{m}{2}}x^m(6+2m)\sin(bx)}{\sqrt{\pi}\,(1+m)(9+3m)b} + \frac{(b^2)^{\frac{1}{2}+\frac{m}{2}}x^m 2^{-m}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,(1+m)b} + \frac{2^{-m}x^{2+m}(b^2)^{\frac{1}{2}+\frac{m}{2}}b}{}\right.$

```
[In] int(x^m*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^m*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-1-m)/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m
)*x^m*(6+2*m)/(9+3*m)/b*sin(b*x)+1/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m)*x^m*2^(
-m)/b*(cos(b*x)*x*b-sin(b*x))+2^(-m)/Pi^(1/2)/(1+m)*x^(2+m)*(b^2)^(1/2+1/2*
m)*b*m*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2,b*x)*sin(b*x)-2^(-m)/Pi^(1/2)/(1+m
)*x^(2+m)*(b^2)^(1/2+1/2*m)*b*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lommel
S1(3/2+m,1/2,b*x))*cos(a)-2^m*b^(-1-m)*Pi^(1/2)*(1/Pi^(1/2)/(2+m)*x^(1+m)*b
^(1+m)*2^(-m)*sin(b*x)-2^(-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)*(b*x)^(-3/2-m)
*LommelS1(3/2+m,3/2,b*x)*sin(b*x)-3*2^(-1-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)
```

```
*(4/3+2/3*m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(1/2+m,1/2,b*x)
)*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

$$\int x^m \cos(a+bx)\,dx = \frac{i\,e^{(-m\log(i\,b)-i\,a)}\Gamma(m+1,i\,bx) - i\,e^{(-m\log(-i\,b)+i\,a)}\Gamma(m+1,-i\,bx)}{2\,b}$$

```
[In] integrate(x^m*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) - I*e^(-m*log(-I*b) + I*a)
*gamma(m + 1, -I*b*x))/b
```

Sympy [F]

$$\int x^m \cos(a+bx)\,dx = \int x^m \cos\left(a+bx\right)\,dx$$

```
[In] integrate(x**m*cos(b*x+a),x)
```

```
[Out] Integral(x**m*cos(a + b*x), x)
```

Maxima [F]

$$\int x^m \cos(a+bx)\,dx = \int x^m \cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cos(b*x + a), x)
```

Giac [F]

$$\int x^m \cos(a+bx)\,dx = \int x^m \cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cos(a+bx)\,dx = \int x^m\,\cos\left(a+b\,x\right)\,dx$$

```
[In] int(x^m*cos(a + b*x),x)
[Out] int(x^m*cos(a + b*x), x)
```

3.108 $\int x^{-1+m}\cos(a+bx)\,dx$

Optimal result . . . 705
Rubi [A] (verified) . . . 705
Mathematica [A] (verified) . . . 706
Maple [C] (verified) . . . 706
Fricas [A] (verification not implemented) . . . 707
Sympy [F] . . . 707
Maxima [F] . . . 707
Giac [F] . . . 708
Mupad [F(-1)] . . . 708

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x^{-1+m}\cos(a+bx)\,dx = -\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx) - \frac{1}{2}e^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)$$

```
[Out] -1/2*exp(I*a)*x^m*GAMMA(m,-I*b*x)/((-I*b*x)^m)-1/2*x^m*GAMMA(m,I*b*x)/exp(I
*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$\int x^{-1+m}\cos(a+bx)\,dx = -\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx) - \frac{1}{2}e^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)$$

```
[In] Int[x^(-1 + m)*Cos[a + b*x],x]
```

```
[Out] -1/2*(E^(I*a)*x^m*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - (x^m*Gamma[m, I*b*x])/
(2*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-i(a+bx)}x^{-1+m}\,dx + \frac{1}{2}\int e^{i(a+bx)}x^{-1+m}\,dx \\ &= -\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx) - \frac{1}{2}e^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int x^{-1+m}\cos(a+bx)\,dx = \frac{1}{2}e^{-ia}x^m\left(-e^{2ia}(-ibx)^{-m}\Gamma(m,-ibx) - (ibx)^{-m}\Gamma(m,ibx)\right)$$

```
[In] Integrate[x^(-1 + m)*Cos[a + b*x],x]
```

```
[Out] (x^m*(-((E^((2*I)*a)*Gamma[m, (-I)*b*x])/((-I)*b*x)^m) - Gamma[m, I*b*x]/(I
*b*x)^m))/(2*E^(I*a))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 427, normalized size of antiderivative = 6.57

method	result
meijerg	$2^{-1+m}(b^2)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{3x^{-1+m}2^{-m}(b^2)^{\frac{m}{2}}(2x^2b^2+2m+4)\sin(bx)}{\sqrt{\pi}\,m(6+3m)b} + \frac{2^{1-m}x^{-1+m}(b^2)^{\frac{m}{2}}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,mb} - \frac{3x^{2+m}2^{1-m}($

```
[In] int(x^(-1+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(-1+m)*(b^2)^(-1/2*m)*Pi^(1/2)*(3/Pi^(1/2)/m*x^(-1+m)*2^(-m)*(b^2)^(1/2*m
)*(2*b^2*x^2+2*m+4)/(6+3*m)/b*sin(b*x)+2^(1-m)/Pi^(1/2)/m*x^(-1+m)*(b^2)^(1
/2*m)/b*(cos(b*x)*x*b-sin(b*x))-3/Pi^(1/2)/m*x^(2+m)*2^(1-m)*(b^2)^(1/2*m)*
b^2/(6+3*m)*(b*x)^(-3/2-m)*LommelS1(3/2+m,3/2,b*x)*sin(b*x)-1/Pi^(1/2)/m*x^
(2+m)*2^(1-m)*(b^2)^(1/2*m)*b^2*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*Lomm
elS1(1/2+m,1/2,b*x))*cos(a)-2^(-1+m)*b^(-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)/(1+m
)*x^m*b^m*sin(b*x)-2^(1-m)/Pi^(1/2)/(1+m)*x^m*b^m/m*(cos(b*x)*x*b-sin(b*x))
-1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2
```

```
,b*x)*sin(b*x)+1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)/m*(b*x)^(-5/2-m)*(c
os(b*x)*x*b-sin(b*x))*LommelS1(3/2+m,1/2,b*x))*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int x^{-1+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m-1)\log(i\,b)-i\,a)}\Gamma(m,i\,bx) - i\,e^{(-(m-1)\log(-i\,b)+i\,a)}\Gamma(m,-i\,bx)}{2\,b}$$

```
[In] integrate(x^(-1+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m - 1)*log(I*b) - I*a)*gamma(m, I*b*x) - I*e^(-(m - 1)*log(-I*b
) + I*a)*gamma(m, -I*b*x))/b
```

Sympy [F]

$$\int x^{-1+m}\cos(a+bx)\,dx = \int x^{m-1}\cos\left(a+bx\right)dx$$

```
[In] integrate(x**(-1+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m - 1)*cos(a + b*x), x)
```

Maxima [F]

$$\int x^{-1+m}\cos(a+bx)\,dx = \int x^{m-1}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(-1+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)*cos(b*x + a), x)
```

Giac [F]

$$\int x^{-1+m}\cos(a+bx)\,dx = \int x^{m-1}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(-1+m)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 1)*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\cos(a+bx)\,dx = \int x^{m-1}\,\cos\left(a+b\,x\right)\,dx$$

```
[In] int(x^(m - 1)*cos(a + b*x),x)
[Out] int(x^(m - 1)*cos(a + b*x), x)
```

3.109 $\int x^{-2+m}\cos(a+bx)\,dx$

Optimal result . . . 709
Rubi [A] (verified) . . . 709
Mathematica [A] (verified) . . . 710
Maple [C] (verified) . . . 710
Fricas [A] (verification not implemented) . . . 711
Sympy [F] . . . 711
Maxima [F] . . . 711
Giac [F] . . . 712
Mupad [F(-1)] . . . 712

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^{-2+m}\cos(a+bx)\,dx = \frac{1}{2}ibe^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx) - \frac{1}{2}ibe^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)$$

```
[Out] 1/2*I*b*exp(I*a)*x^m*GAMMA(-1+m,-I*b*x)/((-I*b*x)^m)-1/2*I*b*x^m*GAMMA(-1+m
,I*b*x)/exp(I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$\int x^{-2+m}\cos(a+bx)\,dx = \frac{1}{2}ie^{ia}bx^m(-ibx)^{-m}\Gamma(m-1,-ibx) - \frac{1}{2}ie^{-ia}bx^m(ibx)^{-m}\Gamma(m-1,ibx)$$

```
[In] Int[x^(-2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*b*x^m*G
amma[-1 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-i(a+bx)}x^{-2+m}\,dx + \frac{1}{2}\int e^{i(a+bx)}x^{-2+m}\,dx \\ &= \frac{1}{2}ibe^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx) - \frac{1}{2}ibe^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^{-2+m}\cos(a+bx)\,dx = &\frac{1}{2}ibe^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx) \\ &- \frac{1}{2}ibe^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)\end{aligned}$$

```
[In] Integrate[x^(-2 + m)*Cos[a + b*x],x]
```

```
[Out] ((I/2)*b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*b*x^m*G
amma[-1 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 530, normalized size of antiderivative = 7.07

method	result
meijerg	$2^{-2+m}b^2(b^2)^{-\frac{1}{2}-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{1-m}x^{-2+m}(b^2)^{-\frac{1}{2}+\frac{m}{2}}(2x^2b^2+2m+2)\sin(bx)}{\sqrt{\pi}\,(-1+m)(3+3m)b} - \frac{2^{2-m}x^{-2+m}(b^2)^{-\frac{1}{2}+\frac{m}{2}}(x^2b^2-m^2-m)(\cos($

```
[In] int(x^(-2+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(-2+m)*b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(1-m)/Pi^(1/2)/(-1+m)*x^(-2+m
)*(b^2)^(-1/2+1/2*m)*(2*b^2*x^2+2*m+2)/(3+3*m)/b*sin(b*x)-2^(2-m)/Pi^(1/2)/
(-1+m)*x^(-2+m)*(b^2)^(-1/2+1/2*m)/b*(b^2*x^2-m^2-m)/(1+m)/m*(cos(b*x)*x*b-
sin(b*x))-3*2^(2-m)/Pi^(1/2)/(-1+m)*x^(2+m)*(b^2)^(-1/2+1/2*m)*b^3/(3+3*m)*
(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2,b*x)*sin(b*x)+2^(2-m)/Pi^(1/2)/(-1+m)*x^(
```

```
2+m)*(b^2)^(-1/2+1/2*m)*b^3/(1+m)/m*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*
LommelS1(3/2+m,1/2,b*x))*cos(a)-2^(-2+m)*b^(1-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2)
/m*x^(-1+m)*b^(-1+m)*(-2*b^2*x^2+2*m^2+2*m-4)/(2+m)/(-1+m)*sin(b*x)-3*2^(2-
m)/Pi^(1/2)/m*x^(-1+m)*b^(-1+m)/(-3+3*m)*(cos(b*x)*x*b-sin(b*x))+2^(2-m)/Pi
^(1/2)/m*x^(2+m)*b^(2+m)/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(3/2+m,3/2,b*x
)*sin(b*x)+3*2^(2-m)/Pi^(1/2)/m*x^(2+m)*b^(2+m)/(-3+3*m)*(b*x)^(-5/2-m)*(co
s(b*x)*x*b-sin(b*x))*LommelS1(1/2+m,1/2,b*x))*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int x^{-2+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m-2)\log(i\,b)-i\,a)}\Gamma(m-1,i\,bx) - i\,e^{(-(m-2)\log(-i\,b)+i\,a)}\Gamma(m-1,-i\,bx)}{2\,b}$$

```
[In] integrate(x^(-2+m)*cos(b*x+a),x, algorithm="fricas")
[Out] 1/2*(I*e^(-(m - 2)*log(I*b) - I*a)*gamma(m - 1, I*b*x) - I*e^(-(m - 2)*log(
-I*b) + I*a)*gamma(m - 1, -I*b*x))/b
```

Sympy [F]

$$\int x^{-2+m}\cos(a+bx)\,dx = \int x^{m-2}\cos\left(a+bx\right)dx$$

```
[In] integrate(x**(-2+m)*cos(b*x+a),x)
[Out] Integral(x**(m - 2)*cos(a + b*x), x)
```

Maxima [F]

$$\int x^{-2+m}\cos(a+bx)\,dx = \int x^{m-2}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(-2+m)*cos(b*x+a),x, algorithm="maxima")
[Out] integrate(x^(m - 2)*cos(b*x + a), x)
```

Giac [F]

$$\int x^{-2+m}\cos(a+bx)\,dx = \int x^{m-2}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(-2+m)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 2)*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\cos(a+bx)\,dx = \int x^{m-2}\,\cos\left(a+b\,x\right)\,dx$$

```
[In] int(x^(m - 2)*cos(a + b*x),x)
[Out] int(x^(m - 2)*cos(a + b*x), x)
```

3.110 $\int x^{-3+m}\cos(a+bx)\,dx$

Optimal result . . . 713
Rubi [A] (verified) . . . 713
Mathematica [A] (verified) . . . 714
Maple [C] (verified) . . . 714
Fricas [A] (verification not implemented) . . . 715
Sympy [F] . . . 715
Maxima [F] . . . 715
Giac [F] . . . 716
Mupad [F(-1)] . . . 716

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^{-3+m}\cos(a+bx)\,dx = \frac{1}{2}b^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx) + \frac{1}{2}b^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)$$

```
[Out] 1/2*b^2*exp(I*a)*x^m*GAMMA(-2+m,-I*b*x)/((-I*b*x)^m)+1/2*b^2*x^m*GAMMA(-2+m
,I*b*x)/exp(I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$\int x^{-3+m}\cos(a+bx)\,dx = \frac{1}{2}e^{ia}b^2x^m(-ibx)^{-m}\Gamma(m-2,-ibx)+\frac{1}{2}e^{-ia}b^2x^m(ibx)^{-m}\Gamma(m-2,ibx)$$

```
[In] Int[x^(-3 + m)*Cos[a + b*x],x]
```

```
[Out] (b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b^2*x^m*Gamma
[-2 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-i(a+bx)}x^{-3+m}\,dx + \frac{1}{2}\int e^{i(a+bx)}x^{-3+m}\,dx \\ &= \frac{1}{2}b^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx) + \frac{1}{2}b^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^{-3+m}\cos(a+bx)\,dx = &\frac{1}{2}b^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx) \\ &+ \frac{1}{2}b^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)\end{aligned}$$

```
[In] Integrate[x^(-3 + m)*Cos[a + b*x],x]
```

```
[Out] (b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b^2*x^m*Gamma
[-2 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 600, normalized size of antiderivative = 8.00

method	result
meijerg	$2^{-3+m}b^2(b^2)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{2^{2-m}x^{-3+m}(b^2)^{\frac{m}{2}}(-2x^4b^4+2x^2b^2m^2+2x^2b^2m-4x^2b^2+2m^3+2m^2-4m)\sin(bx)}{\sqrt{\pi}(-2+m)b^3m(2+m)(-1+m)} - \frac{2^{3-m}x^{-3+m}(b^2$

```
[In] int(x^(-3+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(-3+m)*b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-2+m)*x^(-3+m)/b^3*
(b^2)^(1/2*m)*(-2*b^4*x^4+2*b^2*m^2*x^2+2*b^2*m*x^2-4*b^2*x^2+2*m^3+2*m^2-4
*m)/m/(2+m)/(-1+m)*sin(b*x)-2^(3-m)/Pi^(1/2)/(-2+m)*x^(-3+m)/b^3*(b^2)^(1/2
*m)*(b^2*x^2-m^2+m)/m/(-1+m)*(cos(b*x)*x*b-sin(b*x))+2^(3-m)/Pi^(1/2)/(-2+m
)*x^(2+m)*b^2*(b^2)^(1/2*m)/m/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(3/2+m,3/
```

```
2,b*x)*sin(b*x)+2^(3-m)/Pi^(1/2)/(-2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)/m/(-1+m)*
(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(1/2+m,1/2,b*x))*cos(a)-2^(-
3+m)*b^(2-m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-1+m)*x^(-2+m)*b^(-2+m)*(-2*b^2*x^
2+2*m^2-2*m-4)/(1+m)/(-2+m)*sin(b*x)+2^(3-m)/Pi^(1/2)/(-1+m)*x^(-2+m)*b^(-2
+m)*(b^2*x^2-m^2-m)/(1+m)/(-2+m)/m*(cos(b*x)*x*b-sin(b*x))+2^(3-m)/Pi^(1/2)
/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(-2+m)*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2,b*x)
*sin(b*x)-2^(3-m)/Pi^(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(-2+m)/m*(b*x)^(-5/
2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(3/2+m,1/2,b*x))*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int x^{-3+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m-3)\log(i\,b)-i\,a)}\Gamma(m-2,i\,bx) - i\,e^{(-(m-3)\log(-i\,b)+i\,a)}\Gamma(m-2,-i\,bx)}{2\,b}$$

```
[In] integrate(x^(-3+m)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(-(m - 3)*log(I*b) - I*a)*gamma(m - 2, I*b*x) - I*e^(-(m - 3)*log(
-I*b) + I*a)*gamma(m - 2, -I*b*x))/b
```

Sympy [F]

$$\int x^{-3+m}\cos(a+bx)\,dx = \int x^{m-3}\cos\left(a+bx\right)\,dx$$

```
[In] integrate(x**(-3+m)*cos(b*x+a),x)
```

```
[Out] Integral(x**(m - 3)*cos(a + b*x), x)
```

Maxima [F]

$$\int x^{-3+m}\cos(a+bx)\,dx = \int x^{m-3}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(-3+m)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 3)*cos(b*x + a), x)
```

Giac [F]

$$\int x^{-3+m}\cos(a+bx)\,dx = \int x^{m-3}\cos\left(bx+a\right)\,dx$$

```
[In] integrate(x^(-3+m)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 3)*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\cos(a+bx)\,dx = \int x^{m-3}\,\cos\left(a+b\,x\right)\,dx$$

```
[In] int(x^(m - 3)*cos(a + b*x),x)
[Out] int(x^(m - 3)*cos(a + b*x), x)
```

3.111 $\int x^{3+m}\cos^2(a+bx)\,dx$

Optimal result . . . 717
Rubi [A] (verified) . . . 717
Mathematica [A] (verified) . . . 718
Maple [F] . . . 719
Fricas [A] (verification not implemented) . . . 719
Sympy [F] . . . 719
Maxima [F] . . . 719
Giac [F] . . . 720
Mupad [F(-1)] . . . 720

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int x^{3+m}\cos^2(a+bx)\,dx = \frac{x^{4+m}}{2(4+m)} - \frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4} - \frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(4+m,2ibx)}{b^4}$$

```
[Out] 1/2*x^(4+m)/(4+m)-2^(-6-m)*exp(2*I*a)*x^m*GAMMA(4+m,-2*I*b*x)/b^4/((-I*b*x)
^m)-2^(-6-m)*x^m*GAMMA(4+m,2*I*b*x)/b^4/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$\int x^{3+m}\cos^2(a+bx)\,dx = -\frac{e^{2ia}2^{-m-6}x^m(-ibx)^{-m}\Gamma(m+4,-2ibx)}{b^4} - \frac{e^{-2ia}2^{-m-6}x^m(ibx)^{-m}\Gamma(m+4,2ibx)}{b^4} + \frac{x^{m+4}}{2(m+4)}$$

```
[In] Int[x^(3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^((2*I)*a)*x^m*Gamma[4 + m, (-2*I)*b*x
])/(b^4*((-I)*b*x)^m) - (2^(-6 - m)*x^m*Gamma[4 + m, (2*I)*b*x])/(b^4*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^{3+m}}{2} + \frac{1}{2}x^{3+m}\cos(2a+2bx)\right)\,dx \\
&= \frac{x^{4+m}}{2(4+m)} + \frac{1}{2}\int x^{3+m}\cos(2a+2bx)\,dx \\
&= \frac{x^{4+m}}{2(4+m)} + \frac{1}{4}\int e^{-i(2a+2bx)}x^{3+m}\,dx + \frac{1}{4}\int e^{i(2a+2bx)}x^{3+m}\,dx \\
&= \frac{x^{4+m}}{2(4+m)} - \frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4} - \frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(4+m,2ibx)}{b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int x^{3+m}\cos^2(a+bx)\,dx = \frac{1}{64}x^m\left(\frac{32x^4}{4+m} - \frac{2^{-m}e^{2ia}(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4} - \frac{2^{-m}e^{-2ia}(ibx)^{-m}\Gamma(4+m,2ibx)}{b^4}\right)$$

```
[In] Integrate[x^(3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((32*x^4)/(4 + m) - (E^((2*I)*a)*Gamma[4 + m, (-2*I)*b*x])/(2^m*b^4*((
-I)*b*x)^m) - Gamma[4 + m, (2*I)*b*x]/(2^m*b^4*E^((2*I)*a)*(I*b*x)^m)))/64
```

Maple [F]

$$\int x^{3+m}\left(\cos^2\left(bx+a\right)\right)\,dx$$

```
[In] int(x^(3+m)*cos(b*x+a)^2,x)
[Out] int(x^(3+m)*cos(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int x^{3+m}\cos^2(a+bx)\,dx$$
$$=\frac{4\,bxx^{m+3}+(i\,m+4i)e^{(-(m+3)\log(2i\,b)-2i\,a)}\Gamma(m+4,2i\,bx)+(-i\,m-4i)e^{(-(m+3)\log(-2i\,b)+2i\,a)}\Gamma(m+4,-2i}{8\,(bm+4\,b)}$$

```
[In] integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(4*b*x*x^(m + 3) + (I*m + 4*I)*e^(-(m + 3)*log(2*I*b) - 2*I*a)*gamma(m
+ 4, 2*I*b*x) + (-I*m - 4*I)*e^(-(m + 3)*log(-2*I*b) + 2*I*a)*gamma(m + 4,
-2*I*b*x))/(b*m + 4*b)
```

Sympy [F]

$$\int x^{3+m}\cos^2(a+bx)\,dx=\int x^{m+3}\cos^2\left(a+bx\right)\,dx$$

```
[In] integrate(x**(3+m)*cos(b*x+a)**2,x)
[Out] Integral(x**(m + 3)*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{3+m}\cos^2(a+bx)\,dx=\int x^{m+3}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((m + 4)*integrate(x^3*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 4*log(x
)))/(m + 4)
```

Giac [F]

$$\int x^{3+m}\cos^2(a+bx)\,dx = \int x^{m+3}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^(m + 3)*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\cos^2(a+bx)\,dx = \int x^{m+3}\cos\left(a+b\,x\right)^2\,dx$$

```
[In] int(x^(m + 3)*cos(a + b*x)^2,x)
[Out] int(x^(m + 3)*cos(a + b*x)^2, x)
```

3.112 $\int x^{2+m}\cos^2(a+bx)\,dx$

Optimal result . . . 721
Rubi [A] (verified) . . . 721
Mathematica [A] (verified) . . . 722
Maple [F] . . . 723
Fricas [A] (verification not implemented) . . . 723
Sympy [F] . . . 723
Maxima [F] . . . 723
Giac [F] . . . 724
Mupad [F(-1)] . . . 724

Optimal result

Integrand size = 14, antiderivative size = 103

$$\int x^{2+m}\cos^2(a+bx)\,dx = \frac{x^{3+m}}{2(3+m)} + \frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3} - \frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(3+m,2ibx)}{b^3}$$

```
[Out] 1/2*x^(3+m)/(3+m)+I*2^(-5-m)*exp(2*I*a)*x^m*GAMMA(3+m,-2*I*b*x)/b^3/((-I*b*
x)^m)-I*2^(-5-m)*x^m*GAMMA(3+m,2*I*b*x)/b^3/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$\int x^{2+m}\cos^2(a+bx)\,dx = \frac{ie^{2ia}2^{-m-5}x^m(-ibx)^{-m}\Gamma(m+3,-2ibx)}{b^3} - \frac{ie^{-2ia}2^{-m-5}x^m(ibx)^{-m}\Gamma(m+3,2ibx)}{b^3} + \frac{x^{m+3}}{2(m+3)}$$

```
[In] Int[x^(2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^(3 + m)/(2*(3 + m)) + (I*2^(-5 - m)*E^((2*I)*a)*x^m*Gamma[3 + m, (-2*I)*b
*x])/(b^3*((-I)*b*x)^m) - (I*2^(-5 - m)*x^m*Gamma[3 + m, (2*I)*b*x])/(b^3*E
^((2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^{2+m}}{2} + \frac{1}{2}x^{2+m}\cos(2a+2bx)\right)\,dx \\
&= \frac{x^{3+m}}{2(3+m)} + \frac{1}{2}\int x^{2+m}\cos(2a+2bx)\,dx \\
&= \frac{x^{3+m}}{2(3+m)} + \frac{1}{4}\int e^{-i(2a+2bx)}x^{2+m}\,dx + \frac{1}{4}\int e^{i(2a+2bx)}x^{2+m}\,dx \\
&= \frac{x^{3+m}}{2(3+m)} + \frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3} - \frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(3+m,2ibx)}{b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int x^{2+m}\cos^2(a+bx)\,dx = \frac{1}{32}x^m\left(\frac{16x^3}{3+m} + \frac{i2^{-m}e^{2ia}(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3} - \frac{i2^{-m}e^{-2ia}(ibx)^{-m}\Gamma(3+m,2ibx)}{b^3}\right)$$

```
[In] Integrate[x^(2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((16*x^3)/(3 + m) + (I*E^((2*I)*a)*Gamma[3 + m, (-2*I)*b*x])/(2^m*b^3*
((-I)*b*x)^m) - (I*Gamma[3 + m, (2*I)*b*x])/(2^m*b^3*E^((2*I)*a)*(I*b*x)^m)
))/32
```

Maple [F]

$$\int x^{2+m}\left(\cos^2\left(bx+a\right)\right)\,dx$$

```
[In] int(x^(2+m)*cos(b*x+a)^2,x)
[Out] int(x^(2+m)*cos(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int x^{2+m}\cos^2(a+bx)\,dx$$
$$=\frac{4\,bxx^{m+2}+(i\,m+3i)e^{(-(m+2)\log(2i\,b)-2i\,a)}\Gamma(m+3,2i\,bx)+(-i\,m-3i)e^{(-(m+2)\log(-2i\,b)+2i\,a)}\Gamma(m+3,-2i}{8\,(bm+3\,b)}$$

```
[In] integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(4*b*x*x^(m + 2) + (I*m + 3*I)*e^(-(m + 2)*log(2*I*b) - 2*I*a)*gamma(m
+ 3, 2*I*b*x) + (-I*m - 3*I)*e^(-(m + 2)*log(-2*I*b) + 2*I*a)*gamma(m + 3,
-2*I*b*x))/(b*m + 3*b)
```

Sympy [F]

$$\int x^{2+m}\cos^2(a+bx)\,dx=\int x^{m+2}\cos^2\left(a+bx\right)\,dx$$

```
[In] integrate(x**(2+m)*cos(b*x+a)**2,x)
[Out] Integral(x**(m + 2)*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{2+m}\cos^2(a+bx)\,dx=\int x^{m+2}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((m + 3)*integrate(x^2*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 3*log(x
)))/(m + 3)
```

Giac [F]

$$\int x^{2+m}\cos^2(a+bx)\,dx = \int x^{m+2}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^(m + 2)*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2+m}\cos^2(a+bx)\,dx = \int x^{m+2}\cos\left(a+b\,x\right)^2\,dx$$

```
[In] int(x^(m + 2)*cos(a + b*x)^2,x)
[Out] int(x^(m + 2)*cos(a + b*x)^2, x)
```

3.113 $\int x^{1+m}\cos^2(a+bx)\,dx$

Optimal result . . . 725
Rubi [A] (verified) . . . 725
Mathematica [A] (verified) . . . 726
Maple [F] . . . 727
Fricas [A] (verification not implemented) . . . 727
Sympy [F] . . . 727
Maxima [F] . . . 727
Giac [F] . . . 728
Mupad [F(-1)] . . . 728

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int x^{1+m}\cos^2(a+bx)\,dx = \frac{x^{2+m}}{2(2+m)} + \frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2} + \frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(2+m,2ibx)}{b^2}$$

```
[Out] 1/2*x^(2+m)/(2+m)+2^(-4-m)*exp(2*I*a)*x^m*GAMMA(2+m,-2*I*b*x)/b^2/((-I*b*x)
^m)+2^(-4-m)*x^m*GAMMA(2+m,2*I*b*x)/b^2/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$\int x^{1+m}\cos^2(a+bx)\,dx = \frac{e^{2ia}2^{-m-4}x^m(-ibx)^{-m}\Gamma(m+2,-2ibx)}{b^2} + \frac{e^{-2ia}2^{-m-4}x^m(ibx)^{-m}\Gamma(m+2,2ibx)}{b^2} + \frac{x^{m+2}}{2(m+2)}$$

```
[In] Int[x^(1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^(2 + m)/(2*(2 + m)) + (2^(-4 - m)*E^((2*I)*a)*x^m*Gamma[2 + m, (-2*I)*b*x
])/(b^2*((-I)*b*x)^m) + (2^(-4 - m)*x^m*Gamma[2 + m, (2*I)*b*x])/(b^2*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^{1+m}}{2} + \frac{1}{2}x^{1+m}\cos(2a+2bx)\right)\,dx \\
&= \frac{x^{2+m}}{2(2+m)} + \frac{1}{2}\int x^{1+m}\cos(2a+2bx)\,dx \\
&= \frac{x^{2+m}}{2(2+m)} + \frac{1}{4}\int e^{-i(2a+2bx)}x^{1+m}\,dx + \frac{1}{4}\int e^{i(2a+2bx)}x^{1+m}\,dx \\
&= \frac{x^{2+m}}{2(2+m)} + \frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2} + \frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(2+m,2ibx)}{b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^{1+m}\cos^2(a+bx)\,dx = \frac{1}{16}x^m\left(\frac{8x^2}{2+m} + \frac{2^{-m}e^{2ia}(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2} + \frac{2^{-m}e^{-2ia}(ibx)^{-m}\Gamma(2+m,2ibx)}{b^2}\right)$$

```
[In] Integrate[x^(1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((8*x^2)/(2 + m) + (E^((2*I)*a)*Gamma[2 + m, (-2*I)*b*x])/(2^m*b^2*((-
I)*b*x)^m) + Gamma[2 + m, (2*I)*b*x]/(2^m*b^2*E^((2*I)*a)*(I*b*x)^m)))/16
```

Maple [F]

$$\int x^{1+m}\left(\cos^2\left(bx+a\right)\right)\,dx$$

```
[In] int(x^(1+m)*cos(b*x+a)^2,x)
[Out] int(x^(1+m)*cos(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int x^{1+m}\cos^2(a+bx)\,dx$$
$$=\frac{4\,bxx^{m+1}+(i\,m+2i)e^{(-(m+1)\log(2i\,b)-2i\,a)}\Gamma(m+2,2i\,bx)+(-i\,m-2i)e^{(-(m+1)\log(-2i\,b)+2i\,a)}\Gamma(m+2,-2i}{8\,(bm+2\,b)}$$

```
[In] integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(4*b*x*x^(m + 1) + (I*m + 2*I)*e^(-(m + 1)*log(2*I*b) - 2*I*a)*gamma(m
+ 2, 2*I*b*x) + (-I*m - 2*I)*e^(-(m + 1)*log(-2*I*b) + 2*I*a)*gamma(m + 2,
-2*I*b*x))/(b*m + 2*b)
```

Sympy [F]

$$\int x^{1+m}\cos^2(a+bx)\,dx=\int x^{m+1}\cos^2\left(a+bx\right)\,dx$$

```
[In] integrate(x**(1+m)*cos(b*x+a)**2,x)
[Out] Integral(x**(m + 1)*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{1+m}\cos^2(a+bx)\,dx=\int x^{m+1}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((m + 2)*integrate(x*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 2*log(x))
)/(m + 2)
```

Giac [F]

$$\int x^{1+m}\cos^2(a+bx)\,dx = \int x^{m+1}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^(m + 1)*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\cos^2(a+bx)\,dx = \int x^{m+1}\cos\left(a+b\,x\right)^2\,dx$$

```
[In] int(x^(m + 1)*cos(a + b*x)^2,x)
[Out] int(x^(m + 1)*cos(a + b*x)^2, x)
```

3.114 $\int x^m \cos^2(a+bx)\,dx$

Optimal result . . . 729
Rubi [A] (verified) . . . 729
Mathematica [A] (verified) . . . 730
Maple [F] . . . 731
Fricas [A] (verification not implemented) . . . 731
Sympy [F] . . . 731
Maxima [F] . . . 731
Giac [F] . . . 732
Mupad [F(-1)] . . . 732

Optimal result

Integrand size = 12, antiderivative size = 103

$$\int x^m \cos^2(a+bx)\,dx = \frac{x^{1+m}}{2(1+m)} - \frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(1+m,-2ibx)}{b} + \frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(1+m,2ibx)}{b}$$

```
[Out] 1/2*x^(1+m)/(1+m)-I*2^(-3-m)*exp(2*I*a)*x^m*GAMMA(1+m,-2*I*b*x)/b/((-I*b*x)
^m)+I*2^(-3-m)*x^m*GAMMA(1+m,2*I*b*x)/b/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3388, 2212}

$$\int x^m \cos^2(a+bx)\,dx = -\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\Gamma(m+1,-2ibx)}{b} + \frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\Gamma(m+1,2ibx)}{b} + \frac{x^{m+1}}{2(m+1)}$$

```
[In] Int[x^m*Cos[a + b*x]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamma[1 + m, (-2*I)*b
*x])/(b*((-I)*b*x)^m) + (I*2^(-3 - m)*x^m*Gamma[1 + m, (2*I)*b*x])/(b*E^((2
*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^m}{2} + \frac{1}{2}x^m \cos(2a+2bx)\right)\,dx \\
&= \frac{x^{1+m}}{2(1+m)} + \frac{1}{2}\int x^m \cos(2a+2bx)\,dx \\
&= \frac{x^{1+m}}{2(1+m)} + \frac{1}{4}\int e^{-i(2a+2bx)}x^m\,dx + \frac{1}{4}\int e^{i(2a+2bx)}x^m\,dx \\
&= \frac{x^{1+m}}{2(1+m)} - \frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(1+m,-2ibx)}{b} + \frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(1+m,2ibx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int x^m \cos^2(a+bx)\,dx = \frac{1}{8}x^m\left(\frac{4x}{1+m} - 2^{-m}e^{2ia}x(-ibx)^{-1-m}\Gamma(1+m,-2ibx) - 2^{-m}e^{-2ia}x(ibx)^{-1-m}\Gamma(1+m,2ibx)\right)$$

```
[In] Integrate[x^m*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*((4*x)/(1 + m) - (E^((2*I)*a)*x*((-I)*b*x)^(-1 - m)*Gamma[1 + m, (-2*I
)*b*x])/2^m - (x*(I*b*x)^(-1 - m)*Gamma[1 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)
)))/8
```

Maple [F]

$$\int x^m \left(\cos^2\left(bx+a\right)\right) dx$$

```
[In] int(x^m*cos(b*x+a)^2,x)
[Out] int(x^m*cos(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.67

$$\int x^m \cos^2(a+bx)\, dx$$
$$= \frac{4\, bx x^m + (i\, m + i) e^{(-m \log(2 i\, b) - 2 i\, a)} \Gamma(m+1, 2 i\, bx) + (-i\, m - i) e^{(-m \log(-2 i\, b) + 2 i\, a)} \Gamma(m+1, -2 i\, bx)}{8\, (bm + b)}$$

```
[In] integrate(x^m*cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(4*b*x*x^m + (I*m + I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x)
+ (-I*m - I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))/(b*m + b)
```

Sympy [F]

$$\int x^m \cos^2(a+bx)\, dx = \int x^m \cos^2\left(a+bx\right) dx$$

```
[In] integrate(x**m*cos(b*x+a)**2,x)
[Out] Integral(x**m*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int x^m \cos^2(a+bx)\, dx = \int x^m \cos\left(bx+a\right)^2\, dx$$

```
[In] integrate(x^m*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + log(x)))/(m
 + 1)
```

Giac [F]

$$\int x^m \cos^2(a+bx)\,dx = \int x^m \cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^m*cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cos^2(a+bx)\,dx = \int x^m \cos\left(a+b\,x\right)^2\,dx$$

```
[In] int(x^m*cos(a + b*x)^2,x)
[Out] int(x^m*cos(a + b*x)^2, x)
```

3.115 $\int x^{-1+m}\cos^2(a+bx)\,dx$

Optimal result . . . 733
Rubi [A] (verified) . . . 733
Mathematica [A] (verified) . . . 734
Maple [F] . . . 735
Fricas [A] (verification not implemented) . . . 735
Sympy [F] . . . 735
Maxima [F] . . . 735
Giac [F] . . . 736
Mupad [F(-1)] . . . 736

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int x^{-1+m}\cos^2(a+bx)\,dx = \frac{x^m}{2m} - 2^{-2-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(m,-2ibx) - 2^{-2-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(m,2ibx)$$

```
[Out] 1/2*x^m/m-2^(-2-m)*exp(2*I*a)*x^m*GAMMA(m,-2*I*b*x)/((-I*b*x)^m)-2^(-2-m)*x
^m*GAMMA(m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$\int x^{-1+m}\cos^2(a+bx)\,dx = e^{2ia}\left(-2^{-m-2}\right)x^m(-ibx)^{-m}\Gamma(m,-2ibx) - e^{-2ia}2^{-m-2}x^m(ibx)^{-m}\Gamma(m,2ibx) + \frac{x^m}{2m}$$

```
[In] Int[x^(-1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] x^m/(2*m) - (2^(-2 - m)*E^((2*I)*a)*x^m*Gamma[m, (-2*I)*b*x])/((-I)*b*x)^m
- (2^(-2 - m)*x^m*Gamma[m, (2*I)*b*x])/(E^((2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
```

```
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{x^{-1+m}}{2} + \frac{1}{2}x^{-1+m}\cos(2a+2bx)\right)dx \\ &= \frac{x^m}{2m} + \frac{1}{2}\int x^{-1+m}\cos(2a+2bx)\,dx \\ &= \frac{x^m}{2m} + \frac{1}{4}\int e^{-i(2a+2bx)}x^{-1+m}\,dx + \frac{1}{4}\int e^{i(2a+2bx)}x^{-1+m}\,dx \\ &= \frac{x^m}{2m} - 2^{-2-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(m,-2ibx) - 2^{-2-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(m,2ibx)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{-1+m}\cos^2(a+bx)\,dx = \frac{1}{4}x^m\left(\frac{2}{m} - 2^{-m}e^{2ia}(-ibx)^{-m}\Gamma(m,-2ibx) - 2^{-m}e^{-2ia}(ibx)^{-m}\Gamma(m,2ibx)\right)$$

```
[In] Integrate[x^(-1 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*(2/m - (E^((2*I)*a)*Gamma[m, (-2*I)*b*x])/(2^m*((-I)*b*x)^m) - Gamma[m
, (2*I)*b*x]/(2^m*E^((2*I)*a)*(I*b*x)^m)))/4
```

Maple [F]

$$\int x^{-1+m}\left(\cos^2(bx+a)\right)\,dx$$

```
[In] int(x^(-1+m)*cos(b*x+a)^2,x)
[Out] int(x^(-1+m)*cos(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int x^{-1+m}\cos^2(a+bx)\,dx = \frac{4\,bxx^{m-1}+i\,me^{(-(m-1)\log(2i\,b)-2i\,a)}\Gamma(m,2i\,bx)-i\,me^{(-(m-1)\log(-2i\,b)+2i\,a)}\Gamma(m,-2i\,bx)}{8\,bm}$$

```
[In] integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(4*b*x*x^(m - 1) + I*m*e^(-(m - 1)*log(2*I*b) - 2*I*a)*gamma(m, 2*I*b*x
) - I*m*e^(-(m - 1)*log(-2*I*b) + 2*I*a)*gamma(m, -2*I*b*x))/(b*m)
```

Sympy [F]

$$\int x^{-1+m}\cos^2(a+bx)\,dx = \int x^{m-1}\cos^2(a+bx)\,dx$$

```
[In] integrate(x**(-1+m)*cos(b*x+a)**2,x)
[Out] Integral(x**(m - 1)*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{-1+m}\cos^2(a+bx)\,dx = \int x^{m-1}\cos(bx+a)^2\,dx$$

```
[In] integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*(m*integrate(x^m*cos(2*b*x + 2*a)/x, x) + x^m)/m
```

Giac [F]

$$\int x^{-1+m}\cos^2(a+bx)\,dx = \int x^{m-1}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^(m - 1)*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\cos^2(a+bx)\,dx = \int x^{m-1}\cos\left(a+b\,x\right)^2\,dx$$

```
[In] int(x^(m - 1)*cos(a + b*x)^2,x)
[Out] int(x^(m - 1)*cos(a + b*x)^2, x)
```

3.116 $\int x^{-2+m}\cos^2(a+bx)\,dx$

Optimal result . . . 737
Rubi [A] (verified) . . . 737
Mathematica [A] (verified) . . . 738
Maple [F] . . . 739
Fricas [A] (verification not implemented) . . . 739
Sympy [F] . . . 739
Maxima [F] . . . 739
Giac [F] . . . 740
Mupad [F(-1)] . . . 740

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int x^{-2+m}\cos^2(a+bx)\,dx = -\frac{x^{-1+m}}{2(1-m)} + i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\Gamma(-1+m,-2ibx) - i2^{-1-m}be^{-2ia}x^m(ibx)^{-m}\Gamma(-1+m,2ibx)$$

```
[Out] -1/2*x^(-1+m)/(1-m)+I*2^(-1-m)*b*exp(2*I*a)*x^m*GAMMA(-1+m,-2*I*b*x)/((-I*b
*x)^m)-I*2^(-1-m)*b*x^m*GAMMA(-1+m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$\int x^{-2+m}\cos^2(a+bx)\,dx = ie^{2ia}b2^{-m-1}x^m(-ibx)^{-m}\Gamma(m-1,-2ibx) - ie^{-2ia}b2^{-m-1}x^m(ibx)^{-m}\Gamma(m-1,2ibx) - \frac{x^{m-1}}{2(1-m)}$$

```
[In] Int[x^(-2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-1 + m)/(1 - m) + (I*2^(-1 - m)*b*E^((2*I)*a)*x^m*Gamma[-1 + m, (-2
*I)*b*x])/((-I)*b*x)^m - (I*2^(-1 - m)*b*x^m*Gamma[-1 + m, (2*I)*b*x])/(E^(
(2*I)*a)*(I*b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^{-2+m}}{2} + \frac{1}{2}x^{-2+m}\cos(2a+2bx)\right)\,dx \\
&= -\frac{x^{-1+m}}{2(1-m)} + \frac{1}{2}\int x^{-2+m}\cos(2a+2bx)\,dx \\
&= -\frac{x^{-1+m}}{2(1-m)} + \frac{1}{4}\int e^{-i(2a+2bx)}x^{-2+m}\,dx + \frac{1}{4}\int e^{i(2a+2bx)}x^{-2+m}\,dx \\
&= -\frac{x^{-1+m}}{2(1-m)} + i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\Gamma(-1+m,-2ibx) \\
&\quad - i2^{-1-m}be^{-2ia}x^m(ibx)^{-m}\Gamma(-1+m,2ibx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int x^{-2+m}\cos^2(a+bx)\,dx = \frac{1}{2}x^m\left(\frac{1}{(-1+m)x} + i2^{-m}be^{2ia}(-ibx)^{-m}\Gamma(-1+m,-2ibx) - i2^{-m}be^{-2ia}(ibx)^{-m}\Gamma(-1+m,2ibx)\right)$$

```
[In] Integrate[x^(-2 + m)*Cos[a + b*x]^2,x]
```

```
[Out] (x^m*(1/((-1 + m)*x) + (I*b*E^((2*I)*a)*Gamma[-1 + m, (-2*I)*b*x])/(2^m*((-
I)*b*x)^m) - (I*b*Gamma[-1 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*b*x)^m)))/2
```

Maple [F]

$$\int x^{-2+m}\left(\cos^2\left(bx+a\right)\right)\,dx$$

```
[In] int(x^(-2+m)*cos(b*x+a)^2,x)
[Out] int(x^(-2+m)*cos(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int x^{-2+m}\cos^2(a+bx)\,dx$$
$$=\frac{4\,bxx^{m-2}+(i\,m-i)e^{(-(m-2)\log(2i\,b)-2i\,a)}\Gamma(m-1,2i\,bx)+(-i\,m+i)e^{(-(m-2)\log(-2i\,b)+2i\,a)}\Gamma(m-1,-2i\,bx}{8\,(bm-b)}$$

```
[In] integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(4*b*x*x^(m - 2) + (I*m - I)*e^(-(m - 2)*log(2*I*b) - 2*I*a)*gamma(m -
1, 2*I*b*x) + (-I*m + I)*e^(-(m - 2)*log(-2*I*b) + 2*I*a)*gamma(m - 1, -2*I
*b*x))/(b*m - b)
```

Sympy [F]

$$\int x^{-2+m}\cos^2(a+bx)\,dx=\int x^{m-2}\cos^2\left(a+bx\right)\,dx$$

```
[In] integrate(x**(-2+m)*cos(b*x+a)**2,x)
[Out] Integral(x**(m - 2)*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{-2+m}\cos^2(a+bx)\,dx=\int x^{m-2}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((m - 1)*x*integrate(x^m*cos(2*b*x + 2*a)/x^2, x) + x^m)/((m - 1)*x)
```

Giac [F]

$$\int x^{-2+m}\cos^2(a+bx)\,dx = \int x^{m-2}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^(m - 2)*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\cos^2(a+bx)\,dx = \int x^{m-2}\cos\left(a+b\,x\right)^2\,dx$$

```
[In] int(x^(m - 2)*cos(a + b*x)^2,x)
[Out] int(x^(m - 2)*cos(a + b*x)^2, x)
```

3.117 $\int x^{-3+m}\cos^2(a+bx)\,dx$

Optimal result . . . 741
Rubi [A] (verified) . . . 741
Mathematica [A] (verified) . . . 742
Maple [F] . . . 743
Fricas [A] (verification not implemented) . . . 743
Sympy [F] . . . 743
Maxima [F] . . . 743
Giac [F] . . . 744
Mupad [F(-1)] . . . 744

Optimal result

Integrand size = 14, antiderivative size = 95

$$\int x^{-3+m}\cos^2(a+bx)\,dx = -\frac{x^{-2+m}}{2(2-m)} + 2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx) + 2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\Gamma(-2+m,2ibx)$$

```
[Out] -1/2*x^(-2+m)/(2-m)+b^2*exp(2*I*a)*x^m*GAMMA(-2+m,-2*I*b*x)/(2^m)/((-I*b*x)
^m)+b^2*x^m*GAMMA(-2+m,2*I*b*x)/(2^m)/exp(2*I*a)/((I*b*x)^m)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$\int x^{-3+m}\cos^2(a+bx)\,dx = e^{2ia}b^2 2^{-m}x^m(-ibx)^{-m}\Gamma(m-2,-2ibx) + e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\Gamma(m-2,2ibx) - \frac{x^{m-2}}{2(2-m)}$$

```
[In] Int[x^(-3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-2 + m)/(2 - m) + (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) + (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^{-3+m}}{2} + \frac{1}{2}x^{-3+m}\cos(2a+2bx)\right)\,dx \\
&= -\frac{x^{-2+m}}{2(2-m)} + \frac{1}{2}\int x^{-3+m}\cos(2a+2bx)\,dx \\
&= -\frac{x^{-2+m}}{2(2-m)} + \frac{1}{4}\int e^{-i(2a+2bx)}x^{-3+m}\,dx + \frac{1}{4}\int e^{i(2a+2bx)}x^{-3+m}\,dx \\
&= -\frac{x^{-2+m}}{2(2-m)} + 2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx) \\
&\quad + 2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\Gamma(-2+m,2ibx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int x^{-3+m}\cos^2(a+bx)\,dx = &-\frac{x^{-2+m}}{2(2-m)} + 2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx) \\
&+ 2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\Gamma(-2+m,2ibx)
\end{aligned}$$

```
[In] Integrate[x^(-3 + m)*Cos[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-2 + m)/(2 - m) + (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/(
2^m*((-I)*b*x)^m) + (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*
b*x)^m)
```

Maple [F]

$$\int x^{-3+m}\left(\cos^2\left(bx+a\right)\right)\,dx$$

```
[In] int(x^(-3+m)*cos(b*x+a)^2,x)
[Out] int(x^(-3+m)*cos(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int x^{-3+m}\cos^2(a+bx)\,dx$$
$$=\frac{4\,bxx^{m-3}+(i\,m-2i)e^{(-(m-3)\log(2i\,b)-2i\,a)}\Gamma(m-2,2i\,bx)+(-i\,m+2i)e^{(-(m-3)\log(-2i\,b)+2i\,a)}\Gamma(m-2,-2i}{8\,(bm-2\,b)}$$

```
[In] integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(4*b*x*x^(m - 3) + (I*m - 2*I)*e^(-(m - 3)*log(2*I*b) - 2*I*a)*gamma(m
- 2, 2*I*b*x) + (-I*m + 2*I)*e^(-(m - 3)*log(-2*I*b) + 2*I*a)*gamma(m - 2,
-2*I*b*x))/(b*m - 2*b)
```

Sympy [F]

$$\int x^{-3+m}\cos^2(a+bx)\,dx=\int x^{m-3}\cos^2\left(a+bx\right)\,dx$$

```
[In] integrate(x**(-3+m)*cos(b*x+a)**2,x)
[Out] Integral(x**(m - 3)*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int x^{-3+m}\cos^2(a+bx)\,dx=\int x^{m-3}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((m - 2)*x^2*integrate(x^m*cos(2*b*x + 2*a)/x^3, x) + x^m)/((m - 2)*x^2
)
```

Giac [F]

$$\int x^{-3+m}\cos^2(a+bx)\,dx = \int x^{m-3}\cos\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^(m - 3)*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\cos^2(a+bx)\,dx = \int x^{m-3}\cos\left(a+b\,x\right)^2\,dx$$

```
[In] int(x^(m - 3)*cos(a + b*x)^2,x)
[Out] int(x^(m - 3)*cos(a + b*x)^2, x)
```

3.118 $\int (c+dx)^3(a+a\cos(e+fx))\,dx$

Optimal result . . . 745
Rubi [A] (verified) . . . 745
Mathematica [A] (verified) . . . 747
Maple [A] (verified) . . . 747
Fricas [A] (verification not implemented) . . . 748
Sympy [B] (verification not implemented) . . . 748
Maxima [B] (verification not implemented) . . . 749
Giac [A] (verification not implemented) . . . 749
Mupad [B] (verification not implemented) . . . 750

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx = \frac{a(c+dx)^4}{4d} - \frac{6ad^3\cos(e+fx)}{f^4} + \frac{3ad(c+dx)^2\cos(e+fx)}{f^2} - \frac{6ad^2(c+dx)\sin(e+fx)}{f^3} + \frac{a(c+dx)^3\sin(e+fx)}{f}$$

```
[Out] 1/4*a*(d*x+c)^4/d-6*a*d^3*cos(f*x+e)/f^4+3*a*d*(d*x+c)^2*cos(f*x+e)/f^2-6*a
*d^2*(d*x+c)*sin(f*x+e)/f^3+a*(d*x+c)^3*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2718}

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx = -\frac{6ad^2(c+dx)\sin(e+fx)}{f^3} + \frac{3ad(c+dx)^2\cos(e+fx)}{f^2} + \frac{a(c+dx)^3\sin(e+fx)}{f} + \frac{a(c+dx)^4}{4d} - \frac{6ad^3\cos(e+fx)}{f^4}$$

```
[In] Int[(c + d*x)^3*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*a*d^3*Cos[e + f*x])/f^4 + (3*a*d*(c + d*x)^2*Cos
[e + f*x])/f^2 - (6*a*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (a*(c + d*x)^3*Sin[
e + f*x])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a(c+dx)^3 + a(c+dx)^3\cos(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^4}{4d} + a\int (c+dx)^3\cos(e+fx)\,dx \\
&= \frac{a(c+dx)^4}{4d} + \frac{a(c+dx)^3\sin(e+fx)}{f} - \frac{(3ad)\int (c+dx)^2\sin(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^4}{4d} + \frac{3ad(c+dx)^2\cos(e+fx)}{f^2} \\
&\quad + \frac{a(c+dx)^3\sin(e+fx)}{f} - \frac{(6ad^2)\int (c+dx)\cos(e+fx)\,dx}{f^2} \\
&= \frac{a(c+dx)^4}{4d} + \frac{3ad(c+dx)^2\cos(e+fx)}{f^2} - \frac{6ad^2(c+dx)\sin(e+fx)}{f^3} \\
&\quad + \frac{a(c+dx)^3\sin(e+fx)}{f} + \frac{(6ad^3)\int \sin(e+fx)\,dx}{f^3} \\
&= \frac{a(c+dx)^4}{4d} - \frac{6ad^3\cos(e+fx)}{f^4} + \frac{3ad(c+dx)^2\cos(e+fx)}{f^2} \\
&\quad - \frac{6ad^2(c+dx)\sin(e+fx)}{f^3} + \frac{a(c+dx)^3\sin(e+fx)}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.37

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx$$
$$= a\left(\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+\frac{3d(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)}{f^4}\right.$$
$$\left.+\frac{(c+dx)\,(c^2f^2+2cdf^2x+d^2(-6+f^2x^2))\sin(e+fx)}{f^3}\right)$$

```
[In] Integrate[(c + d*x)^3*(a + a*Cos[e + f*x]),x]
```

```
[Out] a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + (3*d*(c^2*f^2 + 2*c*
d*f^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x])/f^4 + ((c + d*x)*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(-6 + f^2*x^2))*Sin[e + f*x])/f^3)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

method	result
parallelrisch	$\frac{\left((dx+c)\left((dx+c)^2f^2-6d^2\right)f\sin(fx+e)+3\left((dx+c)^2f^2-2d^2\right)d\cos(fx+e)+\left(\frac{dx}{2}+c\right)x\left(\frac{1}{2}x^2d^2+cdx+c^2\right)f^4-3c^2df^2+6$
risch	$\frac{a\,d^3x^4}{4}+ac\,d^2x^3+\frac{3adc^2x^2}{2}+a\,c^3x+\frac{a\,c^4}{4d}+\frac{3ad(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2)\cos(fx+e)}{f^4}+\frac{a(d^3f^2x^3+$
norman	$\frac{6a\,c^2d\,f^2-12a\,d^3}{f^4}+ac\,d^2x^3+ac\,d^2x^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{ac\left(c^2f^2+6d^2\right)x}{f^2}+\frac{ac\left(c^2f^2-6d^2\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a\,d^3x^4}{4}+\frac{a\,d^3x^4\left($
parts	$\frac{a(dx+c)^4}{4d}+\frac{a\left(\frac{d^3\left((fx+e)^3\sin(fx+e)+3(fx+e)^2\cos(fx+e)-6\cos(fx+e)-6(fx+e)\sin(fx+e)\right)}{f^3}+\frac{3c\,d^2\left((fx+e)^2\sin(fx+e)-2s\right.}{f}$
derivativedivides	$a\,c^3\sin(fx+e)-\frac{3a\,c^2de\sin(fx+e)}{f}+\frac{3a\,c^2d(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{3ac\,d^2e^2\sin(fx+e)}{f^2}-\frac{6ac\,d^2e(\cos(fx+e)+(fx+e)\sin$
default	$a\,c^3\sin(fx+e)-\frac{3a\,c^2de\sin(fx+e)}{f}+\frac{3a\,c^2d(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{3ac\,d^2e^2\sin(fx+e)}{f^2}-\frac{6ac\,d^2e(\cos(fx+e)+(fx+e)\sin$

```
[In] int((d*x+c)^3*(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x+c)*((d*x+c)^2*f^2-6*d^2)*f*sin(f*x+e)+3*((d*x+c)^2*f^2-2*d^2)*d*cos(f
*x+e)+(1/2*d*x+c)*x*(1/2*x^2*d^2+c*d*x+c^2)*f^4-3*c^2*d*f^2+6*d^3)*a/f^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx$$
$$= \frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x + 12\,(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 - 2\,ad^3)\cos(fx+e) +}{4\,f^4}$$

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
+ 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a*d^3)*cos(f*x + e)
 + 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 - 6*a*c*d^2*f + 3*(a*c^
2*d*f^3 - 2*a*d^3*f)*x)*sin(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(88) = 176.

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.97

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx$$
$$= \begin{cases} ac^3x + \frac{ac^3\sin(e+fx)}{f} + \frac{3ac^2dx^2}{2} + \frac{3ac^2dx\sin(e+fx)}{f} + \frac{3ac^2d\cos(e+fx)}{f^2} + acd^2x^3 + \frac{3acd^2x^2\sin(e+fx)}{f} + \frac{6acd^2x\cos(e+fx)}{f^2} \\ (a\cos(e)+a)\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

```
[In] integrate((d*x+c)**3*(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + a*c**3*sin(e + f*x)/f + 3*a*c**2*d*x**2/2 + 3*a*c**2*
d*x*sin(e + f*x)/f + 3*a*c**2*d*cos(e + f*x)/f**2 + a*c*d**2*x**3 + 3*a*c*d
**2*x**2*sin(e + f*x)/f + 6*a*c*d**2*x*cos(e + f*x)/f**2 - 6*a*c*d**2*sin(e
 + f*x)/f**3 + a*d**3*x**4/4 + a*d**3*x**3*sin(e + f*x)/f + 3*a*d**3*x**2*c
os(e + f*x)/f**2 - 6*a*d**3*x*sin(e + f*x)/f**3 - 6*a*d**3*cos(e + f*x)/f**
4, Ne(f, 0)), ((a*cos(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**
3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(87) = 174$.

Time = 0.31 (sec) , antiderivative size = 456, normalized size of antiderivative = 5.12

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx$$

$$= \frac{4\,(fx+e)ac^3 + \frac{(fx+e)^4ad^3}{f^3} - \frac{4\,(fx+e)^3ad^3e}{f^3} + \frac{6\,(fx+e)^2ad^3e^2}{f^3} - \frac{4\,(fx+e)ad^3e^3}{f^3} + \frac{4\,(fx+e)^3acd^2}{f^2} - \frac{12\,(fx+e)^2acd^2e}{f^2} + \frac{12\,(}{}$$

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 - 4*(f*x + e)^3*a*d^3*e/f^3
+ 6*(f*x + e)^2*a*d^3*e^2/f^3 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*(f*x + e)^3*a
*c*d^2/f^2 - 12*(f*x + e)^2*a*c*d^2*e/f^2 + 12*(f*x + e)*a*c*d^2*e^2/f^2 +
6*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)*a*c^2*d*e/f + 4*a*c^3*sin(f*x + e) -
 4*a*d^3*e^3*sin(f*x + e)/f^3 + 12*a*c*d^2*e^2*sin(f*x + e)/f^2 - 12*a*c^2*
d*e*sin(f*x + e)/f + 12*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*d^3*e^2/f
^3 - 24*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*c*d^2*e/f^2 + 12*((f*x +
e)*sin(f*x + e) + cos(f*x + e))*a*c^2*d/f - 12*(2*(f*x + e)*cos(f*x + e) +
((f*x + e)^2 - 2)*sin(f*x + e))*a*d^3*e/f^3 + 12*(2*(f*x + e)*cos(f*x + e)
+ ((f*x + e)^2 - 2)*sin(f*x + e))*a*c*d^2/f^2 + 4*(3*((f*x + e)^2 - 2)*cos(
f*x + e) + ((f*x + e)^3 - 6*f*x - 6*e)*sin(f*x + e))*a*d^3/f^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.73

$$\begin{aligned}
&\int (c+dx)^3(a+a\cos(e+fx))\,dx \\
&= \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x \\
&\quad + \frac{3\,(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 - 2\,ad^3)\cos(fx+e)}{f^4} \\
&\quad + \frac{(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x + ac^3f^3 - 6\,ad^3fx - 6\,acd^2f)\sin(fx+e)}{f^4}
\end{aligned}$$

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3*(a*d^3*f^2*x^2
+ 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a*d^3)*cos(f*x + e)/f^4 + (a*d^3*f^3*x^
3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + a*c^3*f^3 - 6*a*d^3*f*x - 6*a*c*d
^2*f)*sin(f*x + e)/f^4
```

Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.12

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx = \frac{\sin(e+fx)\,(a\,c^3\,f^2-6\,a\,c\,d^2)}{f^3} - \frac{3\,\cos(e+fx)\,(2\,a\,d^3-a\,c^2\,d\,f^2)}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x - \frac{3\,x\,\sin(e+fx)\,(2\,a\,d^3-a\,c^2\,d\,f^2)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 + \frac{3\,a\,d^3\,x^2\,\cos(e+fx)}{f^2} + \frac{a\,d^3\,x^3\,\sin(e+fx)}{f} + \frac{6\,a\,c\,d^2\,x\,\cos(e+fx)}{f^2} + \frac{3\,a\,c\,d^2\,x^2\,\sin(e+fx)}{f}$$

```
[In] int((a + a*cos(e + f*x))*(c + d*x)^3,x)
```

```
[Out] (sin(e + f*x)*(a*c^3*f^2 - 6*a*c*d^2))/f^3 - (3*cos(e + f*x)*(2*a*d^3 - a*c
^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x - (3*x*sin(e + f*x)*(2*a*d^3 - a*c
^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (3*a*d^3*x^2*cos(e + f*x
))/f^2 + (a*d^3*x^3*sin(e + f*x))/f + (6*a*c*d^2*x*cos(e + f*x))/f^2 + (3*a
*c*d^2*x^2*sin(e + f*x))/f
```

3.119 $\int (c+dx)^2(a+a\cos(e+fx))\,dx$

Optimal result . . . 751
Rubi [A] (verified) . . . 751
Mathematica [A] (verified) . . . 752
Maple [A] (verified) . . . 753
Fricas [A] (verification not implemented) . . . 753
Sympy [B] (verification not implemented) . . . 754
Maxima [B] (verification not implemented) . . . 754
Giac [A] (verification not implemented) . . . 755
Mupad [B] (verification not implemented) . . . 755

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx = \frac{a(c+dx)^3}{3d} + \frac{2ad(c+dx)\cos(e+fx)}{f^2} - \frac{2ad^2\sin(e+fx)}{f^3} + \frac{a(c+dx)^2\sin(e+fx)}{f}$$

```
[Out] 1/3*a*(d*x+c)^3/d+2*a*d*(d*x+c)*cos(f*x+e)/f^2-2*a*d^2*sin(f*x+e)/f^3+a*(d*
x+c)^2*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2717}

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx = \frac{2ad(c+dx)\cos(e+fx)}{f^2} + \frac{a(c+dx)^2\sin(e+fx)}{f} + \frac{a(c+dx)^3}{3d} - \frac{2ad^2\sin(e+fx)}{f^3}$$

```
[In] Int[(c + d*x)^2*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*a*d*(c + d*x)*Cos[e + f*x])/f^2 - (2*a*d^2*Sin[e
 + f*x])/f^3 + (a*(c + d*x)^2*Sin[e + f*x])/f
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(c+dx)^2 + a(c+dx)^2\cos(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^3}{3d} + a\int (c+dx)^2\cos(e+fx)\,dx \\
&= \frac{a(c+dx)^3}{3d} + \frac{a(c+dx)^2\sin(e+fx)}{f} - \frac{(2ad)\int (c+dx)\sin(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^3}{3d} + \frac{2ad(c+dx)\cos(e+fx)}{f^2} + \frac{a(c+dx)^2\sin(e+fx)}{f} - \frac{(2ad^2)\int \cos(e+fx)\,dx}{f^2} \\
&= \frac{a(c+dx)^3}{3d} + \frac{2ad(c+dx)\cos(e+fx)}{f^2} - \frac{2ad^2\sin(e+fx)}{f^3} + \frac{a(c+dx)^2\sin(e+fx)}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx = a\left(c^2x + cdx^2 + \frac{d^2x^3}{3} + \frac{2d(c+dx)\cos(e+fx)}{f^2} + \frac{(c^2f^2 + 2cdf^2x + d^2(-2+f^2x^2))\sin(e+fx)}{f^3}\right)$$

```
[In] Integrate[(c + d*x)^2*(a + a*Cos[e + f*x]),x]
```

```
[Out] a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 + (2*d*(c + d*x)*Cos[e + f*x])/f^2 + ((c^2
*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^3)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{\left(\left((dx+c)^2f^2-2d^2\right)\sin(fx+e)+\left((2d^2x+2cd)\cos(fx+e)+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)f^2-2cd\right)f\right)a}{f^3}$
risch	$\frac{a\,d^2x^3}{3}+acd\,x^2+a\,c^2x+\frac{a\,c^3}{3d}+\frac{2ad(dx+c)\cos(fx+e)}{f^2}+\frac{a\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\sin(fx+e)}{f^3}$
parts	$\frac{a(dx+c)^3}{3d}+\frac{a\left(\frac{d^2\left((fx+e)^2\sin(fx+e)-2\sin(fx+e)+2(fx+e)\cos(fx+e)\right)}{f^2}+\frac{2cd(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}-\frac{2d^2e(\cos(fx+$
norman	$\frac{acd\,x^2+\frac{a\left(c^2f^2+2d^2\right)x}{f^2}+acd\,x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{a\left(c^2f^2-2d^2\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a\,d^2x^3}{3}-\frac{4acd\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a\,d^2x^3\left(t}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$
derivativedivides	$a\,c^2\sin(fx+e)-\frac{2acde\sin(fx+e)}{f}+\frac{2acd(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{a\,d^2e^2\sin(fx+e)}{f^2}-\frac{2a\,d^2e(\cos(fx+e)+(fx+e)\sin(fx+e)}{f^2}$
default	$a\,c^2\sin(fx+e)-\frac{2acde\sin(fx+e)}{f}+\frac{2acd(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{a\,d^2e^2\sin(fx+e)}{f^2}-\frac{2a\,d^2e(\cos(fx+e)+(fx+e)\sin(fx+e)}{f^2}$

```
[In] int((d*x+c)^2*(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

```
[Out] (((d*x+c)^2*f^2-2*d^2)*sin(f*x+e)+((2*d^2*x+2*c*d)*cos(f*x+e)+x*(1/3*x^2*d^
2+c*d*x+c^2)*f^2-2*c*d)*f)*a/f^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx$$
$$=\frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x+6\,(ad^2fx+acdf)\cos(fx+e)+3\,(ad^2f^2x^2+2\,acdf^2x+ac^2f^2-2\,ad^2}{3\,f^3}$$

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x + 6*(a*d^2*f*x + a*c*d
*f)*cos(f*x + e) + 3*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*
sin(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx$$

$$= \begin{cases} ac^2x + \frac{ac^2\sin(e+fx)}{f} + acdx^2 + \frac{2acdx\sin(e+fx)}{f} + \frac{2acd\cos(e+fx)}{f^2} + \frac{ad^2x^3}{3} + \frac{ad^2x^2\sin(e+fx)}{f} + \frac{2ad^2x\cos(e+fx)}{f^2} - \frac{2ad^2}{} \\ (a\cos(e)+a)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((d*x+c)**2*(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c**2*sin(e + f*x)/f + a*c*d*x**2 + 2*a*c*d*x*sin(e
+ f*x)/f + 2*a*c*d*cos(e + f*x)/f**2 + a*d**2*x**3/3 + a*d**2*x**2*sin(e +
f*x)/f + 2*a*d**2*x*cos(e + f*x)/f**2 - 2*a*d**2*sin(e + f*x)/f**3, Ne(f, 0
)), ((a*cos(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.51

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx$$

$$= \frac{3\,(fx+e)ac^2 + \frac{(fx+e)^3ad^2}{f^2} - \frac{3\,(fx+e)^2ad^2e}{f^2} + \frac{3\,(fx+e)ad^2e^2}{f^2} + \frac{3\,(fx+e)^2acd}{f} - \frac{6\,(fx+e)acde}{f} + 3\,ac^2\sin(fx+e) + \frac{3\,ac}{}}{}$$

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 - 3*(f*x + e)^2*a*d^2*e/f^2
+ 3*(f*x + e)*a*d^2*e^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 6*(f*x + e)*a*c*d*e/f
 + 3*a*c^2*sin(f*x + e) + 3*a*d^2*e^2*sin(f*x + e)/f^2 - 6*a*c*d*e*sin(f*x
+ e)/f - 6*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*d^2*e/f^2 + 6*((f*x +
e)*sin(f*x + e) + cos(f*x + e))*a*c*d/f + 3*(2*(f*x + e)*cos(f*x + e) + ((f
*x + e)^2 - 2)*sin(f*x + e))*a*d^2/f^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx = \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + \frac{2\,(ad^2fx+acdf)\cos(fx+e)}{f^3} + \frac{(ad^2f^2x^2+2\,acdf^2x+ac^2f^2-2\,ad^2)\sin(fx+e)}{f^3}$$

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e)/
f^3 + (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*sin(f*x + e)/f^
3
```

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx = \frac{a\,d^2\,x^3}{3} - \frac{\sin(e+f\,x)\,(2\,a\,d^2-a\,c^2\,f^2)}{f^3} + a\,c^2\,x + a\,c\,d\,x^2 + \frac{2\,a\,d^2\,x\,\cos(e+f\,x)}{f^2} + \frac{a\,d^2\,x^2\,\sin(e+f\,x)}{f} + \frac{2\,a\,c\,d\,\cos(e+f\,x)}{f^2} + \frac{2\,a\,c\,d\,x\,\sin(e+f\,x)}{f}$$

```
[In] int((a + a*cos(e + f*x))*(c + d*x)^2,x)
```

```
[Out] (a*d^2*x^3)/3 - (sin(e + f*x)*(2*a*d^2 - a*c^2*f^2))/f^3 + a*c^2*x + a*c*d*
x^2 + (2*a*d^2*x*cos(e + f*x))/f^2 + (a*d^2*x^2*sin(e + f*x))/f + (2*a*c*d*
cos(e + f*x))/f^2 + (2*a*c*d*x*sin(e + f*x))/f
```

3.120 $\int (c+dx)(a+a\cos(e+fx))\,dx$

Optimal result . . . 756
Rubi [A] (verified) . . . 756
Mathematica [A] (verified) . . . 757
Maple [A] (verified) . . . 757
Fricas [A] (verification not implemented) . . . 758
Sympy [A] (verification not implemented) . . . 758
Maxima [B] (verification not implemented) . . . 759
Giac [A] (verification not implemented) . . . 759
Mupad [B] (verification not implemented) . . . 759

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \frac{a(c+dx)^2}{2d} + \frac{ad\cos(e+fx)}{f^2} + \frac{a(c+dx)\sin(e+fx)}{f}$$

```
[Out] 1/2*a*(d*x+c)^2/d+a*d*cos(f*x+e)/f^2+a*(d*x+c)*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3398, 3377, 2718}

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \frac{a(c+dx)\sin(e+fx)}{f} + \frac{a(c+dx)^2}{2d} + \frac{ad\cos(e+fx)}{f^2}$$

```
[In] Int[(c + d*x)*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) + (a*d*Cos[e + f*x])/f^2 + (a*(c + d*x)*Sin[e + f*x])
/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (a(c+dx) + a(c+dx)\cos(e+fx))\,dx \\ &= \frac{a(c+dx)^2}{2d} + a\int (c+dx)\cos(e+fx)\,dx \\ &= \frac{a(c+dx)^2}{2d} + \frac{a(c+dx)\sin(e+fx)}{f} - \frac{(ad)\int \sin(e+fx)\,dx}{f} \\ &= \frac{a(c+dx)^2}{2d} + \frac{ad\cos(e+fx)}{f^2} + \frac{a(c+dx)\sin(e+fx)}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\begin{aligned}&\int (c+dx)(a+a\cos(e+fx))\,dx \\ &= \frac{a(-2(e+fx)(de-2cf-dfx)+4d\cos(e+fx)+4f(c+dx)\sin(e+fx))}{4f^2}\end{aligned}$$

```
[In] Integrate[(c + d*x)*(a + a*Cos[e + f*x]),x]
```

```
[Out] (a*(-2*(e + f*x)*(d*e - 2*c*f - d*f*x) + 4*d*Cos[e + f*x] + 4*f*(c + d*x)*S
in[e + f*x]))/(4*f^2)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
parallelrisch	$\frac{\left(f(dx+c)\sin(fx+e)+\cos(fx+e)d+x\left(\frac{dx}{2}+c\right)f^2+d\right)a}{f^2}$	40
risch	$\frac{ad\,x^2}{2}+acx+\frac{ad\cos(fx+e)}{f^2}+\frac{a(dx+c)\sin(fx+e)}{f}$	41
parts	$a\left(\frac{1}{2}d\,x^2+cx\right)+\frac{a\left(\frac{d(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+c\sin(fx+e)-\frac{de\sin(fx+e)}{f}\right)}{f}$	65
derivativedivides	$\frac{\sin(fx+e)ac-\frac{ade\sin(fx+e)}{f}+\frac{ad(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+ac(fx+e)-\frac{ade(fx+e)}{f}+\frac{ad(fx+e)^2}{2f}}{f}$	89
default	$\frac{\sin(fx+e)ac-\frac{ade\sin(fx+e)}{f}+\frac{ad(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+ac(fx+e)-\frac{ade(fx+e)}{f}+\frac{ad(fx+e)^2}{2f}}{f}$	89
norman	$\frac{acx+acx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{ad\,x^2}{2}-\frac{2ad\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{2ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{ad\,x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{2adx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$	113

```
[In] int((d*x+c)*(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

```
[Out] (f*(d*x+c)*sin(f*x+e)+cos(f*x+e)*d+x*(1/2*d*x+c)*f^2+d)*a/f^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \frac{adf^2x^2+2\,acf^2x+2\,ad\cos(fx+e)+2\,(adfx+acf)\sin(fx+e)}{2\,f^2}$$

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*a*d*cos(f*x + e) + 2*(a*d*f*x + a*c*f)*s
in(f*x + e))/f^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \begin{cases} acx+\frac{ac\sin(e+fx)}{f}+\frac{adx^2}{2}+\frac{adx\sin(e+fx)}{f}+\frac{ad\cos(e+fx)}{f^2} & \text{for } f\neq 0 \\ (a\cos(e)+a)\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*c*sin(e + f*x)/f + a*d*x**2/2 + a*d*x*sin(e + f*x)/f +
 a*d*cos(e + f*x)/f**2, Ne(f, 0)), ((a*cos(e) + a)*(c*x + d*x**2/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.07

$$\int (c+dx)(a+a\cos(e+fx))\,dx$$
$$= \frac{2\,(fx+e)ac + \frac{(fx+e)^2 ad}{f} - \frac{2\,(fx+e)ade}{f} + 2\,ac\sin\left(fx+e\right) - \frac{2\,ade\sin(fx+e)}{f} + \frac{2\,((fx+e)\sin(fx+e)+\cos(fx+e))ad}{f}}{2\,f}$$

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*(f*x + e)*a*d*e/f + 2*a*c*sin(
f*x + e) - 2*a*d*e*sin(f*x + e)/f + 2*((f*x + e)*sin(f*x + e) + cos(f*x + e
))*a*d/f)/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \frac{1}{2}\,adx^2 + acx + \frac{ad\cos\left(fx+e\right)}{f^2} + \frac{(adfx+acf)\sin\left(fx+e\right)}{f^2}$$

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + a*d*cos(f*x + e)/f^2 + (a*d*f*x + a*c*f)*sin(f*x + e)
/f^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \frac{\frac{a\,f\,(2\,c\,\sin(e+f\,x)+2\,d\,x\,\sin(e+f\,x))}{2} + a\,d\,\cos\left(e+f\,x\right)}{f^2} + \frac{a\,(d\,x^2+2\,c\,x)}{2}$$

```
[In] int((a + a*cos(e + f*x))*(c + d*x),x)
```

```
[Out] ((a*f*(2*c*sin(e + f*x) + 2*d*x*sin(e + f*x)))/2 + a*d*cos(e + f*x))/f^2 +
(a*(2*c*x + d*x^2))/2
```

3.121 $\int \frac{a+a\cos(e+fx)}{c+dx}\,dx$

Optimal result . . . 760
Rubi [A] (verified) . . . 760
Mathematica [A] (verified) . . . 761
Maple [A] (verified) . . . 762
Fricas [A] (verification not implemented) . . . 762
Sympy [F] . . . 762
Maxima [C] (verification not implemented) . . . 763
Giac [C] (verification not implemented) . . . 763
Mupad [F(-1)] . . . 764

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = \frac{a\cos\left(e-\frac{cf}{d}\right)\operatorname{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a\log(c+dx)}{d} - \frac{a\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(\frac{cf}{d}+fx\right)}{d}$$

```
[Out] a*Ci(c*f/d+f*x)*cos(-e+c*f/d)/d+a*ln(d*x+c)/d+a*Si(c*f/d+f*x)*sin(-e+c*f/d)
/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3384, 3380, 3383}

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = \frac{a\operatorname{CosIntegral}\left(xf+\frac{cf}{d}\right)\cos\left(e-\frac{cf}{d}\right)}{d} - \frac{a\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(xf+\frac{cf}{d}\right)}{d} + \frac{a\log(c+dx)}{d}$$

```
[In] Int[(a + a*Cos[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d - (a
*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{c+dx} + \frac{a\cos(e+fx)}{c+dx}\right) dx \\
&= \frac{a\log(c+dx)}{d} + a\int \frac{\cos(e+fx)}{c+dx}\, dx \\
&= \frac{a\log(c+dx)}{d} + \left(a\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\, dx \\
&\quad - \left(a\sin\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\, dx \\
&= \frac{a\cos\left(e-\frac{cf}{d}\right)\operatorname{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a\log(c+dx)}{d} - \frac{a\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(\frac{cf}{d}+fx\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.22$ (sec) , antiderivative size $= 55$, normalized size of antiderivative $= 0.85$

$$\begin{aligned}
&\int \frac{a+a\cos(e+fx)}{c+dx}\, dx \\
&= \frac{a\left(\cos\left(e-\frac{cf}{d}\right)\operatorname{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right) + \log(c+dx) - \sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}
\end{aligned}$$

```
[In] Integrate[(a + a*Cos[e + f*x])/(c + d*x),x]
```

```
[Out] (a*(Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] + Log[c + d*x] - Sin[e - (c*f
)/d]*SinIntegral[f*(c/d + x)]))/d
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

method	result	size
parts	$\frac{a\ln(dx+c)}{d}+a\left(\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}\right)$	86
derivativedivides	$\frac{af\left(\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{af\ln(cf-de+d(fx+e))}{d}}{f}$	102
default	$\frac{af\left(\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{af\ln(cf-de+d(fx+e))}{d}}{f}$	102
risch	$\frac{a\ln(dx+c)}{d}-\frac{a\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d}-\frac{a\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{2d}$	109

```
[In] int((a+cos(f*x+e)*a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(d*x+c)/d+a*(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e
)/d)*cos((c*f-d*e)/d)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = \frac{a\cos\left(-\frac{de-cf}{d}\right)\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+a\sin\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right)+a\log\left(dx+c\right)}{d}$$

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*cos(-(d*e - c*f)/d)*cos_integral((d*f*x + c*f)/d) + a*sin(-(d*e - c*f)/d
)*sin_integral((d*f*x + c*f)/d) + a*log(d*x + c))/d
```

Sympy [F]

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = a\left(\int \frac{\cos\left(e+fx\right)}{c+dx}\,dx+\int \frac{1}{c+dx}\,dx\right)$$

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x)
```

```
[Out] a*(Integral(cos(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.65

$$\int \frac{a + a\cos(e+fx)}{c+dx}\,dx$$
$$= \frac{\frac{2af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d} - \frac{\left(f\left(E_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(iE_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)-iE_1\left(-\frac{i}{d}\right.\right.\right.}{d}}{2f}$$

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d - (f*(exp_integral_e(1, (I*(f*x
 + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I
*c*f)/d))*cos(-(d*e - c*f)/d) + f*(I*exp_integral_e(1, (I*(f*x + e)*d - I*d
*e + I*c*f)/d) - I*exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/d))*s
in(-(d*e - c*f)/d))*a/d)/f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 673, normalized size of antiderivative = 10.35

$$\int \frac{a + a\cos(e+fx)}{c+dx}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*log(abs(d*x + c))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + a*real_part(cos_
integral(f*x + c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + a*real_part(cos_inte
gral(-f*x - c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 - 2*a*imag_part(cos_integ
ral(f*x + c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) + 2*a*imag_part(cos_integral(
-f*x - c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) - 4*a*sin_integral((d*f*x + c*f)
/d)*tan(1/2*e)^2*tan(1/2*c*f/d) + 2*a*imag_part(cos_integral(f*x + c*f/d))*
tan(1/2*e)*tan(1/2*c*f/d)^2 - 2*a*imag_part(cos_integral(-f*x - c*f/d))*tan
(1/2*e)*tan(1/2*c*f/d)^2 + 4*a*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*tan
(1/2*c*f/d)^2 + 2*a*log(abs(d*x + c))*tan(1/2*e)^2 - a*real_part(cos_integr
al(f*x + c*f/d))*tan(1/2*e)^2 - a*real_part(cos_integral(-f*x - c*f/d))*tan
(1/2*e)^2 + 4*a*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(1/2*c*f
/d) + 4*a*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(1/2*c*f/d) +
 2*a*log(abs(d*x + c))*tan(1/2*c*f/d)^2 - a*real_part(cos_integral(f*x + c*
f/d))*tan(1/2*c*f/d)^2 - a*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*
f/d)^2 - 2*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e) + 2*a*imag_par
t(cos_integral(-f*x - c*f/d))*tan(1/2*e) - 4*a*sin_integral((d*f*x + c*f)/d
```

```
)*tan(1/2*e) + 2*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 2*
a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 4*a*sin_integral((
d*f*x + c*f)/d)*tan(1/2*c*f/d) + 2*a*log(abs(d*x + c)) + a*real_part(cos_in
tegral(f*x + c*f/d)) + a*real_part(cos_integral(-f*x - c*f/d)))/(d*tan(1/2*
e)^2*tan(1/2*c*f/d)^2 + d*tan(1/2*e)^2 + d*tan(1/2*c*f/d)^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\cos(e + fx)}{c + dx}\,dx = \int \frac{a + a\,\cos\left(e + f\,x\right)}{c + d\,x}\,dx$$

```
[In] int((a + a*cos(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + a*cos(e + f*x))/(c + d*x), x)
```

3.122 $\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx$

Optimal result . . . 765
Rubi [A] (verified) . . . 765
Mathematica [A] (verified) . . . 767
Maple [A] (verified) . . . 767
Fricas [A] (verification not implemented) . . . 768
Sympy [F] . . . 768
Maxima [C] (verification not implemented) . . . 768
Giac [B] (verification not implemented) . . . 769
Mupad [F(-1)] . . . 769

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)} - \frac{a\cos(e+fx)}{d(c+dx)} - \frac{af\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2} - \frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2}$$

```
[Out] -a/d/(d*x+c)-a*cos(f*x+e)/d/(d*x+c)-a*f*cos(-e+c*f/d)*Si(c*f/d+f*x)/d^2+a*f
*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3398, 3378, 3384, 3380, 3383}

$$\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx = -\frac{af\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{d^2} - \frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2} - \frac{a\cos(e+fx)}{d(c+dx)} - \frac{a}{d(c+dx)}$$

```
[In] Int[(a + a*Cos[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) - (a*Cos[e + f*x])/(d*(c + d*x)) - (a*f*CosIntegral[(c*f
)/d + f*x]*Sin[e - (c*f)/d])/d^2 - (a*f*Cos[e - (c*f)/d]*SinIntegral[(c*f)/
d + f*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a}{(c+dx)^2} + \frac{a\cos(e+fx)}{(c+dx)^2} \right) dx \\
&= -\frac{a}{d(c+dx)} + a \int \frac{\cos(e+fx)}{(c+dx)^2}\, dx \\
&= -\frac{a}{d(c+dx)} - \frac{a\cos(e+fx)}{d(c+dx)} - \frac{(af)\int \frac{\sin(e+fx)}{c+dx}\, dx}{d} \\
&= -\frac{a}{d(c+dx)} - \frac{a\cos(e+fx)}{d(c+dx)} - \frac{\left(af\cos\left(e-\frac{cf}{d}\right)\right)\int \frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\, dx}{d} \\
&\quad - \frac{\left(af\sin\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\, dx}{d}
\end{aligned}
$$

$$= -\frac{a}{d(c+dx)} - \frac{a\cos(e+fx)}{d(c+dx)} - \frac{af\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2} - \frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx = -\frac{a\left(d(1+\cos(e+fx)) + f(c+dx)\,\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right) + f(c+dx)\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}\right.\right.\right.}{d^2(c+dx)}$$

```
[In] Integrate[(a + a*Cos[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -((a*(d*(1 + Cos[e + f*x]) + f*(c + d*x)*CosIntegral[f*(c/d + x)]*Sin[e - (
c*f)/d] + f*(c + d*x)*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)]))/(d^2*(c +
 d*x)))
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

method	result
parts	$-\frac{a}{d(dx+c)} + af\left(-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d} - \frac{\frac{\text{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d} - \frac{\text{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}\right)$
derivativedivides	$\frac{af^2\left(-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d} - \frac{\frac{\text{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d} - \frac{\text{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}\right) - \frac{af^2}{(cf-de+d(fx+e))d}}{f}$
default	$\frac{af^2\left(-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d} - \frac{\frac{\text{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d} - \frac{\text{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}\right) - \frac{af^2}{(cf-de+d(fx+e))d}}{f}$
risch	$-\frac{a}{d(dx+c)} + \frac{iaf\,e^{\frac{i(cf-de)}{d}}\,\text{Ei}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^2} - \frac{ifa\,e^{-\frac{i(cf-de)}{d}}\,\text{Ei}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{2d^2} - \frac{a(-2dfx-2cf)}{2d(dx+c)(-}$

```
[In] int((a+cos(f*x+e)*a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/(d*x+c)+a*f*(-cos(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*
cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

$$\int \frac{a + a\cos(e+fx)}{(c+dx)^2}\,dx = -\frac{ad\cos\left(fx+e\right) - \left(adfx+acf\right)\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right) + \left(adfx+acf\right)\cos\left(-\frac{de-cf}{d}\right)\mathrm{Si}\left(\frac{dfx+cf}{d}\right) + ad}{d^3x + cd^2}$$

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*d*cos(f*x + e) - (a*d*f*x + a*c*f)*cos_integral((d*f*x + c*f)/d)*sin(-(
d*e - c*f)/d) + (a*d*f*x + a*c*f)*cos(-(d*e - c*f)/d)*sin_integral((d*f*x +
 c*f)/d) + a*d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{a + a\cos(e+fx)}{(c+dx)^2}\,dx = a\left(\int \frac{\cos\left(e+fx\right)}{c^2 + 2cdx + d^2x^2}\,dx + \int \frac{1}{c^2 + 2cdx + d^2x^2}\,dx\right)$$

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)**2,x)
```

```
[Out] a*(Integral(cos(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**
2 + 2*c*d*x + d**2*x**2), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.20

$$\int \frac{a + a\cos(e+fx)}{(c+dx)^2}\,dx = -\frac{\frac{2af^2}{(fx+e)d^2 - d^2e + cdf} + \frac{\left(f^2\left(E_2\left(\frac{i\,(fx+e)d - i\,de + i\,cf}{d}\right) + E_2\left(-\frac{i\,(fx+e)d - i\,de + i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right) - f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d - i\,de + i\,cf}{d}\right) + i\,E_2\left(-\frac{i\,(fx+e)d - i\,de + i\,cf}{d}\right)\right)\sin\left(-\frac{de-cf}{d}\right)\right)a}{(fx+e)d^2 - d^2e + cdf}}{2f}$$

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) + (f^2*(exp_integral_e(2, (I*
(f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d*e
 + I*c*f)/d))*cos(-(d*e - c*f)/d) - f^2*(-I*exp_integral_e(2, (I*(f*x + e)*
d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d - I*d*e + I*c*f
)/d))*sin(-(d*e - c*f)/d))*a/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(90) = 180$.

Time $= 0.31$ (sec) , antiderivative size $= 535$, normalized size of antiderivative $= 6.01$

$$\int \frac{a + a\cos(e+fx)}{(c+dx)^2}\,dx$$

$$= \frac{\left((dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right)f^2\,\mathrm{Ci}\left(\frac{(dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right) - de + cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right) - def^2\,\mathrm{Ci}\left(\frac{(dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + \right.\right.\right.}{d}$$

$$- \frac{a}{(dx+c)d}$$

```
[In] integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos_integral(((d*x + c)*
(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-(d*e - c*f)/d) - d
*e*f^2*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*sin(-(d*e - c*f)/d) + c*f^3*cos_integral(((d*x + c)*(d*e/(d*x + c)
- c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-(d*e - c*f)/d) - (d*x + c)*(d*e/(
d*x + c) - c*f/(d*x + c) + f)*f^2*cos(-(d*e - c*f)/d)*sin_integral(((d*x +
c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + d*e*f^2*cos(-(d*e
- c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d) - c*f^3*cos(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x +
c) - c*f/(d*x + c) + f) - d*e + c*f)/d) - d*f^2*cos(-(d*x + c)*(d*e/(d*x +
c) - c*f/(d*x + c) + f)/d))*a*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x
 + c) + f) - d^5*e + c*d^4*f)*f) - a/((d*x + c)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\cos(e+fx)}{(c+dx)^2}\,dx = \int \frac{a + a\,\cos\left(e+f\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int((a + a*cos(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + a*cos(e + f*x))/(c + d*x)^2, x)
```

3.123 $\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$

Optimal result . . . 770
Rubi [A] (verified) . . . 771
Mathematica [A] (verified) . . . 773
Maple [A] (verified) . . . 773
Fricas [A] (verification not implemented) . . . 774
Sympy [B] (verification not implemented) . . . 775
Maxima [B] (verification not implemented) . . . 775
Giac [A] (verification not implemented) . . . 776
Mupad [B] (verification not implemented) . . . 777

Optimal result

Integrand size = 20, antiderivative size = 237

$$\begin{aligned}\int (c+dx)^3(a+a\cos(e+fx))^2\,dx = &-\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}\\&-\frac{12a^2d^3\cos(e+fx)}{f^4}+\frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2}\\&-\frac{3a^2d^3\cos^2(e+fx)}{8f^4}+\frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2}\\&-\frac{12a^2d^2(c+dx)\sin(e+fx)}{f^3}\\&+\frac{2a^2(c+dx)^3\sin(e+fx)}{f}\\&-\frac{3a^2d^2(c+dx)\cos(e+fx)\sin(e+fx)}{4f^3}\\&+\frac{a^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

```
[Out] -3/4*a^2*c*d^2*x/f^2-3/8*a^2*d^3*x^2/f^2+3/8*a^2*(d*x+c)^4/d-12*a^2*d^3*cos
(f*x+e)/f^4+6*a^2*d*(d*x+c)^2*cos(f*x+e)/f^2-3/8*a^2*d^3*cos(f*x+e)^2/f^4+3
/4*a^2*d*(d*x+c)^2*cos(f*x+e)^2/f^2-12*a^2*d^2*(d*x+c)*sin(f*x+e)/f^3+2*a^2
*(d*x+c)^3*sin(f*x+e)/f-3/4*a^2*d^2*(d*x+c)*cos(f*x+e)*sin(f*x+e)/f^3+1/2*a
^2*(d*x+c)^3*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3398, 3377, 2718, 3392, 32, 3391}

$$\begin{aligned}\int (c+dx)^3(a+a\cos(e+fx))^2\,dx = &-\frac{12a^2d^2(c+dx)\sin(e+fx)}{f^3}\\ &-\frac{3a^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3}\\ &-\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2}\\ &+\frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2}\\ &+\frac{2a^2(c+dx)^3\sin(e+fx)}{f}\\ &+\frac{a^2(c+dx)^3\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2(c+dx)^4}{8d}\\ &-\frac{3a^2d^3\cos^2(e+fx)}{8f^4}-\frac{12a^2d^3\cos(e+fx)}{f^4}-\frac{3a^2d^3x^2}{8f^2}\end{aligned}$$

```
[In] Int[(c + d*x)^3*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (-3*a^2*c*d^2*x)/(4*f^2) - (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8
*d) - (12*a^2*d^3*Cos[e + f*x])/f^4 + (6*a^2*d*(c + d*x)^2*Cos[e + f*x])/f^
2 - (3*a^2*d^3*Cos[e + f*x]^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Cos[e + f*x]^
2)/(4*f^2) - (12*a^2*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (2*a^2*(c + d*x)^3*S
in[e + f*x])/f - (3*a^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) +
(a^2*(c + d*x)^3*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2(c+dx)^3 + 2a^2(c+dx)^3\cos(e+fx) + a^2(c+dx)^3\cos^2(e+fx)\right)\,dx \\
&= \frac{a^2(c+dx)^4}{4d} + a^2\int (c+dx)^3\cos^2(e+fx)\,dx + \left(2a^2\right)\int (c+dx)^3\cos(e+fx)\,dx \\
&= \frac{a^2(c+dx)^4}{4d} + \frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2} + \frac{2a^2(c+dx)^3\sin(e+fx)}{f} \\
&\quad + \frac{a^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f} + \frac{1}{2}a^2\int (c+dx)^3\,dx \\
&\quad - \frac{\left(3a^2d^2\right)\int (c+dx)\cos^2(e+fx)\,dx}{2f^2} - \frac{(6a^2d)\int (c+dx)^2\sin(e+fx)\,dx}{f} \\
&= \frac{3a^2(c+dx)^4}{8d} + \frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2} - \frac{3a^2d^3\cos^2(e+fx)}{8f^4} \\
&\quad + \frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2} + \frac{2a^2(c+dx)^3\sin(e+fx)}{f} \\
&\quad - \frac{3a^2d^2(c+dx)\cos(e+fx)\sin(e+fx)}{4f^3} + \frac{a^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f} \\
&\quad - \frac{\left(3a^2d^2\right)\int (c+dx)\,dx}{4f^2} - \frac{\left(12a^2d^2\right)\int (c+dx)\cos(e+fx)\,dx}{f^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3a^2cd^2x}{4f^2} - \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c+dx)^4}{8d} + \frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2} \\
&\quad - \frac{3a^2d^3\cos^2(e+fx)}{8f^4} + \frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2} - \frac{12a^2d^2(c+dx)\sin(e+fx)}{f^3} \\
&\quad + \frac{2a^2(c+dx)^3\sin(e+fx)}{f} - \frac{3a^2d^2(c+dx)\cos(e+fx)\sin(e+fx)}{4f^3} \\
&\quad + \frac{a^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f} + \frac{(12a^2d^3)\int \sin(e+fx)\,dx}{f^3} \\
&= -\frac{3a^2cd^2x}{4f^2} - \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c+dx)^4}{8d} - \frac{12a^2d^3\cos(e+fx)}{f^4} \\
&\quad + \frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2} - \frac{3a^2d^3\cos^2(e+fx)}{8f^4} + \frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2} \\
&\quad - \frac{12a^2d^2(c+dx)\sin(e+fx)}{f^3} + \frac{2a^2(c+dx)^3\sin(e+fx)}{f} \\
&\quad - \frac{3a^2d^2(c+dx)\cos(e+fx)\sin(e+fx)}{4f^3} + \frac{a^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.92

$$
\begin{aligned}
&\int (c+dx)^3(a+a\cos(e+fx))^2\,dx \\
&= \frac{a^2(96d(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)+3d(2c^2f^2+4cdf^2x+d^2(-1+2f^2x^2))\cos(2(e+f}{}
\end{aligned}
$$

```
[In] Integrate[(c + d*x)^3*(a + a*Cos[e + f*x])^2,x]

[Out] (a^2*(96*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x] + 3*d*
(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)] + 2*f*(3*
f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*(c + d*x)*(c^2*f^2 +
 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Sin[e + f*x] + (c + d*x)*(2*c^2*f^2 + 4*
c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e + f*x)])))/(16*f^4)
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{\left((dx+c)\left((dx+c)^2f^2-\frac{3d^2}{2}\right)f\sin(2fx+2e)+\frac{3\left((dx+c)^2f^2-\frac{d^2}{2}\right)d\cos(2fx+2e)}{2}+8(dx+c)\left((dx+c)^2f^2-6d^2\right)f\sin(fx+e)+2\right.}{4f^4}$
risch	$\frac{3a^2d^3x^4}{8}+\frac{3a^2cd^2x^3}{2}+\frac{9a^2dc^2x^2}{4}+\frac{3a^2c^3x}{2}+\frac{3a^2c^4}{8d}+\frac{6a^2d(d^2x^2f^2+2cdf^2x+c^2f^2-2d^2)\cos(fx+e)}{f^4}+\frac{2a^2(c$
norman	$\frac{12a^2c^2df^2-24a^2d^3}{f^4}+\frac{3a^2d^3x^4}{8}+\frac{\left(18a^2c^2df^2-45a^2d^3\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f^4}+\frac{3a^2cd^2x^3}{2}+\frac{3a^2d^3x^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{3a^2d^3x^4\left(\tan^4\left(\right.\right.}{8}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((d*x+c)^3*(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((d*x+c)*((d*x+c)^2*f^2-3/2*d^2)*f*sin(2*f*x+2*e)+3/2*((d*x+c)^2*f^2-1/
2*d^2)*d*cos(2*f*x+2*e)+8*(d*x+c)*((d*x+c)^2*f^2-6*d^2)*f*sin(f*x+e)+24*((d
*x+c)^2*f^2-2*d^2)*d*cos(f*x+e)+(6*c*d^2*x^3+9*x^2*c^2*d+3/2*d^3*x^4+6*x*c^
3)*f^4+45/2*c^2*d*f^2-189/4*d^3)*a^2/f^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.56

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$$
$$=\frac{3\,a^2d^3f^4x^4+12\,a^2cd^2f^4x^3+3\,(6\,a^2c^2df^4-a^2d^3f^2)x^2+3\,(2\,a^2d^3f^2x^2+4\,a^2cd^2f^2x+2\,a^2c^2df^2-a^2d^3)\cos$$

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^2*d^3*f^4*x^4 + 12*a^2*c*d^2*f^4*x^3 + 3*(6*a^2*c^2*d*f^4 - a^2*d^
3*f^2)*x^2 + 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a
^2*d^3)*cos(f*x + e)^2 + 6*(2*a^2*c^3*f^4 - a^2*c*d^2*f^2)*x + 48*(a^2*d^3*
f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2 - 2*a^2*d^3)*cos(f*x + e) + 2*(
8*a^2*d^3*f^3*x^3 + 24*a^2*c*d^2*f^3*x^2 + 8*a^2*c^3*f^3 - 48*a^2*c*d^2*f +
 24*(a^2*c^2*d*f^3 - 2*a^2*d^3*f)*x + (2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*
x^2 + 2*a^2*c^3*f^3 - 3*a^2*c*d^2*f + 3*(2*a^2*c^2*d*f^3 - a^2*d^3*f)*x)*co
s(f*x + e))*sin(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(243) = 486$.

Time = 0.44 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.29

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$$

$$= \begin{cases} \frac{a^2c^3x\sin^2(e+fx)}{2} + \frac{a^2c^3x\cos^2(e+fx)}{2} + a^2c^3x + \frac{a^2c^3\sin(e+fx)\cos(e+fx)}{2f} + \frac{2a^2c^3\sin(e+fx)}{f} + \frac{3a^2c^2dx^2\sin^2(e+fx)}{4} + \frac{3a^2c} \\ (a\cos(e)+a)^2\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

```
[In] integrate((d*x+c)**3*(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 +
a**2*c**3*x + a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c**3*sin(e
 + f*x)/f + 3*a**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*a**2*c**2*d*x**2*cos(e
 + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 + 3*a**2*c**2*d*x*sin(e + f*x)*cos(e +
f*x)/(2*f) + 6*a**2*c**2*d*x*sin(e + f*x)/f - 3*a**2*c**2*d*sin(e + f*x)**2
/(4*f**2) + 6*a**2*c**2*d*cos(e + f*x)/f**2 + a**2*c*d**2*x**3*sin(e + f*x)
**2/2 + a**2*c*d**2*x**3*cos(e + f*x)**2/2 + a**2*c*d**2*x**3 + 3*a**2*c*d*
*2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 6*a**2*c*d**2*x**2*sin(e + f*x)/f
 - 3*a**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) + 3*a**2*c*d**2*x*cos(e + f*x)*
*2/(4*f**2) + 12*a**2*c*d**2*x*cos(e + f*x)/f**2 - 3*a**2*c*d**2*sin(e + f*
x)*cos(e + f*x)/(4*f**3) - 12*a**2*c*d**2*sin(e + f*x)/f**3 + a**2*d**3*x**
4*sin(e + f*x)**2/8 + a**2*d**3*x**4*cos(e + f*x)**2/8 + a**2*d**3*x**4/4 +
 a**2*d**3*x**3*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*d**3*x**3*sin(e +
f*x)/f - 3*a**2*d**3*x**2*sin(e + f*x)**2/(8*f**2) + 3*a**2*d**3*x**2*cos(e
 + f*x)**2/(8*f**2) + 6*a**2*d**3*x**2*cos(e + f*x)/f**2 - 3*a**2*d**3*x*si
n(e + f*x)*cos(e + f*x)/(4*f**3) - 12*a**2*d**3*x*sin(e + f*x)/f**3 + 3*a**
2*d**3*sin(e + f*x)**2/(8*f**4) - 12*a**2*d**3*cos(e + f*x)/f**4, Ne(f, 0))
, ((a*cos(e) + a)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(223) = 446$.

Time = 0.40 (sec) , antiderivative size = 949, normalized size of antiderivative = 4.00

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c^3 + 16*(f*x + e)*a^2*c^3 + 4
*(f*x + e)^4*a^2*d^3/f^3 - 16*(f*x + e)^3*a^2*d^3*e/f^3 + 24*(f*x + e)^2*a^
```

```
2*d^3*e^2/f^3 - 4*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*d^3*e^3/f^3 - 16*(f*
x + e)*a^2*d^3*e^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 - 48*(f*x + e)^2*a^2*
c*d^2*e/f^2 + 12*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c*d^2*e^2/f^2 + 48*(f
*x + e)*a^2*c*d^2*e^2/f^2 + 24*(f*x + e)^2*a^2*c^2*d/f - 12*(2*f*x + 2*e +
sin(2*f*x + 2*e))*a^2*c^2*d*e/f - 48*(f*x + e)*a^2*c^2*d*e/f + 32*a^2*c^3*s
in(f*x + e) - 32*a^2*d^3*e^3*sin(f*x + e)/f^3 + 96*a^2*c*d^2*e^2*sin(f*x +
e)/f^2 - 96*a^2*c^2*d*e*sin(f*x + e)/f + 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin
(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*d^3*e^2/f^3 + 96*((f*x + e)*sin(f*x +
 e) + cos(f*x + e))*a^2*d^3*e^2/f^3 - 12*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2
*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*c*d^2*e/f^2 - 192*((f*x + e)*sin(f*x +
e) + cos(f*x + e))*a^2*c*d^2*e/f^2 + 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2*f
*x + 2*e) + cos(2*f*x + 2*e))*a^2*c^2*d/f + 96*((f*x + e)*sin(f*x + e) + co
s(f*x + e))*a^2*c^2*d/f - 2*(4*(f*x + e)^3 + 6*(f*x + e)*cos(2*f*x + 2*e) +
 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^3*e/f^3 - 96*(2*(f*x + e)*co
s(f*x + e) + ((f*x + e)^2 - 2)*sin(f*x + e))*a^2*d^3*e/f^3 + 2*(4*(f*x + e)
^3 + 6*(f*x + e)*cos(2*f*x + 2*e) + 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))
*a^2*c*d^2/f^2 + 96*(2*(f*x + e)*cos(f*x + e) + ((f*x + e)^2 - 2)*sin(f*x +
 e))*a^2*c*d^2/f^2 + (2*(f*x + e)^4 + 3*(2*(f*x + e)^2 - 1)*cos(2*f*x + 2*e
) + 2*(2*(f*x + e)^3 - 3*f*x - 3*e)*sin(2*f*x + 2*e))*a^2*d^3/f^3 + 32*(3*(
(f*x + e)^2 - 2)*cos(f*x + e) + ((f*x + e)^3 - 6*f*x - 6*e)*sin(f*x + e))*a
^2*d^3/f^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.41

$$\begin{aligned}\int (c+dx)^3(a+a\cos(e+fx))^2\,dx &= \frac{3}{8}\,a^2d^3x^4+\frac{3}{2}\,a^2cd^2x^3+\frac{9}{4}\,a^2c^2dx^2\\ &+\frac{3}{2}\,a^2c^3x+\frac{3\,(2\,a^2d^3f^2x^2+4\,a^2cd^2f^2x+2\,a^2c^2df^2-a^2d^3)\cos(2\,fx+2\,e)}{16\,f^4}\\ &+\frac{6\,(a^2d^3f^2x^2+2\,a^2cd^2f^2x+a^2c^2df^2-2\,a^2d^3)\cos(fx+e)}{f^4}\\ &+\frac{(2\,a^2d^3f^3x^3+6\,a^2cd^2f^3x^2+6\,a^2c^2df^3x+2\,a^2c^3f^3-3\,a^2d^3fx-3\,a^2cd^2f)\sin(2\,fx+2\,e)}{8\,f^4}\\ &+\frac{2\,(a^2d^3f^3x^3+3\,a^2cd^2f^3x^2+3\,a^2c^2df^3x+a^2c^3f^3-6\,a^2d^3fx-6\,a^2cd^2f)\sin(fx+e)}{f^4}\end{aligned}$$

```
[In] integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x + 3
/16*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a^2*d^3)*cos
(2*f*x + 2*e)/f^4 + 6*(a^2*d^3*f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2
- 2*a^2*d^3)*cos(f*x + e)/f^4 + 1/8*(2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*x^
```

```
2 + 6*a^2*c^2*d*f^3*x + 2*a^2*c^3*f^3 - 3*a^2*d^3*f*x - 3*a^2*c*d^2*f)*sin(
2*f*x + 2*e)/f^4 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f
^3*x + a^2*c^3*f^3 - 6*a^2*d^3*f*x - 6*a^2*c*d^2*f)*sin(f*x + e)/f^4
```

Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.91

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$$

$$= \frac{16\,a^2\,c^3\,f^3\,\sin(e+f\,x) - \frac{3\,a^2\,d^3\,\cos(2\,e+2\,f\,x)}{2} - 96\,a^2\,d^3\,\cos(e+f\,x) + 12\,a^2\,c^3\,f^4\,x + 2\,a^2\,c^3\,f^3\,\sin(2\,e+2$$

```
[In] int((a + a*cos(e + f*x))^2*(c + d*x)^3,x)
```

```
[Out] (16*a^2*c^3*f^3*sin(e + f*x) - (3*a^2*d^3*cos(2*e + 2*f*x))/2 - 96*a^2*d^3*
cos(e + f*x) + 12*a^2*c^3*f^4*x + 2*a^2*c^3*f^3*sin(2*e + 2*f*x) + 3*a^2*d^
3*f^4*x^4 - 96*a^2*c*d^2*f*sin(e + f*x) - 96*a^2*d^3*f*x*sin(e + f*x) + 3*a
^2*d^3*f^2*x^2*cos(2*e + 2*f*x) + 2*a^2*d^3*f^3*x^3*sin(2*e + 2*f*x) + 48*a
^2*c^2*d*f^2*cos(e + f*x) - 3*a^2*c*d^2*f*sin(2*e + 2*f*x) - 3*a^2*d^3*f*x*
sin(2*e + 2*f*x) + 3*a^2*c^2*d*f^2*cos(2*e + 2*f*x) + 18*a^2*c^2*d*f^4*x^2
+ 12*a^2*c*d^2*f^4*x^3 + 48*a^2*d^3*f^2*x^2*cos(e + f*x) + 16*a^2*d^3*f^3*x
^3*sin(e + f*x) + 6*a^2*c*d^2*f^2*x*cos(2*e + 2*f*x) + 6*a^2*c^2*d*f^3*x*si
n(2*e + 2*f*x) + 48*a^2*c*d^2*f^3*x^2*sin(e + f*x) + 6*a^2*c*d^2*f^3*x^2*si
n(2*e + 2*f*x) + 96*a^2*c*d^2*f^2*x*cos(e + f*x) + 48*a^2*c^2*d*f^3*x*sin(e
 + f*x))/(8*f^4)
```

3.124 $\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$

Optimal result . . . 778
Rubi [A] (verified) . . . 778
Mathematica [A] (verified) . . . 781
Maple [A] (verified) . . . 781
Fricas [A] (verification not implemented) . . . 782
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Maxima [B] (verification not implemented) . . . 783
Giac [A] (verification not implemented) . . . 783
Mupad [B] (verification not implemented) . . . 784

Optimal result

Integrand size = 20, antiderivative size = 168

$$\begin{aligned}\int (c+dx)^2(a+a\cos(e+fx))^2\,dx = &-\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}\\&+\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}\\&-\frac{4a^2d^2\sin(e+fx)}{f^3}+\frac{2a^2(c+dx)^2\sin(e+fx)}{f}\\&-\frac{a^2d^2\cos(e+fx)\sin(e+fx)}{4f^3}\\&+\frac{a^2(c+dx)^2\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

```
[Out] -1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d+4*a^2*d*(d*x+c)*cos(f*x+e)/f^2+1/2*a
^2*d*(d*x+c)*cos(f*x+e)^2/f^2-4*a^2*d^2*sin(f*x+e)/f^3+2*a^2*(d*x+c)^2*sin(
f*x+e)/f-1/4*a^2*d^2*cos(f*x+e)*sin(f*x+e)/f^3+1/2*a^2*(d*x+c)^2*cos(f*x+e)
*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used

$= \{3398, 3377, 2717, 3392, 32, 2715, 8\}$

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx = \frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2} + \frac{4a^2d(c+dx)\cos(e+fx)}{f^2} + \frac{2a^2(c+dx)^2\sin(e+fx)}{f} + \frac{a^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f} + \frac{a^2(c+dx)^3}{2d} - \frac{4a^2d^2\sin(e+fx)}{f^3} - \frac{a^2d^2\sin(e+fx)\cos(e+fx)}{4f^3} - \frac{a^2d^2x}{4f^2}$$

```
[In] Int[(c + d*x)^2*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] -1/4*(a^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(2*d) + (4*a^2*d*(c + d*x)*Cos[e +
 f*x])/f^2 + (a^2*d*(c + d*x)*Cos[e + f*x]^2)/(2*f^2) - (4*a^2*d^2*Sin[e +
f*x])/f^3 + (2*a^2*(c + d*x)^2*Sin[e + f*x])/f - (a^2*d^2*Cos[e + f*x]*Sin[
e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
 - Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2(c+dx)^2 + 2a^2(c+dx)^2\cos(e+fx) + a^2(c+dx)^2\cos^2(e+fx)\right)\,dx \\
&= \frac{a^2(c+dx)^3}{3d} + a^2\int (c+dx)^2\cos^2(e+fx)\,dx + \left(2a^2\right)\int (c+dx)^2\cos(e+fx)\,dx \\
&= \frac{a^2(c+dx)^3}{3d} + \frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2} + \frac{2a^2(c+dx)^2\sin(e+fx)}{f} \\
&\quad + \frac{a^2(c+dx)^2\cos(e+fx)\sin(e+fx)}{2f} + \frac{1}{2}a^2\int (c+dx)^2\,dx \\
&\quad - \frac{\left(a^2d^2\right)\int \cos^2(e+fx)\,dx}{2f^2} - \frac{\left(4a^2d\right)\int (c+dx)\sin(e+fx)\,dx}{f} \\
&= \frac{a^2(c+dx)^3}{2d} + \frac{4a^2d(c+dx)\cos(e+fx)}{f^2} + \frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2} \\
&\quad + \frac{2a^2(c+dx)^2\sin(e+fx)}{f} - \frac{a^2d^2\cos(e+fx)\sin(e+fx)}{4f^3} \\
&\quad + \frac{a^2(c+dx)^2\cos(e+fx)\sin(e+fx)}{2f} - \frac{\left(a^2d^2\right)\int 1\,dx}{4f^2} - \frac{\left(4a^2d^2\right)\int \cos(e+fx)\,dx}{f^2} \\
&= -\frac{a^2d^2x}{4f^2} + \frac{a^2(c+dx)^3}{2d} + \frac{4a^2d(c+dx)\cos(e+fx)}{f^2} \\
&\quad + \frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2} - \frac{4a^2d^2\sin(e+fx)}{f^3} + \frac{2a^2(c+dx)^2\sin(e+fx)}{f} \\
&\quad - \frac{a^2d^2\cos(e+fx)\sin(e+fx)}{4f^3} + \frac{a^2(c+dx)^2\cos(e+fx)\sin(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.15

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$
$$= \frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3+32df(c+dx)\cos(e+fx)+2df(c+dx)\cos(2(e+fx))-32d^2\sin(e$$

```
[In] Integrate[(c + d*x)^2*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 + 32*d*f*(c + d*x)*Cos[
e + f*x] + 2*d*f*(c + d*x)*Cos[2*(e + f*x)] - 32*d^2*Sin[e + f*x] + 16*c^2*
f^2*Sin[e + f*x] + 32*c*d*f^2*x*Sin[e + f*x] + 16*d^2*f^2*x^2*Sin[e + f*x]
- d^2*Sin[2*(e + f*x)] + 2*c^2*f^2*Sin[2*(e + f*x)] + 4*c*d*f^2*x*Sin[2*(e
+ f*x)] + 2*d^2*f^2*x^2*Sin[2*(e + f*x)]))/(8*f^3)
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{\left(\left((dx+c)^2f^2-\frac{d^2}{2}\right)\sin(2fx+2e)+df(dx+c)\cos(2fx+2e)+8\left((dx+c)^2f^2-2d^2\right)\sin(fx+e)+6\left(\frac{8d(dx+c)\cos(fx+e)}{3}+x\left(\frac{1}{3}\right.\right.\right.$
risch	$\frac{a^2d^2x^3}{2}+\frac{3a^2cdx^2}{2}+\frac{3a^2c^2x}{2}+\frac{a^2c^3}{2d}+\frac{4a^2d(dx+c)\cos(fx+e)}{f^2}+\frac{2a^2\left(d^2x^2f^2+2cdf^2x+c^2f^2-2d^2\right)\sin(fx+e)}{f^3}$
norman	$\frac{\frac{8a^2cd}{f^2}+a^2d^2x^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{a^2d^2x^3}{2}+\frac{6a^2cd\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{3a^2cdx^2}{2}+\frac{a^2d^2x^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{3a^2\left(2c^2f^2-5d^2\right)\left(\tan\right.}{2f^3}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{a^2\left(\frac{d^2\left((fx+e)^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+\frac{(fx+e)\left(\cos^2(fx+e)\right)}{2}-\frac{\cos(fx+e)\sin(fx+e)}{4}-\frac{fx}{4}-\frac{e}{4}-\frac{(fx+e}{3}\right.\right.}{f^2}$
derivativedivides	$a^2c^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cde\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a^2cd\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)\right.}{f}$
default	$a^2c^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cde\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a^2cd\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)\right.}{f}$

```
[In] int((d*x+c)^2*(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(((d*x+c)^2*f^2-1/2*d^2)*sin(2*f*x+2*e)+d*f*(d*x+c)*cos(2*f*x+2*e)+8*((
d*x+c)^2*f^2-2*d^2)*sin(f*x+e)+6*(8/3*d*(d*x+c)*cos(f*x+e)+x*(1/3*x^2*d^2+c
*d*x+c^2)*f^2-17/6*c*d)*f)*a^2/f^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.26

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$
$$= \frac{2\,a^2d^2f^3x^3 + 6\,a^2cdf^3x^2 + 2\,(a^2d^2fx + a^2cdf)\cos(fx+e)^2 + (6\,a^2c^2f^3 - a^2d^2f)x + 16\,(a^2d^2fx + a^2cdf)\,c}{}$$

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*co
s(f*x + e)^2 + (6*a^2*c^2*f^3 - a^2*d^2*f)*x + 16*(a^2*d^2*f*x + a^2*c*d*f)
*cos(f*x + e) + (8*a^2*d^2*f^2*x^2 + 16*a^2*c*d*f^2*x + 8*a^2*c^2*f^2 - 16*
a^2*d^2 + (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*c
os(f*x + e))*sin(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(163) = 326.

Time = 0.33 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.71

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^2x\sin^2(e+fx)}{2} + \frac{a^2c^2x\cos^2(e+fx)}{2} + a^2c^2x + \frac{a^2c^2\sin(e+fx)\cos(e+fx)}{2f} + \frac{2a^2c^2\sin(e+fx)}{f} + \frac{a^2cdx^2\sin^2(e+fx)}{2} + \frac{a^2cdx^2\,c}{} \\ (a\cos(e)+a)^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((d*x+c)**2*(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
a**2*c**2*x + a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c**2*sin(e
 + f*x)/f + a**2*c*d*x**2*sin(e + f*x)**2/2 + a**2*c*d*x**2*cos(e + f*x)**2
/2 + a**2*c*d*x**2 + a**2*c*d*x*sin(e + f*x)*cos(e + f*x)/f + 4*a**2*c*d*x*
sin(e + f*x)/f - a**2*c*d*sin(e + f*x)**2/(2*f**2) + 4*a**2*c*d*cos(e + f*x
)/f**2 + a**2*d**2*x**3*sin(e + f*x)**2/6 + a**2*d**2*x**3*cos(e + f*x)**2/
6 + a**2*d**2*x**3/3 + a**2*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a
**2*d**2*x**2*sin(e + f*x)/f - a**2*d**2*x*sin(e + f*x)**2/(4*f**2) + a**2*
d**2*x*cos(e + f*x)**2/(4*f**2) + 4*a**2*d**2*x*cos(e + f*x)/f**2 - a**2*d*
*2*sin(e + f*x)*cos(e + f*x)/(4*f**3) - 4*a**2*d**2*sin(e + f*x)/f**3, Ne(f
, 0)), ((a*cos(e) + a)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(158) = 316.

Time = 0.32 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.94

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$

$$= \frac{6\,(2\,fx+2\,e+\sin(2\,fx+2\,e))a^2c^2+24\,(fx+e)a^2c^2+\frac{8\,(fx+e)^3a^2d^2}{f^2}-\frac{24\,(fx+e)^2a^2d^2e}{f^2}+\frac{6\,(2\,fx+2\,e+\sin(2\,fx+2}{f^2}$$

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(6*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c^2 + 24*(f*x + e)*a^2*c^2 + 8
*(f*x + e)^3*a^2*d^2/f^2 - 24*(f*x + e)^2*a^2*d^2*e/f^2 + 6*(2*f*x + 2*e +
sin(2*f*x + 2*e))*a^2*d^2*e^2/f^2 + 24*(f*x + e)*a^2*d^2*e^2/f^2 + 24*(f*x
+ e)^2*a^2*c*d/f - 12*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c*d*e/f - 48*(f*
x + e)*a^2*c*d*e/f + 48*a^2*c^2*sin(f*x + e) + 48*a^2*d^2*e^2*sin(f*x + e)/
f^2 - 96*a^2*c*d*e*sin(f*x + e)/f - 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2*f*
x + 2*e) + cos(2*f*x + 2*e))*a^2*d^2*e/f^2 - 96*((f*x + e)*sin(f*x + e) + c
os(f*x + e))*a^2*d^2*e/f^2 + 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*a^2*c*d/f + 96*((f*x + e)*sin(f*x + e) + cos(f*x + e)
)*a^2*c*d/f + (4*(f*x + e)^3 + 6*(f*x + e)*cos(2*f*x + 2*e) + 3*(2*(f*x + e
)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^2/f^2 + 48*(2*(f*x + e)*cos(f*x + e) + ((f
*x + e)^2 - 2)*sin(f*x + e))*a^2*d^2/f^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.21

$$\begin{aligned}
&\int (c+dx)^2(a+a\cos(e+fx))^2\,dx \\
&\quad = \frac{1}{2}\,a^2d^2x^3+\frac{3}{2}\,a^2cdx^2+\frac{3}{2}\,a^2c^2x+\frac{(a^2d^2fx+a^2cdf)\cos(2\,fx+2\,e)}{4\,f^3} \\
&\qquad + \frac{4\,(a^2d^2fx+a^2cdf)\cos(fx+e)}{f^3} \\
&\qquad + \frac{(2\,a^2d^2f^2x^2+4\,a^2cdf^2x+2\,a^2c^2f^2-a^2d^2)\sin(2\,fx+2\,e)}{8\,f^3} \\
&\qquad + \frac{2\,(a^2d^2f^2x^2+2\,a^2cdf^2x+a^2c^2f^2-2\,a^2d^2)\sin(fx+e)}{f^3}
\end{aligned}$$

```
[In] integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x + 1/4*(a^2*d^2*f*x + a^2*
c*d*f)*cos(2*f*x + 2*e)/f^3 + 4*(a^2*d^2*f*x + a^2*c*d*f)*cos(f*x + e)/f^3
+ 1/8*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*sin(2
*f*x + 2*e)/f^3 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2 - 2*a^
2*d^2)*sin(f*x + e)/f^3
```

Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.52

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$

$$= \frac{8\,a^2\,c^2\,f^2\,\sin(e+f\,x) - \frac{a^2\,d^2\,\sin(2\,e+2\,f\,x)}{2} - 16\,a^2\,d^2\,\sin(e+f\,x) + 6\,a^2\,c^2\,f^3\,x + a^2\,c^2\,f^2\,\sin(2\,e+2\,f\,x) +}{}$$

```
[In] int((a + a*cos(e + f*x))^2*(c + d*x)^2,x)
```

```
[Out] (8*a^2*c^2*f^2*sin(e + f*x) - (a^2*d^2*sin(2*e + 2*f*x))/2 - 16*a^2*d^2*sin
(e + f*x) + 6*a^2*c^2*f^3*x + a^2*c^2*f^2*sin(2*e + 2*f*x) + 2*a^2*d^2*f^3*
x^3 + a^2*c*d*f*cos(2*e + 2*f*x) + 16*a^2*d^2*f*x*cos(e + f*x) + a^2*d^2*f^
2*x^2*sin(2*e + 2*f*x) + 6*a^2*c*d*f^3*x^2 + a^2*d^2*f*x*cos(2*e + 2*f*x) +
 16*a^2*c*d*f*cos(e + f*x) + 8*a^2*d^2*f^2*x^2*sin(e + f*x) + 16*a^2*c*d*f^
2*x*sin(e + f*x) + 2*a^2*c*d*f^2*x*sin(2*e + 2*f*x))/(4*f^3)
```

3.125 $\int (c+dx)(a+a\cos(e+fx))^2\,dx$

Optimal result . . . 785
Rubi [A] (verified) . . . 785
Mathematica [A] (verified) . . . 787
Maple [A] (verified) . . . 787
Fricas [A] (verification not implemented) . . . 788
Sympy [A] (verification not implemented) . . . 788
Maxima [A] (verification not implemented) . . . 788
Giac [A] (verification not implemented) . . . 789
Mupad [B] (verification not implemented) . . . 789

Optimal result

Integrand size = 18, antiderivative size = 118

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx = \frac{1}{2}a^2cx + \frac{1}{4}a^2dx^2 + \frac{a^2(c+dx)^2}{2d} + \frac{2a^2d\cos(e+fx)}{f^2} + \frac{a^2d\cos^2(e+fx)}{4f^2} + \frac{2a^2(c+dx)\sin(e+fx)}{f} + \frac{a^2(c+dx)\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a^2*c*x+1/4*a^2*d*x^2+1/2*a^2*(d*x+c)^2/d+2*a^2*d*cos(f*x+e)/f^2+1/4*a^
2*d*cos(f*x+e)^2/f^2+2*a^2*(d*x+c)*sin(f*x+e)/f+1/2*a^2*(d*x+c)*cos(f*x+e)*
sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3377, 2718, 3391}

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx = \frac{2a^2(c+dx)\sin(e+fx)}{f} + \frac{a^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f} + \frac{a^2(c+dx)^2}{2d} + \frac{1}{2}a^2cx + \frac{a^2d\cos^2(e+fx)}{4f^2} + \frac{2a^2d\cos(e+fx)}{f^2} + \frac{1}{4}a^2dx^2$$

```
[In] Int[(c + d*x)*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + (2*a^2*d*Cos[e + f*
x])/f^2 + (a^2*d*Cos[e + f*x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sin[e + f*x])/f
 + (a^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2(c+dx) + 2a^2(c+dx)\cos(e+fx) + a^2(c+dx)\cos^2(e+fx)\right)\,dx \\
&= \frac{a^2(c+dx)^2}{2d} + a^2\int (c+dx)\cos^2(e+fx)\,dx + \left(2a^2\right)\int (c+dx)\cos(e+fx)\,dx \\
&= \frac{a^2(c+dx)^2}{2d} + \frac{a^2 d\cos^2(e+fx)}{4f^2} + \frac{2a^2(c+dx)\sin(e+fx)}{f} \\
&\quad + \frac{a^2(c+dx)\cos(e+fx)\sin(e+fx)}{2f} + \frac{1}{2}a^2\int (c+dx)\,dx - \frac{\left(2a^2 d\right)\int \sin(e+fx)\,dx}{f} \\
&= \frac{1}{2}a^2cx + \frac{1}{4}a^2dx^2 + \frac{a^2(c+dx)^2}{2d} + \frac{2a^2 d\cos(e+fx)}{f^2} + \frac{a^2 d\cos^2(e+fx)}{4f^2} \\
&\quad + \frac{2a^2(c+dx)\sin(e+fx)}{f} + \frac{a^2(c+dx)\cos(e+fx)\sin(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx$$
$$= \frac{a^2(-6(e+fx)(-2cf+d(e-fx))+16d\cos(e+fx)+d\cos(2(e+fx))+16f(c+dx)\sin(e+fx)+2f}{8f^2}$$

```
[In] Integrate[(c + d*x)*(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (a^2*(-6*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*d*Cos[e + f*x] + d*Cos[2*(e
+ f*x)] + 16*f*(c + d*x)*Sin[e + f*x] + 2*f*(c + d*x)*Sin[2*(e + f*x)]))/(8
*f^2)
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

method	result
risch	$\frac{3a^2dx^2}{4}+\frac{3a^2cx}{2}+\frac{2a^2d\cos(fx+e)}{f^2}+\frac{2a^2(dx+c)\sin(fx+e)}{f}+\frac{a^2d\cos(2fx+2e)}{8f^2}+\frac{a^2(dx+c)\sin(2fx+2e)}{4f}$
parts	$a^2\left(\frac{1}{2}dx^2+cx\right)+\frac{a^2\left(\frac{d\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{(fx+e)^2}{4}-\frac{\left(\sin^2(fx+e)\right)}{4}\right)}{f}+c\left(\frac{\cos(fx+e)\sin(fx+e)}{2}\right.\right.}{f}$
norman	$\frac{\frac{4a^2d}{f^2}+\frac{3a^2d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{3a^2cx}{2}+\frac{3a^2dx^2}{4}+\frac{5a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{3a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+3a^2cx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{3a^2cx\left(\ta}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$
derivativedivides	$a^2c\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2de\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2d\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{(fx}{f}\right.}{f}$
default	$a^2c\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2de\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2d\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{(fx}{f}\right.}{f}$

```
[In] int((d*x+c)*(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*a^2*d*x^2+3/2*a^2*c*x+2*a^2*d*cos(f*x+e)/f^2+2*a^2*(d*x+c)*sin(f*x+e)/f
+1/8*a^2*d/f^2*cos(2*f*x+2*e)+1/4*a^2/f*(d*x+c)*sin(2*f*x+2*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx$$

$$=\frac{3\,a^2df^2x^2+6\,a^2cf^2x+a^2d\cos\left(fx+e\right)^2+8\,a^2d\cos\left(fx+e\right)+2\left(4\,a^2dfx+4\,a^2cf+\left(a^2dfx+a^2cf\right)\cos\left(f\right.\right.}{4\,f^2}$$

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*d*f^2*x^2 + 6*a^2*c*f^2*x + a^2*d*cos(f*x + e)^2 + 8*a^2*d*cos(f
*x + e) + 2*(4*a^2*d*f*x + 4*a^2*c*f + (a^2*d*f*x + a^2*c*f)*cos(f*x + e))*
sin(f*x + e))/f^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.86

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx$$

$$=\begin{cases}\frac{a^2cx\sin^2\left(e+fx\right)}{2}+\frac{a^2cx\cos^2\left(e+fx\right)}{2}+a^2cx+\frac{a^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{2a^2c\sin\left(e+fx\right)}{f}+\frac{a^2dx^2\sin^2\left(e+fx\right)}{4}+\frac{a^2dx^2\cos^2\left(e+f\right.}{4} \\ \left(a\cos\left(e\right)+a\right)^2\left(cx+\frac{dx^2}{2}\right)\end{cases}$$

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*c
*x + a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c*sin(e + f*x)/f + a**
2*d*x**2*sin(e + f*x)**2/4 + a**2*d*x**2*cos(e + f*x)**2/4 + a**2*d*x**2/2
+ a**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*d*x*sin(e + f*x)/f - a*
*2*d*sin(e + f*x)**2/(4*f**2) + 2*a**2*d*cos(e + f*x)/f**2, Ne(f, 0)), ((a*
cos(e) + a)**2*(c*x + d*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.67

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx$$

$$=\frac{2\left(2\,fx+2\,e+\sin\left(2\,fx+2\,e\right)\right)a^2c+8\left(fx+e\right)a^2c+\frac{4\left(fx+e\right)^2a^2d}{f}-\frac{2\left(2\,fx+2\,e+\sin(2\,fx+2\,e)\right)a^2de}{f}-\frac{8\left(fx+e\right)a^2de}{f}+}$$

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c + 8*(f*x + e)*a^2*c + 4*(f*x
+ e)^2*a^2*d/f - 2*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*d*e/f - 8*(f*x + e)
*a^2*d*e/f + 16*a^2*c*sin(f*x + e) - 16*a^2*d*e*sin(f*x + e)/f + (2*(f*x +
e)^2 + 2*(f*x + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*d/f + 16*((f*x
+ e)*sin(f*x + e) + cos(f*x + e))*a^2*d/f)/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx = \frac{3}{4}\,a^2dx^2 + \frac{3}{2}\,a^2cx + \frac{a^2d\cos(2\,fx+2\,e)}{8\,f^2} + \frac{2\,a^2d\cos(fx+e)}{f^2} + \frac{(a^2dfx+a^2cf)\sin(2\,fx+2\,e)}{4\,f^2} + \frac{2\,(a^2dfx+a^2cf)\sin(fx+e)}{f^2}$$

```
[In] integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*d*x^2 + 3/2*a^2*c*x + 1/8*a^2*d*cos(2*f*x + 2*e)/f^2 + 2*a^2*d*cos(
f*x + e)/f^2 + 1/4*(a^2*d*f*x + a^2*c*f)*sin(2*f*x + 2*e)/f^2 + 2*(a^2*d*f*
x + a^2*c*f)*sin(f*x + e)/f^2
```

Mupad [B] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx$$

$$= \frac{3\,a^2\,d\,f^2\,x^2 - 16\,a^2\,d\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 - a^2\,d\sin(e+f\,x)^2 + 8\,a^2\,c\,f\,\sin(e+f\,x) + a^2\,c\,f\,\sin(2\,e+2\,f\,x) +}{4\,f^2}$$

```
[In] int((a + a*cos(e + f*x))^2*(c + d*x),x)
```

```
[Out] (3*a^2*d*f^2*x^2 - 16*a^2*d*sin(e/2 + (f*x)/2)^2 - a^2*d*sin(e + f*x)^2 + 8
*a^2*c*f*sin(e + f*x) + a^2*c*f*sin(2*e + 2*f*x) + 6*a^2*c*f^2*x + a^2*d*f*
x*sin(2*e + 2*f*x) + 8*a^2*d*f*x*sin(e + f*x))/(4*f^2)
```

3.126 $\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx$

Optimal result . . . 790
Rubi [A] (verified) . . . 790
Mathematica [A] (verified) . . . 792
Maple [A] (verified) . . . 792
Fricas [A] (verification not implemented) . . . 793
Sympy [F] . . . 794
Maxima [C] (verification not implemented) . . . 794
Giac [C] (verification not implemented) . . . 794
Mupad [F(-1)] . . . 799

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx = \frac{2a^2\cos\left(e-\frac{cf}{d}\right)\operatorname{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2d} + \frac{3a^2\log(c+dx)}{2d} - \frac{2a^2\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(\frac{cf}{d}+fx\right)}{d} - \frac{a^2\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2cf}{d}+2fx\right)}{2d}$$

```
[Out] 1/2*a^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/d+2*a^2*Ci(c*f/d+f*x)*cos(-e+c*
f/d)/d+3/2*a^2*ln(d*x+c)/d+1/2*a^2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/d+2*
a^2*Si(c*f/d+f*x)*sin(-e+c*f/d)/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3399, 3393, 3384, 3380, 3383}

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx = \frac{2a^2\operatorname{CosIntegral}\left(xf+\frac{cf}{d}\right)\cos\left(e-\frac{cf}{d}\right)}{d} + \frac{a^2\operatorname{CosIntegral}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{2d} - \frac{2a^2\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(xf+\frac{cf}{d}\right)}{d} - \frac{a^2\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(2xf+\frac{2cf}{d}\right)}{2d} + \frac{3a^2\log(c+dx)}{2d}$$

```
[In] Int[(a + a*Cos[e + f*x])^2/(c + d*x),x]
```

```
[Out] (2*a^2*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d + (a^2*Cos[2*e - (2*c
*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2*Log[c + d*x])/(2*d) -
 (2*a^2*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d - (a^2*Sin[2*e - (2*
c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(4a^2\right) \int \frac{\sin^4\left(\frac{e+\pi}{2} + \frac{fx}{2}\right)}{c+dx}\,dx \\
&= \left(4a^2\right) \int \left(\frac{3}{8(c+dx)} + \frac{\cos(e+fx)}{2(c+dx)} + \frac{\cos(2e+2fx)}{8(c+dx)}\right) dx \\
&= \frac{3a^2\log(c+dx)}{2d} + \frac{1}{2}a^2 \int \frac{\cos(2e+2fx)}{c+dx}\,dx + \left(2a^2\right) \int \frac{\cos(e+fx)}{c+dx}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3a^2\log(c+dx)}{2d} + \frac{1}{2}\left(a^2\cos\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx \\
&\quad + \left(2a^2\cos\left(e-\frac{cf}{d}\right)\right)\int\frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx \\
&\quad - \frac{1}{2}\left(a^2\sin\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sin\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx \\
&\quad - \left(2a^2\sin\left(e-\frac{cf}{d}\right)\right)\int\frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx \\
&= \frac{2a^2\cos\left(e-\frac{cf}{d}\right)\operatorname{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2d} \\
&\quad + \frac{3a^2\log(c+dx)}{2d} - \frac{2a^2\sin\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(\frac{cf}{d}+fx\right)}{d} - \frac{a^2\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2cf}{d}+2fx\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$
\begin{aligned}
&\int\frac{(a+a\cos(e+fx))^2}{c+dx}\,dx \\
&= \frac{a^2\left(4\cos\left(e-\frac{cf}{d}\right)\operatorname{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)+\cos\left(2e-\frac{2cf}{d}\right)\operatorname{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+3\log(c+dx)-4\sin\left(\right.\right.}{2d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Cos[e + f*x])^2/(c + d*x),x]
```

```
[Out] (a^2*(4*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] + Cos[2*e - (2*c*f)/d]*Co
sIntegral[(2*f*(c + d*x))/d] + 3*Log[c + d*x] - 4*Sin[e - (c*f)/d]*SinInteg
ral[f*(c/d + x)] - Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d]))/(2
*d)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{f a^2 \left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}+\frac{3f a^2\ln(cf-de+d(fx+e))}{2d}+2f a^2\left($
default	$\frac{f a^2 \left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}+\frac{3f a^2\ln(cf-de+d(fx+e))}{2d}+2f a^2\left($
parts	$\frac{a^2\ln(dx+c)}{d}+\frac{a^2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{2d}+\frac{a^2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{2d}+\frac{a^2\ln(cf-d$
risch	$-\frac{a^2 e^{\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{d}-\frac{a^2 e^{-\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{d}+\frac{3a^2\ln(dx+c)}{2d}-\frac{a^2 e^{\frac{2i(cf-de)}{d}}$

```
[In] int((a+cos(f*x+e)*a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*f*a^2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f
*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d)+3/2*f*a^2*ln(c*f-d*e+d*(f*x+e))
/d+2*f*a^2*(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/d)*
cos((c*f-d*e)/d)/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx$$
$$= \frac{a^2\cos\left(-\frac{2\,(de-cf)}{d}\right)\mathrm{Ci}\left(\frac{2\,(dfx+cf)}{d}\right)+4\,a^2\cos\left(-\frac{de-cf}{d}\right)\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+a^2\sin\left(-\frac{2\,(de-cf)}{d}\right)\mathrm{Si}\left(\frac{2\,(dfx+cf)}{d}\right)+4\,a$$
$$2\,d$$

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cos(-2*(d*e - c*f)/d)*cos_integral(2*(d*f*x + c*f)/d) + 4*a^2*cos(
-(d*e - c*f)/d)*cos_integral((d*f*x + c*f)/d) + a^2*sin(-2*(d*e - c*f)/d)*s
in_integral(2*(d*f*x + c*f)/d) + 4*a^2*sin(-(d*e - c*f)/d)*sin_integral((d*
f*x + c*f)/d) + 3*a^2*log(d*x + c))/d
```

Sympy [F]

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx = a^2\left(\int \frac{2\cos(e+fx)}{c+dx}\,dx + \int \frac{\cos^2(e+fx)}{c+dx}\,dx + \int \frac{1}{c+dx}\,dx\right)$$

```
[In] integrate((a+a*cos(f*x+e))**2/(d*x+c),x)
```

```
[Out] a**2*(Integral(2*cos(e + f*x)/(c + d*x), x) + Integral(cos(e + f*x)**2/(c +
 d*x), x) + Integral(1/(c + d*x), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.34

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx$$
$$= \frac{\frac{4a^2 f\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d} - \frac{4\left(f\left(E_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)+E_1\left(-\frac{i(fx+e)d-ide+icf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(iE_1\left(\frac{i(fx+e)d-ide+icf}{d}\right)-iE_1\left(-\right.\right.\right.}{d}$$

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d - 4*(f*(exp_integral_e(1, (I*
(f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e
 + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(I*exp_integral_e(1, (I*(f*x + e)*d -
 I*d*e + I*c*f)/d) - I*exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/d
))*sin(-(d*e - c*f)/d))*a^2/d - (f*(exp_integral_e(1, 2*(-I*(f*x + e)*d + I
*d*e - I*c*f)/d) + exp_integral_e(1, -2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)
)*cos(-2*(d*e - c*f)/d) - f*(I*exp_integral_e(1, 2*(-I*(f*x + e)*d + I*d*e
- I*c*f)/d) - I*exp_integral_e(1, -2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d))*s
in(-2*(d*e - c*f)/d) - 2*f*log((f*x + e)*d - d*e + c*f))*a^2/d)/f
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 6693, normalized size of antiderivative = 46.16

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(6*a^2*log(abs(d*x + c))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f
/d)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)^2*
tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*
tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*real_part(cos_i
ntegral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2
+ a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c
*f/d)^2*tan(1/2*c*f/d)^2 - 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1
/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) + 8*a^2*imag_part(cos_integral
(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) - 16*a^2*
sin_integral((d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*
f/d) - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)^2
*tan(c*f/d)*tan(1/2*c*f/d)^2 + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/
d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 - 4*a^2*sin_integral(
2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 + 2*a^
2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2
*tan(1/2*c*f/d)^2 - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2
*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*sin_integral(2*(d*f*x +
c*f)/d)*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 8*a^2*imag_part
(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)
^2 - 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(e)^2*tan(c*
f/d)^2*tan(1/2*c*f/d)^2 + 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*t
an(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*e)^
2*tan(e)^2*tan(c*f/d)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(
1/2*e)^2*tan(e)^2*tan(c*f/d)^2 - 4*a^2*real_part(cos_integral(f*x + c*f/d))
*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2 - 4*a^2*real_part(cos_integral(-f*x - c
*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2 + a^2*real_part(cos_integral(-2*f
*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2 + 16*a^2*real_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) + 16*a
^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(e)^2*tan(c*f/d)^2*t
an(1/2*c*f/d) + 6*a^2*log(abs(d*x + c))*tan(1/2*e)^2*tan(e)^2*tan(1/2*c*f/d
)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)^2*ta
n(1/2*c*f/d)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*ta
n(e)^2*tan(1/2*c*f/d)^2 + 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1
/2*e)^2*tan(e)^2*tan(1/2*c*f/d)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c
*f/d))*tan(1/2*e)^2*tan(e)^2*tan(1/2*c*f/d)^2 + 4*a^2*real_part(cos_integra
l(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f/d)*tan(1/2*c*f/d)^2 + 4*a^2
*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f/d)*t
an(1/2*c*f/d)^2 + 6*a^2*log(abs(d*x + c))*tan(1/2*e)^2*tan(c*f/d)^2*tan(1/2
*c*f/d)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(c
*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1
/2*e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*real_part(cos_integral(-f*x -
 c*f/d))*tan(1/2*e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a^2*real_part(cos_int
egral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 6*a^2
*log(abs(d*x + c))*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + a^2*real_part(c
os_integral(2*f*x + 2*c*f/d))*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 4*a^
```

```
2*real_part(cos_integral(f*x + c*f/d))*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)
^2 - 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(e)^2*tan(c*f/d)^2*tan(
1/2*c*f/d)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e)^2*tan(c
*f/d)^2*tan(1/2*c*f/d)^2 - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*t
an(1/2*e)^2*tan(e)^2*tan(c*f/d) + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c
*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d) - 4*a^2*sin_integral(2*(d*f*x + c*f
)/d)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d) + 2*a^2*imag_part(cos_integral(2*f*x
+ 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2 - 2*a^2*imag_part(cos_integral
(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2 + 4*a^2*sin_integral(2
*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2 - 8*a^2*imag_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2*tan(c*f/d)^2 + 8*a^2*imag_part(cos
_integral(-f*x - c*f/d))*tan(1/2*e)*tan(e)^2*tan(c*f/d)^2 - 16*a^2*sin_inte
gral((d*f*x + c*f)/d)*tan(1/2*e)*tan(e)^2*tan(c*f/d)^2 - 8*a^2*imag_part(co
s_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(1/2*c*f/d) + 8*a^2*imag_
part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(1/2*c*f/d) - 16*
a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)^2*tan(1/2*c*f/d) - 8*
a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(c*f/d)^2*tan(1/2*
c*f/d) + 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(c*f/d
)^2*tan(1/2*c*f/d) - 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)^2*tan(
c*f/d)^2*tan(1/2*c*f/d) + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(e)
^2*tan(c*f/d)^2*tan(1/2*c*f/d) - 8*a^2*imag_part(cos_integral(-f*x - c*f/d)
)*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) + 16*a^2*sin_integral((d*f*x + c*f)/
d)*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) - 2*a^2*imag_part(cos_integral(2*f*
x + 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(1/2*c*f/d)^2 + 2*a^2*imag_part(cos_in
tegral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(1/2*c*f/d)^2 - 4*a^2*sin_
integral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)*tan(1/2*c*f/d)^2 + 8*a^2*im
ag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2*tan(1/2*c*f/d)^2 - 8
*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(e)^2*tan(1/2*c*f/
d)^2 + 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*tan(e)^2*tan(1/2*c*f
/d)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(c*f
/d)*tan(1/2*c*f/d)^2 - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(
1/2*e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 + 4*a^2*sin_integral(2*(d*f*x + c*f)/d
)*tan(1/2*e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 - 2*a^2*imag_part(cos_integral(2
*f*x + 2*c*f/d))*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 + 2*a^2*imag_part(cos
_integral(-2*f*x - 2*c*f/d))*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 - 4*a^2*s
in_integral(2*(d*f*x + c*f)/d)*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 + 8*a^2
*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(c*f/d)^2*tan(1/2*c*f/d
)^2 - 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(c*f/d)^2*t
an(1/2*c*f/d)^2 + 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*tan(c*f/d
)^2*tan(1/2*c*f/d)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(e
)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c
*f/d))*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*sin_integral(2*(d*f*x +
 c*f)/d)*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 6*a^2*log(abs(d*x + c))*tan
(1/2*e)^2*tan(e)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e
)^2*tan(e)^2 - 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(
```

```
e)^2 - 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2 -
a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2 + 4*a^2
*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f/d) +
4*a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f
/d) + 6*a^2*log(abs(d*x + c))*tan(1/2*e)^2*tan(c*f/d)^2 - a^2*real_part(cos
_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(c*f/d)^2 - 4*a^2*real_part(cos
_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(c*f/d)^2 - 4*a^2*real_part(cos_int
egral(-f*x - c*f/d))*tan(1/2*e)^2*tan(c*f/d)^2 - a^2*real_part(cos_integral
(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(c*f/d)^2 + 6*a^2*log(abs(d*x + c))*tan
(e)^2*tan(c*f/d)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(e)^2*
tan(c*f/d)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(e)^2*tan(c*f/
d)^2 + 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(e)^2*tan(c*f/d)^2 +
a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e)^2*tan(c*f/d)^2 + 16*a^
2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2*tan(1/2*c*f/d) +
 16*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(e)^2*tan(1/2*c
*f/d) + 16*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(c*f/d)^2
*tan(1/2*c*f/d) + 16*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*t
an(c*f/d)^2*tan(1/2*c*f/d) + 6*a^2*log(abs(d*x + c))*tan(1/2*e)^2*tan(1/2*c
*f/d)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(1/2
*c*f/d)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(1/2
*c*f/d)^2 + 4*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(1/
2*c*f/d)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan
(1/2*c*f/d)^2 + 6*a^2*log(abs(d*x + c))*tan(e)^2*tan(1/2*c*f/d)^2 - a^2*rea
l_part(cos_integral(2*f*x + 2*c*f/d))*tan(e)^2*tan(1/2*c*f/d)^2 - 4*a^2*rea
l_part(cos_integral(f*x + c*f/d))*tan(e)^2*tan(1/2*c*f/d)^2 - 4*a^2*real_pa
rt(cos_integral(-f*x - c*f/d))*tan(e)^2*tan(1/2*c*f/d)^2 - a^2*real_part(co
s_integral(-2*f*x - 2*c*f/d))*tan(e)^2*tan(1/2*c*f/d)^2 + 4*a^2*real_part(c
os_integral(2*f*x + 2*c*f/d))*tan(e)*tan(c*f/d)*tan(1/2*c*f/d)^2 + 4*a^2*re
al_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e)*tan(c*f/d)*tan(1/2*c*f/d)^2
+ 6*a^2*log(abs(d*x + c))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a^2*real_part(cos
_integral(2*f*x + 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 4*a^2*real_part
(cos_integral(f*x + c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 4*a^2*real_part
(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - a^2*real_part(
cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 2*a^2*imag_
part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e) + 2*a^2*imag_part(c
os_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e) - 4*a^2*sin_integral(2*(
d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e) - 8*a^2*imag_part(cos_integral(f*x + c*
f/d))*tan(1/2*e)*tan(e)^2 + 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan
(1/2*e)*tan(e)^2 - 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*tan(e)^2
 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(c*f/d) -
 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(c*f/d) +
4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(c*f/d) - 2*a^2*imag_
part(cos_integral(2*f*x + 2*c*f/d))*tan(e)^2*tan(c*f/d) + 2*a^2*imag_part(c
os_integral(-2*f*x - 2*c*f/d))*tan(e)^2*tan(c*f/d) - 4*a^2*sin_integral(2*(
d*f*x + c*f)/d)*tan(e)^2*tan(c*f/d) - 8*a^2*imag_part(cos_integral(f*x + c*
```

```
f/d))*tan(1/2*e)*tan(c*f/d)^2 + 8*a^2*imag_part(cos_integral(-f*x - c*f/d))
*tan(1/2*e)*tan(c*f/d)^2 - 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*
tan(c*f/d)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(e)*tan(c*
f/d)^2 - 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e)*tan(c*f/d)^
2 + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(e)*tan(c*f/d)^2 - 8*a^2*imag_
part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) + 8*a^2*imag_pa
rt(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) - 16*a^2*sin_int
egral((d*f*x + c*f)/d)*tan(1/2*e)^2*tan(1/2*c*f/d) + 8*a^2*imag_part(cos_in
tegral(f*x + c*f/d))*tan(e)^2*tan(1/2*c*f/d) - 8*a^2*imag_part(cos_integral
(-f*x - c*f/d))*tan(e)^2*tan(1/2*c*f/d) + 16*a^2*sin_integral((d*f*x + c*f)
/d)*tan(e)^2*tan(1/2*c*f/d) + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*ta
n(c*f/d)^2*tan(1/2*c*f/d) - 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan
(c*f/d)^2*tan(1/2*c*f/d) + 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(c*f/d)^
2*tan(1/2*c*f/d) + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*ta
n(1/2*c*f/d)^2 - 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan
(1/2*c*f/d)^2 + 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)*tan(1/2*c*f
/d)^2 - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(e)*tan(1/2*c*f/d
)^2 + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e)*tan(1/2*c*f/d)
^2 - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(e)*tan(1/2*c*f/d)^2 + 2*a^2*
imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2 - 2*a^
2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan(c*f/d)*tan(1/2*c*f/d)^2 + 4
*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d)*tan(1/2*c*f/d)^2 + 6*a^2*lo
g(abs(d*x + c))*tan(1/2*e)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))
*tan(1/2*e)^2 - 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)^2 - 4
*a^2*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)^2 + a^2*real_part(cos
_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2 + 6*a^2*log(abs(d*x + c))*tan(e)^
2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(e)^2 + 4*a^2*real_part
(cos_integral(f*x + c*f/d))*tan(e)^2 + 4*a^2*real_part(cos_integral(-f*x -
c*f/d))*tan(e)^2 - a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e)^2 +
 4*a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(e)*tan(c*f/d) + 4*a^2*r
eal_part(cos_integral(-2*f*x - 2*c*f/d))*tan(e)*tan(c*f/d) + 6*a^2*log(abs(
d*x + c))*tan(c*f/d)^2 - a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(c
*f/d)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d))*tan(c*f/d)^2 + 4*a^2*r
eal_part(cos_integral(-f*x - c*f/d))*tan(c*f/d)^2 - a^2*real_part(cos_integ
ral(-2*f*x - 2*c*f/d))*tan(c*f/d)^2 + 16*a^2*real_part(cos_integral(f*x + c
*f/d))*tan(1/2*e)*tan(1/2*c*f/d) + 16*a^2*real_part(cos_integral(-f*x - c*f
/d))*tan(1/2*e)*tan(1/2*c*f/d) + 6*a^2*log(abs(d*x + c))*tan(1/2*c*f/d)^2 +
 a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*c*f/d)^2 - 4*a^2*real
_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d)^2 - 4*a^2*real_part(cos_int
egral(-f*x - c*f/d))*tan(1/2*c*f/d)^2 + a^2*real_part(cos_integral(-2*f*x -
 2*c*f/d))*tan(1/2*c*f/d)^2 - 8*a^2*imag_part(cos_integral(f*x + c*f/d))*ta
n(1/2*e) + 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) - 16*a^2*
sin_integral((d*f*x + c*f)/d)*tan(1/2*e) - 2*a^2*imag_part(cos_integral(2*f
*x + 2*c*f/d))*tan(e) + 2*a^2*imag_part(cos_integral(-2*f*x - 2*c*f/d))*tan
(e) - 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(e) + 2*a^2*imag_part(cos_in
```

```
tegral(2*f*x + 2*c*f/d))*tan(c*f/d) - 2*a^2*imag_part(cos_integral(-2*f*x -
 2*c*f/d))*tan(c*f/d) + 4*a^2*sin_integral(2*(d*f*x + c*f)/d)*tan(c*f/d) +
8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*c*f/d) - 8*a^2*imag_part
(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) + 16*a^2*sin_integral((d*f*x +
c*f)/d)*tan(1/2*c*f/d) + 6*a^2*log(abs(d*x + c)) + a^2*real_part(cos_integr
al(2*f*x + 2*c*f/d)) + 4*a^2*real_part(cos_integral(f*x + c*f/d)) + 4*a^2*r
eal_part(cos_integral(-f*x - c*f/d)) + a^2*real_part(cos_integral(-2*f*x -
2*c*f/d)))/(d*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + d*tan(1
/2*e)^2*tan(e)^2*tan(c*f/d)^2 + d*tan(1/2*e)^2*tan(e)^2*tan(1/2*c*f/d)^2 +
d*tan(1/2*e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + d*tan(e)^2*tan(c*f/d)^2*tan(
1/2*c*f/d)^2 + d*tan(1/2*e)^2*tan(e)^2 + d*tan(1/2*e)^2*tan(c*f/d)^2 + d*ta
n(e)^2*tan(c*f/d)^2 + d*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + d*tan(e)^2*tan(1/2*
c*f/d)^2 + d*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + d*tan(1/2*e)^2 + d*tan(e)^2 +
d*tan(c*f/d)^2 + d*tan(1/2*c*f/d)^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx = \int \frac{(a+a\,\cos(e+f\,x))^2}{c+d\,x}\,dx$$

```
[In] int((a + a*cos(e + f*x))^2/(c + d*x),x)
```

```
[Out] int((a + a*cos(e + f*x))^2/(c + d*x), x)
```

3.127 $\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx$

Optimal result . . . 800
Rubi [A] (verified) . . . 800
Mathematica [A] (verified) . . . 802
Maple [A] (verified) . . . 803
Fricas [A] (verification not implemented) . . . 804
Sympy [F] . . . 804
Maxima [C] (verification not implemented) . . . 804
Giac [B] (verification not implemented) . . . 805
Mupad [F(-1)] . . . 806

Optimal result

Integrand size = 20, antiderivative size = 159

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx = -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{a^2 f\,\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2} - \frac{2a^2 f\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2} - \frac{2a^2 f\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{a^2 f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2cf}{d}+2fx\right)}{d^2}$$

```
[Out] -4*a^2*cos(1/2*f*x+1/2*e)^4/d/(d*x+c)-2*a^2*f*cos(-e+c*f/d)*Si(c*f/d+f*x)/d
^2-a^2*f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/d^2+a^2*f*Ci(2*c*f/d+2*f*x)*si
n(-2*e+2*c*f/d)/d^2+2*a^2*f*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{3399, 3394, 3384, 3380, 3383\}$

$$\int \frac{(a + a\cos(e + fx))^2}{(c + dx)^2}\,dx = -\frac{a^2 f\,\text{CosIntegral}\left(2xf + \frac{2cf}{d}\right)\sin\left(2e - \frac{2cf}{d}\right)}{d^2} - \frac{2a^2 f\,\text{CosIntegral}\left(xf + \frac{cf}{d}\right)\sin\left(e - \frac{cf}{d}\right)}{d^2} - \frac{2a^2 f\cos\left(e - \frac{cf}{d}\right)\text{Si}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{a^2 f\cos\left(2e - \frac{2cf}{d}\right)\text{Si}\left(2xf + \frac{2cf}{d}\right)}{d^2} - \frac{4a^2\cos^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)}$$

```
[In] Int[(a + a*Cos[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-4*a^2*Cos[e/2 + (f*x)/2]^4)/(d*(c + d*x)) - (a^2*f*CosIntegral[(2*c*f)/d
+ 2*f*x]*Sin[2*e - (2*c*f)/d])/d^2 - (2*a^2*f*CosIntegral[(c*f)/d + f*x]*Si
n[e - (c*f)/d])/d^2 - (2*a^2*f*Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])
/d^2 - (a^2*f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)}{(c+dx)^2}\,dx \\
&= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{\left(8a^2f\right)\int\left(-\frac{\sin(e+fx)}{4(c+dx)}-\frac{\sin(2e+2fx)}{8(c+dx)}\right)\,dx}{d} \\
&= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{\left(a^2f\right)\int\frac{\sin(2e+2fx)}{c+dx}\,dx}{d} - \frac{\left(2a^2f\right)\int\frac{\sin(e+fx)}{c+dx}\,dx}{d} \\
&= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{\left(a^2f\cos\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sin\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{d} \\
&\quad - \frac{\left(2a^2f\cos\left(e-\frac{cf}{d}\right)\right)\int\frac{\sin\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d} \\
&\quad - \frac{\left(a^2f\sin\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{d} - \frac{\left(2a^2f\sin\left(e-\frac{cf}{d}\right)\right)\int\frac{\cos\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d} \\
&= -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{a^2f\,\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2} \\
&\quad - \frac{2a^2f\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2} \\
&\quad - \frac{2a^2f\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{a^2f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2cf}{d}+2fx\right)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx = -\frac{a^2\Big(3d+4d\cos(e+fx)+d\cos(2(e+fx))+2f(c+dx)\,\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)+4f(c+$$

```
[In] Integrate[(a + a*Cos[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] -1/2*(a^2*(3*d + 4*d*Cos[e + f*x] + d*Cos[2*(e + f*x)] + 2*f*(c + d*x)*CosI
ntegral[(2*f*(c + d*x))/d]*Sin[2*e - (2*c*f)/d] + 4*f*(c + d*x)*CosIntegral
[f*(c/d + x)]*Sin[e - (c*f)/d] + 4*c*f*Cos[e - (c*f)/d]*SinIntegral[f*(c/d
+ x)] + 4*d*f*x*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 2*c*f*Cos[2*e -
 (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cos[2*e - (2*c*f)/d]*S
inIntegral[(2*f*(c + d*x))/d]))/(d^2*(c + d*x))
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.74

method	result
derivativedivides	$\frac{a^2 f^2 \left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d} - \frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d} - \frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d} \right)}{d} \right)}{4} - \frac{}{2(cf-}$ f
default	$\frac{a^2 f^2 \left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d} - \frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d} - \frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d} \right)}{d} \right)}{4} - \frac{}{2(cf-}$ f
parts	$-\frac{a^2}{d(dx+c)} + \frac{a^2 \left(\frac{f^2 \left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d} - \frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d} - \frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-}{d}\right)}{d} \right)}{d} \right)}{4}}{f}$
risch	$\frac{ia^2 f\, \mathrm{e}^{\frac{i(cf-de)}{d}}\, \mathrm{Ei}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{d^2} - \frac{if\,a^2 \mathrm{e}^{-\frac{i(cf-de)}{d}}\, \mathrm{Ei}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{d^2} - \frac{3a^2}{2d(dx+c)} + \frac{ia^2 f\, \mathrm{e}^{\frac{2i(cf-d}{d}}}{}$

```
[In] int((a+cos(f*x+e)*a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*a^2*f^2*(-2*cos(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e
+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c
*f-d*e)/d)/d)/d)-3/2*a^2*f^2/(c*f-d*e+d*(f*x+e))/d+2*a^2*f^2*(-cos(f*x+e)/(
c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*
f-d*e)/d)*sin((c*f-d*e)/d)/d)/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.38

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx =$$

$$-\frac{a^2 d\cos(fx+e)^2 + 2\,a^2 d\cos(fx+e) + a^2 d - 2\,(a^2 dfx + a^2 cf)\,\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right) - (a^2 dfx + a^2 c}{}$$

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*cos(f*x + e)^2 + 2*a^2*d*cos(f*x + e) + a^2*d - 2*(a^2*d*f*x + a^2*
c*f)*cos_integral((d*f*x + c*f)/d)*sin(-(d*e - c*f)/d) - (a^2*d*f*x + a^2*c
*f)*cos_integral(2*(d*f*x + c*f)/d)*sin(-2*(d*e - c*f)/d) + (a^2*d*f*x + a^
2*c*f)*cos(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) + 2*(a^2*d*f*x
 + a^2*c*f)*cos(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d))/(d^3*x + c*d
^2)
```

Sympy [F]

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx = a^2\left(\int \frac{2\cos(e+fx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{\cos^2(e+fx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

```
[In] integrate((a+a*cos(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*cos(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(c
os(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*d*
x + d**2*x**2), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.34

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx =$$

$$-\frac{\frac{4\,a^2 f^2}{(fx+e)d^2-d^2e+cdf} + \frac{4\left(f^2\left(E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E_2\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)-f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_2\left(-\right.\right.\right.}{(fx+e)d^2-d^2e+cdf}$$

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^2*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) + 4*(f^2*(exp_integral_e(2,
 (I*(f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I
*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) - f^2*(-I*exp_integral_e(2, (I*(f*x +
 e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d - I*d*e + I
*c*f)/d))*sin(-(d*e - c*f)/d))*a^2/((f*x + e)*d^2 - d^2*e + c*d*f) + (f^2*(
exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) + exp_integral_e(2,
 -2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d))*cos(-2*(d*e - c*f)/d) - f^2*(I*exp
_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - I*exp_integral_e(2,
-2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d))*sin(-2*(d*e - c*f)/d) + 2*f^2)*a^2/
((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1049 vs. $2(156) = 312$.

Time $= 0.45$ (sec) , antiderivative size $= 1049$, normalized size of antiderivative $= 6.60$

$$\int \frac{(a + a\cos(e + fx))^2}{(c + dx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos_integral((
(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-(d*e - c
*f)/d) - 4*a^2*d*e*f^2*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x +
c) + f) - d*e + c*f)/d)*sin(-(d*e - c*f)/d) + 4*a^2*c*f^3*cos_integral(((d*
x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-(d*e - c*f)
/d) + 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos_integral(
2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-2*(d*
e - c*f)/d) - 2*a^2*d*e*f^2*cos_integral(2*((d*x + c)*(d*e/(d*x + c) - c*f/
(d*x + c) + f) - d*e + c*f)/d)*sin(-2*(d*e - c*f)/d) + 2*a^2*c*f^3*cos_inte
gral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-
2*(d*e - c*f)/d) - 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*
cos(-2*(d*e - c*f)/d)*sin_integral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x +
 c) + f) - d*e + c*f)/d) + 2*a^2*d*e*f^2*cos(-2*(d*e - c*f)/d)*sin_integral
(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) - 2*a^2*c
*f^3*cos(-2*(d*e - c*f)/d)*sin_integral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(
d*x + c) + f) - d*e + c*f)/d) - 4*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x +
 c) + f)*f^2*cos(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x + c) - c
*f/(d*x + c) + f) - d*e + c*f)/d) + 4*a^2*d*e*f^2*cos(-(d*e - c*f)/d)*sin_i
ntegral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) - 4*
a^2*c*f^3*cos(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x + c) - c*f/
(d*x + c) + f) - d*e + c*f)/d) - 4*a^2*d*f^2*cos(-(d*x + c)*(d*e/(d*x + c)
- c*f/(d*x + c) + f)/d) - a^2*d*f^2*cos(-2*(d*x + c)*(d*e/(d*x + c) - c*f/(
```

```
d*x + c) + f)/d) - 3*a^2*d*f^2)*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d^5*e + c*d^4*f)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+a\,\cos(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

```
[In] int((a + a*cos(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + a*cos(e + f*x))^2/(c + d*x)^2, x)
```

3.128 $\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$

Optimal result . . . 807
Rubi [A] (verified) . . . 807
Mathematica [A] (verified) . . . 810
Maple [B] (verified) . . . 810
Fricas [B] (verification not implemented) . . . 811
Sympy [F] . . . 811
Maxima [B] (verification not implemented) . . . 812
Giac [F] . . . 812
Mupad [F(-1)] . . . 813

Optimal result

Integrand size = 20, antiderivative size = 134

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3} + \frac{12d^3\,\mathrm{PolyLog}\left(3,-e^{i(e+fx)}\right)}{af^4} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[Out] -I*(d*x+c)^3/a/f+6*d*(d*x+c)^2*ln(1+exp(I*(f*x+e)))/a/f^2-12*I*d^2*(d*x+c)*
polylog(2,-exp(I*(f*x+e)))/a/f^3+12*d^3*polylog(3,-exp(I*(f*x+e)))/a/f^4+(d
*x+c)^3*tan(1/2*f*x+1/2*e)/a/f
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3399, 4269, 3800, 2221, 2611, 2320, 6724}

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = -\frac{12id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3} + \frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{i(c+dx)^3}{af} + \frac{12d^3\,\mathrm{PolyLog}\left(3,-e^{i(e+fx)}\right)}{af^4}$$

```
[In] Int[(c + d*x)^3/(a + a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3)/(a*f) + (6*d*(c + d*x)^2*Log[1 + E^(I*(e + f*x))])/(a*f^
2) - ((12*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(e + f*x))])/(a*f^3) + (12*d^3*
PolyLog[3, -E^(I*(e + f*x))])/(a*f^4) + ((c + d*x)^3*Tan[e/2 + (f*x)/2])/(a
*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (c+dx)^3 \csc^2\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{2a} \\
&= \frac{(c+dx)^3 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(3d)\int (c+dx)^2 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af} \\
&= -\frac{i(c+dx)^3}{af} + \frac{(c+dx)^3 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(6id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af} \\
&= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2 \log\left(1+e^{i(e+fx)}\right)}{af^2} \\
&\quad + \frac{(c+dx)^3 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(12d^2)\int (c+dx)\log\left(1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\,dx}{af^2} \\
&= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2 \log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3} \\
&\quad + \frac{(c+dx)^3 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(12id^3)\int \text{PolyLog}\left(2,-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\,dx}{af^3} \\
&= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2 \log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3} \\
&\quad + \frac{(c+dx)^3 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(12d^3)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{af^4} \\
&= -\frac{i(c+dx)^3}{af} + \frac{6d(c+dx)^2 \log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3} \\
&\quad + \frac{12d^3\,\text{PolyLog}\left(3,-e^{i(e+fx)}\right)}{af^4} + \frac{(c+dx)^3 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-\frac{i\cos\left(\frac{1}{2}(e+fx)\right)(f^2(c+dx)^2(f(c+dx)+6id\log(1+e^{i(e+fx)}))+12d^2f(c+dx)\,\mathrm{PolyLog}(2,-e^{i(e+fx)})+12id^3\,\mathrm{PolyLog}(3,}{f^3}\right.}{af(1+\cos(e+fx))}$$

```
[In] Integrate[(c + d*x)^3/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(((-I)*Cos[(e + f*x)/2]*(f^2*(c + d*x)^2*(f*(c + d*x) +
 (6*I)*d*Log[1 + E^(I*(e + f*x))]) + 12*d^2*f*(c + d*x)*PolyLog[2, -E^(I*(e
 + f*x))] + (12*I)*d^3*PolyLog[3, -E^(I*(e + f*x))]))/f^3 + (c + d*x)^3*Sin
[(e + f*x)/2]))/(a*f*(1 + Cos[e + f*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(122) = 244.

Time = 1.62 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.72

method	result
risch	$-\frac{12id^3\,\mathrm{Li}_2\left(-\mathrm{e}^{i(fx+e)}\right)x}{af^3}+\frac{12d^2c\ln\left(\mathrm{e}^{i(fx+e)}+1\right)x}{af^2}-\frac{6id^2ce^2}{af^3}+\frac{4id^3e^3}{af^4}+\frac{12d^2ce\ln\left(\mathrm{e}^{i(fx+e)}\right)}{af^3}-\frac{12id^2cex}{af^2}-\frac{12id^2c\,\mathrm{Li}_2\left(-\mathrm{e}\right.}{af^3}$

```
[In] int((d*x+c)^3/(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

```
[Out] -12*I/a/f^3*d^3*polylog(2,-exp(I*(f*x+e)))*x+12/a/f^2*d^2*c*ln(exp(I*(f*x+e
))+1)*x-6*I/a/f^3*d^2*c*e^2+4*I/a/f^4*d^3*e^3+12/a/f^3*d^2*c*e*ln(exp(I*(f*
x+e)))-12*I/a/f^2*d^2*c*e*x-12*I/a/f^3*d^2*c*polylog(2,-exp(I*(f*x+e)))+2*I
*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))+1)+6*I/a/f^3*d^3*e
^2*x-2*I/a/f*d^3*x^3-6/a/f^4*d^3*e^2*ln(exp(I*(f*x+e)))-6*I/a/f*d^2*c*x^2+6
/a/f^2*d^3*ln(exp(I*(f*x+e))+1)*x^2+12*d^3*polylog(3,-exp(I*(f*x+e)))/a/f^4
-6/a/f^2*d*c^2*ln(exp(I*(f*x+e)))+6/a/f^2*d*c^2*ln(exp(I*(f*x+e))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(119) = 238$.

Time $= 0.31$ (sec) , antiderivative size $= 422$, normalized size of antiderivative $= 3.15$

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx =$$
$$-\frac{6\left(-i\,d^3fx - i\,cd^2f + \left(-i\,d^3fx - i\,cd^2f\right)\cos\left(fx+e\right)\right)\mathrm{Li}_2\left(-\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)+6\left(i\,d^3fx\right.}{}$$

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(6*(-I*d^3*f*x - I*c*d^2*f + (-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e))*dilog(
-cos(f*x + e) + I*sin(f*x + e)) + 6*(I*d^3*f*x + I*c*d^2*f + (I*d^3*f*x + I
*c*d^2*f)*cos(f*x + e))*dilog(-cos(f*x + e) - I*sin(f*x + e)) - 3*(d^3*f^2*
x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)
*cos(f*x + e))*log(cos(f*x + e) + I*sin(f*x + e) + 1) - 3*(d^3*f^2*x^2 + 2*
c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(f*x
 + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) - 6*(d^3*cos(f*x + e) + d^3)*
polylog(3, -cos(f*x + e) + I*sin(f*x + e)) - 6*(d^3*cos(f*x + e) + d^3)*pol
ylog(3, -cos(f*x + e) - I*sin(f*x + e)) - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 +
3*c^2*d*f^3*x + c^3*f^3)*sin(f*x + e))/(a*f^4*cos(f*x + e) + a*f^4)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$$
$$= \frac{\int \frac{c^3}{\cos\left(e+fx\right)+1}\,dx + \int \frac{d^3x^3}{\cos\left(e+fx\right)+1}\,dx + \int \frac{3cd^2x^2}{\cos\left(e+fx\right)+1}\,dx + \int \frac{3c^2dx}{\cos\left(e+fx\right)+1}\,dx}{a}$$

```
[In] integrate((d*x+c)**3/(a+a*cos(f*x+e)),x)
```

```
[Out] (Integral(c**3/(cos(e + f*x) + 1), x) + Integral(d**3*x**3/(cos(e + f*x) +
1), x) + Integral(3*c*d**2*x**2/(cos(e + f*x) + 1), x) + Integral(3*c**2*d*
x/(cos(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(119) = 238.

Time = 0.45 (sec) , antiderivative size = 935, normalized size of antiderivative = 6.98

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f*x + e
)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e))*c*d^
2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 + 2*a*f^2*cos(f*x + e) + a
*f^2) - 3*((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f
*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e)
)*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*cos(f*x + e) + a*f
) - c^3*sin(f*x + e)/(a*(cos(f*x + e) + 1)) - 3*c*d^2*e^2*sin(f*x + e)/(a*f
^2*(cos(f*x + e) + 1)) + 3*c^2*d*e*sin(f*x + e)/(a*f*(cos(f*x + e) + 1)) +
(2*d^3*e^3 - 6*((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e) +
 ((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x + e) -
 (-I*(f*x + e)^2*d^3 - I*d^3*e^2 + 2*(I*d^3*e - I*c*d^2*f)*(f*x + e))*sin(f
*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*((f*x + e)^3*d^3 + 3*(
f*x + e)*d^3*e^2 - 3*(d^3*e - c*d^2*f)*(f*x + e)^2)*cos(f*x + e) + 12*((f*x
 + e)*d^3 - d^3*e + c*d^2*f + ((f*x + e)*d^3 - d^3*e + c*d^2*f)*cos(f*x + e
) + (I*(f*x + e)*d^3 - I*d^3*e + I*c*d^2*f)*sin(f*x + e))*dilog(-e^(I*f*x +
 I*e)) + 3*(I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x +
 e) + (I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x + e))*
cos(f*x + e) - ((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*
sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 1
2*(I*d^3*cos(f*x + e) - d^3*sin(f*x + e) + I*d^3)*polylog(3, -e^(I*f*x + I*
e)) + 2*(I*(f*x + e)^3*d^3 + 3*I*(f*x + e)*d^3*e^2 + 3*(-I*d^3*e + I*c*d^2*
f)*(f*x + e)^2)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) -
 I*a*f^3))/f
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = \int \frac{(dx+c)^3}{a\cos(fx+e)+a}\,dx$$

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cos(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+a\,\cos{(e+f\,x)}}\,dx$$

```
[In] int((c + d*x)^3/(a + a*cos(e + f*x)),x)
[Out] int((c + d*x)^3/(a + a*cos(e + f*x)), x)
```

3.129 $\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx$

Optimal result . . . 814
Rubi [A] (verified) . . . 814
Mathematica [A] (verified) . . . 816
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Mupad [F(-1)] . . . 819

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{4id^2\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[Out] -I*(d*x+c)^2/a/f+4*d*(d*x+c)*ln(1+exp(I*(f*x+e)))/a/f^2-4*I*d^2*polylog(2,-
exp(I*(f*x+e)))/a/f^3+(d*x+c)^2*tan(1/2*f*x+1/2*e)/a/f
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3399, 4269, 3800, 2221, 2317, 2438}

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{i(c+dx)^2}{af} - \frac{4id^2\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3}$$

```
[In] Int[(c + d*x)^2/(a + a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) + (4*d*(c + d*x)*Log[1 + E^(I*(e + f*x))])/(a*f^2)
 - ((4*I)*d^2*PolyLog[2, -E^(I*(e + f*x))])/(a*f^3) + ((c + d*x)^2*Tan[e/2
+ (f*x)/2])/(a*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (c+dx)^2 \csc^2\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{2a} \\ &= \frac{(c+dx)^2 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(2d)\int (c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{i(c+dx)^2}{af} + \frac{(c+dx)^2 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(4id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af} \\
&= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} \\
&\quad + \frac{(c+dx)^2 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(4d^2)\int \log\left(1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\,dx}{af^2} \\
&= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} + \frac{(c+dx)^2 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} \\
&\quad + \frac{(4id^2)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{af^3} \\
&= -\frac{i(c+dx)^2}{af} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} \\
&\quad - \frac{4id^2\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3} + \frac{(c+dx)^2 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$
\begin{aligned}
&\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx \\
&= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-4id^2\cos\left(\frac{1}{2}(e+fx)\right)\text{PolyLog}\left(2,-e^{i(e+fx)}\right) + f(c+dx)\left(\cos\left(\frac{1}{2}(e+fx)\right)\left(-if(c+dx\right.\right.}{af^3(1+\cos(e+fx))}
\end{aligned}
$$

```
[In] Integrate[(c + d*x)^2/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*((-4*I)*d^2*Cos[(e + f*x)/2]*PolyLog[2, -E^(I*(e + f*x)
)] + f*(c + d*x)*(Cos[(e + f*x)/2]*((-I)*f*(c + d*x) + 4*d*Log[1 + E^(I*(e
+ f*x))]) + f*(c + d*x)*Sin[(e + f*x)/2])))/(a*f^3*(1 + Cos[e + f*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(91) = 182$.

Time = 1.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

method	result
risch	$\frac{2i(x^2d^2+2cdx+c^2)}{fa(e^{i(fx+e)}+1)} - \frac{4dc\ln(e^{i(fx+e)})}{af^2} + \frac{4dc\ln(e^{i(fx+e)}+1)}{af^2} - \frac{2id^2x^2}{af} - \frac{4id^2ex}{af^2} - \frac{2id^2e^2}{af^3} + \frac{4d^2\ln(e^{i(fx+e)}+1)x}{af^2} - \frac{4i}$

```
[In] int((d*x+c)^2/(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))+1)-4/a/f^2*d*c*ln(exp(I*(f*x+
e)))+4/a/f^2*d*c*ln(exp(I*(f*x+e))+1)-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-2*I
/a/f^3*d^2*e^2+4/a/f^2*d^2*ln(exp(I*(f*x+e))+1)*x-4*I*d^2*polylog(2,-exp(I*
(f*x+e)))/a/f^3+4/a/f^3*d^2*e*ln(exp(I*(f*x+e)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(88) = 176$.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.24

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx =$$
$$-\frac{2\left(-i\,d^2\cos\left(fx+e\right)-i\,d^2\right)\mathrm{Li}_2(-\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+2\left(i\,d^2\cos\left(fx+e\right)+i\,d^2\right)\mathrm{Li}_2(-\cos($$

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(2*(-I*d^2*cos(f*x + e) - I*d^2)*dilog(-cos(f*x + e) + I*sin(f*x + e)) + 2
*(I*d^2*cos(f*x + e) + I*d^2)*dilog(-cos(f*x + e) - I*sin(f*x + e)) - 2*(d^
2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) + I*sin(f*
x + e) + 1) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(
f*x + e) - I*sin(f*x + e) + 1) - (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*sin(
f*x + e))/(a*f^3*cos(f*x + e) + a*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \frac{\int \frac{c^2}{\cos(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cos(e+fx)+1}\,dx + \int \frac{2cdx}{\cos(e+fx)+1}\,dx}{a}$$

```
[In] integrate((d*x+c)**2/(a+a*cos(f*x+e)),x)
```

```
[Out] (Integral(c**2/(cos(e + f*x) + 1), x) + Integral(d**2*x**2/(cos(e + f*x) +
1), x) + Integral(2*c*d*x/(cos(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(88) = 176$.

Time $= 0.37$ (sec) , antiderivative size $= 284$, normalized size of antiderivative $= 2.81$

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx$$
$$= \frac{2\left(c^2f^2 + 2\left(d^2fx + cdf + \left(d^2fx + cdf\right)\cos\left(fx+e\right) - \left(-i\,d^2fx - i\,cdf\right)\sin\left(fx+e\right)\right)\arctan\left(\sin\left(fx+e\right),\right.\right.}{}$$

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(c^2*f^2 + 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e) - (-I*d^2*
f*x - I*c*d*f)*sin(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - (d^2
*f^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) - 2*(d^2*cos(f*x + e) + I*d^2*sin(f*x
+ e) + d^2)*dilog(-e^(I*f*x + I*e)) - (I*d^2*f*x + I*c*d*f + (I*d^2*f*x + I
*c*d*f)*cos(f*x + e) - (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 +
 sin(f*x + e)^2 + 2*cos(f*x + e) + 1) - (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin
(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) - I*a*f^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \int \frac{(dx+c)^2}{a\cos\left(fx+e\right)+a}\,dx$$

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cos(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+a\,\cos{(e+f\,x)}}\,dx$$

```
[In] int((c + d*x)^2/(a + a*cos(e + f*x)),x)
[Out] int((c + d*x)^2/(a + a*cos(e + f*x)), x)
```

3.130 $\int \frac{c+dx}{a+a\cos(e+fx)}\,dx$

Optimal result . . . 820
Rubi [A] (verified) . . . 820
Mathematica [A] (verified) . . . 821
Maple [A] (verified) . . . 821
Fricas [A] (verification not implemented) . . . 822
Sympy [A] (verification not implemented) . . . 822
Maxima [B] (verification not implemented) . . . 822
Giac [B] (verification not implemented) . . . 823
Mupad [B] (verification not implemented) . . . 823

Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2} + \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[Out] 2*d*ln(cos(1/2*f*x+1/2*e))/a/f^2+(d*x+c)*tan(1/2*f*x+1/2*e)/a/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3399, 4269, 3556}

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

```
[In] Int[(c + d*x)/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*d*Log[Cos[e/2 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tan[e/2 + (f*x)/2])/(a*f
)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (c+dx)\csc^2\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{2a} \\ &= \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{d\int \tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af} \\ &= \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2} + \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\begin{aligned}&\int \frac{c+dx}{a+a\cos(e+fx)}\,dx \\ &= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(2d\cos\left(\frac{1}{2}(e+fx)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+f(c+dx)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{af^2(1+\cos(e+fx))}\end{aligned}$$

```
[In] Integrate[(c + d*x)/(a + a*Cos[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(2*d*Cos[(e + f*x)/2]*Log[Cos[(e + f*x)/2]] + f*(c + d*
x)*Sin[(e + f*x)/2]))/(a*f^2*(1 + Cos[e + f*x]))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$\frac{-d\ln\left(\sec^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)f(dx+c)}{a\,f^2}$	40
norman	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a\,f^2}$	60
risch	$-\frac{2idx}{af}-\frac{2ide}{a\,f^2}+\frac{2i(dx+c)}{fa\left(e^{i(fx+e)}+1\right)}+\frac{2d\ln\left(e^{i(fx+e)}+1\right)}{a\,f^2}$	72

```
[In] int((d*x+c)/(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*ln(sec(1/2*f*x+1/2*e)^2)+tan(1/2*f*x+1/2*e)*f*(d*x+c))/a/f^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \frac{(d\cos(fx+e)+d)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+(dfx+cf)\sin(fx+e)}{af^2\cos(fx+e)+af^2}$$

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((d*cos(f*x + e) + d)*log(1/2*cos(f*x + e) + 1/2) + (d*f*x + c*f)*sin(f*x +
 e))/(a*f^2*cos(f*x + e) + a*f^2)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \begin{cases} \frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{dx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2} & \text{for } f\neq 0 \\ \frac{cx+\frac{dx^2}{2}}{a\cos(e)+a} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x)
```

```
[Out] Piecewise((c*tan(e/2 + f*x/2)/(a*f) + d*x*tan(e/2 + f*x/2)/(a*f) - d*log(ta
n(e/2 + f*x/2)**2 + 1)/(a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cos(e) + a
), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(41) = 82.

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.27

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \frac{\left(\left(\cos(fx+e)^2+\sin(fx+e)^2+2\cos(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2+2\cos(fx+e)+1\right)+2(fx+e)\sin(fx+e)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2+2af\cos(fx+e)+af}+\frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}-$$

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] (((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f*x + e)^2
 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e))*d/(a*f*
cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*cos(f*x + e) + a*f) + c*sin(f*x
 + e)/(a*(cos(f*x + e) + 1)) - d*e*sin(f*x + e)/(a*f*(cos(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(41) = 82$.

Time $= 0.33$ (sec) , antiderivative size $= 194$, normalized size of antiderivative $= 3.96$

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = -\frac{dfx\tan\left(\frac{1}{2}\,fx\right)+dfx\tan\left(\frac{1}{2}\,e\right)-d\log\left(\frac{4\left(\tan\left(\frac{1}{2}\,fx\right)^2\tan\left(\frac{1}{2}\,e\right)^2-2\tan\left(\frac{1}{2}\,fx\right)\tan\left(\frac{1}{2}\,e\right)+1\right)}{\tan\left(\frac{1}{2}\,fx\right)^2\tan\left(\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx\right)^2+\tan\left(\frac{1}{2}\,e\right)^2+1}\right)\tan\left(\frac{1}{2}\,fx\right)\tan\left(\frac{1}{2}\,e\right)+}{af^2\tan\left(\frac{1}{2}\,fx\right)\tan\left(\frac{1}{2}\,e\right)-}$$

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] -(d*f*x*tan(1/2*f*x) + d*f*x*tan(1/2*e) - d*log(4*(tan(1/2*f*x)^2*tan(1/2*e
)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2
*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x)*tan(1/2*e) + c*f*tan(1/2*f*x) + c
*f*tan(1/2*e) + d*log(4*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1
/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1
)))/(a*f^2*tan(1/2*f*x)*tan(1/2*e) - a*f^2)
```

Mupad [B] (verification not implemented)

Time $= 15.46$ (sec) , antiderivative size $= 65$, normalized size of antiderivative $= 1.33$

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\right)}{a\,f^2}+\frac{(c+d\,x)\;2\mathrm{i}}{a\,f\;\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}-\frac{d\,x\,2\mathrm{i}}{a\,f}$$

```
[In] int((c + d*x)/(a + a*cos(e + f*x)),x)
```

```
[Out] (2*d*log(exp(e*1i)*exp(f*x*1i) + 1))/(a*f^2) + ((c + d*x)*2i)/(a*f*(exp(e*1
i + f*x*1i) + 1)) - (d*x*2i)/(a*f)
```

3.131 $\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$

Optimal result . . . 824
Rubi [N/A] . . . 824
Mathematica [N/A] . . . 825
Maple [N/A] (verified) . . . 825
Fricas [N/A] . . . 825
Sympy [N/A] . . . 825
Maxima [N/A] . . . 826
Giac [N/A] . . . 826
Mupad [N/A] . . . 826

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+a\cos(e+fx))}, x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

```
[In] Int[1/((c + d*x)*(a + a*Cos[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + a*Cos[e + f*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

```
[In] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a+\cos(fx+e)\,a)}dx$$

```
[In] int(1/(d*x+c)/(a+cos(f*x+e)*a),x)
[Out] int(1/(d*x+c)/(a+cos(f*x+e)*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)}\,dx$$

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="fricas")
[Out] integral(1/(a*d*x + a*c + (a*d*x + a*c)*cos(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \frac{\int \frac{1}{c\cos(e+fx)+c+dx\cos(e+fx)+dx}\,dx}{a}$$

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x)
[Out] Integral(1/(c*cos(e + f*x) + c + d*x*cos(e + f*x) + d*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 285, normalized size of antiderivative = 14.25

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)}\,dx$$

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f*x
 + a*c*d*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e))*integrat
e(sin(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*a*c*
d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*sin
(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)), x) + s
in(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (a*d*f*x
 + a*c*f)*sin(f*x + e)^2 + 2*(a*d*f*x + a*c*f)*cos(f*x + e))
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)}\,dx$$

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*cos(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 14.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(a+a\,\cos(e+f\,x))\,(c+d\,x)}\,dx$$

```
[In] int(1/((a + a*cos(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cos(e + f*x))*(c + d*x)), x)
```

3.132 $\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$

Optimal result . . . 827
Rubi [N/A] . . . 827
Mathematica [N/A] . . . 828
Maple [N/A] (verified) . . . 828
Fricas [N/A] . . . 828
Sympy [N/A] . . . 829
Maxima [N/A] . . . 829
Giac [N/A] . . . 829
Mupad [N/A] . . . 830

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+a\cos(e+fx))}, x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+a*cos(f*x+e)),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

```
[In] Int[1/((c + d*x)^2*(a + a*Cos[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cos[e + f*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\,(a+\cos(fx+e)\,a)}dx$$

```
[In] int(1/(d*x+c)^2/(a+cos(f*x+e)*a),x)
[Out] int(1/(d*x+c)^2/(a+cos(f*x+e)*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="fricas")
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*cos(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \frac{\int \frac{1}{c^2\cos(e+fx)+c^2+2cdx\cos(e+fx)+2cdx+d^2x^2\cos(e+fx)+d^2x^2}\,dx}{a}$$

```
[In] integrate(1/(d*x+c)**2/(a+a*cos(f*x+e)),x)
```

```
[Out] Integral(1/(c**2*cos(e + f*x) + c**2 + 2*c*d*x*cos(e + f*x) + 2*c*d*x + d**
2*x**2*cos(e + f*x) + d**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 442, normalized size of antiderivative = 22.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*f*
x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*s
in(f*x + e)^2 + 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*cos(f*x + e))*i
ntegrate(sin(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^
3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x + e
)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f*x + e
)^2 + 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x +
 e)), x) + sin(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^
2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*
c^2*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e
))
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*cos(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 14.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(a+a\,\cos(e+f\,x))\,(c+d\,x)^2}\,dx$$

```
[In] int(1/((a + a*cos(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a + a*cos(e + f*x))*(c + d*x)^2), x)
```

3.133 $\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx$

Optimal result . . . 831
Rubi [A] (verified) . . . 832
Mathematica [A] (verified) . . . 835
Maple [B] (verified) . . . 836
Fricas [B] (verification not implemented) . . . 836
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Maxima [B] (verification not implemented) . . . 837
Giac [F] . . . 839
Mupad [F(-1)] . . . 840

Optimal result

Integrand size = 20, antiderivative size = 271

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = &-\frac{i(c+dx)^3}{3a^2f}+\frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2}\\&+\frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4}-\frac{4id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{a^2f^3}\\&+\frac{4d^3\,\mathrm{PolyLog}\left(3,-e^{i(e+fx)}\right)}{a^2f^4}-\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2}\\&+\frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}\\&+\frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}\end{aligned}$$

```
[Out] -1/3*I*(d*x+c)^3/a^2/f+2*d*(d*x+c)^2*ln(1+exp(I*(f*x+e)))/a^2/f^2+4*d^3*ln(
cos(1/2*f*x+1/2*e))/a^2/f^4-4*I*d^2*(d*x+c)*polylog(2,-exp(I*(f*x+e)))/a^2/
f^3+4*d^3*polylog(3,-exp(I*(f*x+e)))/a^2/f^4-1/2*d*(d*x+c)^2*sec(1/2*f*x+1/
2*e)^2/a^2/f^2+2*d^2*(d*x+c)*tan(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^3*tan(1
/2*f*x+1/2*e)/a^2/f+1/6*(d*x+c)^3*sec(1/2*f*x+1/2*e)^2*tan(1/2*f*x+1/2*e)/a
^2/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3399, 4271, 4269, 3556, 3800, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = &-\frac{4id^2(c+dx)\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{a^2f^3} \\ &+\frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3}+\frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} \\ &-\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} \\ &+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}-\frac{i(c+dx)^3}{3a^2f} \\ &+\frac{4d^3\,\text{PolyLog}\left(3,-e^{i(e+fx)}\right)}{a^2f^4}+\frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4}\end{aligned}$$

```
[In] Int[(c + d*x)^3/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] ((-1/3*I)*(c + d*x)^3)/(a^2*f) + (2*d*(c + d*x)^2*Log[1 + E^(I*(e + f*x))])
/(a^2*f^2) + (4*d^3*Log[Cos[e/2 + (f*x)/2]])/(a^2*f^4) - ((4*I)*d^2*(c + d*
x)*PolyLog[2, -E^(I*(e + f*x))])/(a^2*f^3) + (4*d^3*PolyLog[3, -E^(I*(e + f
*x))])/(a^2*f^4) - (d*(c + d*x)^2*Sec[e/2 + (f*x)/2]^2)/(2*a^2*f^2) + (2*d^
2*(c + d*x)*Tan[e/2 + (f*x)/2])/(a^2*f^3) + ((c + d*x)^3*Tan[e/2 + (f*x)/2]
)/(3*a^2*f) + ((c + d*x)^3*Sec[e/2 + (f*x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*
f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
 - 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (c+dx)^3 \csc^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{4a^2} \\
&= -\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} \\
&\quad + \frac{\int (c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{6a^2} + \frac{d^2\int (c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{a^2f^2} \\
&= -\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} \\
&\quad + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} \\
&\quad - \frac{(2d^3)\int \tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{a^2f^3} - \frac{d\int (c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{a^2f} \\
&= -\frac{i(c+dx)^3}{3a^2f} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} \\
&\quad + \frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} \\
&\quad + \frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{(2id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{a^2f} \\
&= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} \\
&\quad - \frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} \\
&\quad + \frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} - \frac{(4d^2)\int (c+dx)\log\left(1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\,dx}{a^2f^2} \\
&= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} \\
&\quad - \frac{4id^2(c+dx)\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{a^2f^3} - \frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} \\
&\quad + \frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} \\
&\quad + \frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{(4id^3)\int \text{PolyLog}\left(2,-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\,dx}{a^2f^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} \\
&\quad + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{a^2f^3} \\
&\quad - \frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} \\
&\quad + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} \\
&\quad + \frac{(4d^3)\,\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(2,-x)}{x}\,dx, x, e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{a^2f^4} \\
&= -\frac{i(c+dx)^3}{3a^2f} + \frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2} + \frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} \\
&\quad - \frac{4id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{a^2f^3} + \frac{4d^3\,\mathrm{PolyLog}\left(3,-e^{i(e+fx)}\right)}{a^2f^4} \\
&\quad - \frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} \\
&\quad + \frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.92

$$
\begin{aligned}
&\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx \\
&= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-3df^2(c+dx)^2\cos\left(\frac{1}{2}(e+fx)\right)+f^3(c+dx)^3\sin\left(\frac{1}{2}(e+fx)\right)+12d^2\cos^3\left(\frac{1}{2}(e+fx)\right)\right.}
\end{aligned}
$$

```
[In] Integrate[(c + d*x)^3/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-3*d*f^2*(c + d*x)^2*Cos[(e + f*x)/2] + f^3*(c + d*x)^
3*Sin[(e + f*x)/2] + 12*d^2*Cos[(e + f*x)/2]^3*(2*d*Log[Cos[(e + f*x)/2]] +
 f*(c + d*x)*Tan[(e + f*x)/2]) - 2*Cos[(e + f*x)/2]^3*(I*f^3*(c + d*x)^3 -
6*d*(f^2*(c + d*x)^2*Log[1 + E^(I*(e + f*x))] - (2*I)*d*f*(c + d*x)*PolyLog
[2, -E^(I*(e + f*x))] + 2*d^2*PolyLog[3, -E^(I*(e + f*x))]) - f^3*(c + d*x)
^3*Tan[(e + f*x)/2])))/(3*a^2*f^4*(1 + Cos[e + f*x])^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(231) = 462$.

Time = 2.92 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.50

method	result
risch	$-\frac{2id^3x^3}{3a^2f} - \frac{4id^2cex}{a^2f^2} + \frac{4d^2c\ln\left(e^{i(fx+e)}+1\right)x}{a^2f^2} - \frac{4id^2c\,\mathrm{Li}_2\left(-e^{i(fx+e)}\right)}{a^2f^3} + \frac{4d^2ce\ln\left(e^{i(fx+e)}\right)}{a^2f^3} - \frac{2id^2ce^2}{a^2f^3} - \frac{2id^2cx^2}{a^2f} + \frac{4id^3e}{3a^2f}$

```
[In] int((d*x+c)^3/(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/a^2/f*d^3*x^3-4*I/a^2/f^2*d^2*c*e*x+4/a^2/f^2*d^2*c*ln(exp(I*(f*x+e)
)+1)*x-4*I/a^2/f^3*d^2*c*polylog(2,-exp(I*(f*x+e)))+4/a^2/f^3*d^2*c*e*ln(ex
p(I*(f*x+e)))-2*I/a^2/f^3*d^2*c*e^2-2*I/a^2/f*d^2*c*x^2+4/3*I/a^2/f^4*d^3*e
^3+2/3*I*(3*I*c^2*d*f*exp(I*(f*x+e))+3*d^3*f^2*x^3*exp(I*(f*x+e))+6*I*c*d^2
*f*x*exp(2*I*(f*x+e))+3*I*d^3*f*x^2*exp(I*(f*x+e))+9*c*d^2*f^2*x^2*exp(I*(f
*x+e))+d^3*x^3*f^2+3*I*d^3*f*x^2*exp(2*I*(f*x+e))+3*I*c^2*d*f*exp(2*I*(f*x+
e))+9*c^2*d*f^2*x*exp(I*(f*x+e))+3*c*d^2*f^2*x^2+6*I*c*d^2*f*x*exp(I*(f*x+e
))+3*c^3*f^2*exp(I*(f*x+e))+3*c^2*d*f^2*x+6*d^3*x*exp(2*I*(f*x+e))+c^3*f^2+
6*c*d^2*exp(2*I*(f*x+e))+12*d^3*x*exp(I*(f*x+e))+12*c*d^2*exp(I*(f*x+e))+6*
d^3*x+6*d^2*c)/f^3/a^2/(exp(I*(f*x+e))+1)^3+2*I/a^2/f^3*d^3*e^2*x+2/a^2/f^2
*d^3*ln(exp(I*(f*x+e))+1)*x^2-2/a^2/f^2*d*c^2*ln(exp(I*(f*x+e)))+2/a^2/f^2*
d*c^2*ln(exp(I*(f*x+e))+1)-2/a^2/f^4*d^3*e^2*ln(exp(I*(f*x+e)))-4*I/a^2/f^3
*d^3*polylog(2,-exp(I*(f*x+e)))*x+4*d^3*polylog(3,-exp(I*(f*x+e)))/a^2/f^4-
4/a^2/f^4*d^3*ln(exp(I*(f*x+e)))+4/a^2/f^4*d^3*ln(exp(I*(f*x+e))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(228) = 456$.

Time = 0.31 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.85

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx =$$
$$-\frac{3\,d^3f^2x^2 + 6\,cd^2f^2x + 3\,c^2df^2 + 3\left(d^3f^2x^2 + 2\,cd^2f^2x + c^2df^2\right)\cos\left(fx+e\right) + 6\left(-i\,d^3fx - i\,cd^2f + (-i\right.$$

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*d^3*f^2*x^2 + 6*c*d^2*f^2*x + 3*c^2*d*f^2 + 3*(d^3*f^2*x^2 + 2*c*d^
2*f^2*x + c^2*d*f^2)*cos(f*x + e) + 6*(-I*d^3*f*x - I*c*d^2*f + (-I*d^3*f*x
 - I*c*d^2*f)*cos(f*x + e)^2 + 2*(-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e))*dil
og(-cos(f*x + e) + I*sin(f*x + e)) + 6*(I*d^3*f*x + I*c*d^2*f + (I*d^3*f*x
```

```
+ I*c*d^2*f)*cos(f*x + e)^2 + 2*(I*d^3*f*x + I*c*d^2*f)*cos(f*x + e))*dilog
(-cos(f*x + e) - I*sin(f*x + e)) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f
^2 + 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f*x + e)
^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f*x + e))*log(
cos(f*x + e) + I*sin(f*x + e) + 1) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d
*f^2 + 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f*x +
e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f*x + e))*lo
g(cos(f*x + e) - I*sin(f*x + e) + 1) - 6*(d^3*cos(f*x + e)^2 + 2*d^3*cos(f*
x + e) + d^3)*polylog(3, -cos(f*x + e) + I*sin(f*x + e)) - 6*(d^3*cos(f*x +
 e)^2 + 2*d^3*cos(f*x + e) + d^3)*polylog(3, -cos(f*x + e) - I*sin(f*x + e)
) - (2*d^3*f^3*x^3 + 6*c*d^2*f^3*x^2 + 2*c^3*f^3 + 6*c*d^2*f + 6*(c^2*d*f^3
 + d^3*f)*x + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + c^3*f^3 + 6*c*d^2*f + 3*(c^2
*d*f^3 + 2*d^3*f)*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f^4*cos(f*x + e)^2 +
2*a^2*f^4*cos(f*x + e) + a^2*f^4)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx$$
$$= \frac{\int \frac{c^3}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cos^2(e+fx)+2\cos}}{a^2}$$

```
[In] integrate((d*x+c)**3/(a+a*cos(f*x+e))**2,x)
```

```
[Out] (Integral(c**3/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(d**3*x
**3/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(co
s(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(3*c**2*d*x/(cos(e + f*x)
**2 + 2*cos(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3275 vs. $2(228) = 456$.

Time = 1.00 (sec) , antiderivative size = 3275, normalized size of antiderivative = 12.08

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(12*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*e) + cos(f*x + e))*cos
(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f*x + e) + 1)*cos(2*f*x
 + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (2*(3*cos(2*f*x + 2*e)
```

```
+ 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x + 3*e)^2 + 6*(3*cos(f*x
+ e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9*cos(f*x + e)^2 + 6*(s
in(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + sin(3*f*x + 3*e)^2 + 9*s
in(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e) + 9*sin(f*x + e)^2 + 6
*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1
) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x + 2*e) - sin(f*x + e)
)*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e) + e - 2*sin(f*x + e)
)*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x + e)^2 + 2*cos(f*x +
e))*c*d^2*e/(a^2*f^2*cos(3*f*x + 3*e)^2 + 9*a^2*f^2*cos(2*f*x + 2*e)^2 + 9*
a^2*f^2*cos(f*x + e)^2 + a^2*f^2*sin(3*f*x + 3*e)^2 + 9*a^2*f^2*sin(2*f*x +
 2*e)^2 + 18*a^2*f^2*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f^2*sin(f*x + e)
^2 + 6*a^2*f^2*cos(f*x + e) + a^2*f^2 + 2*(3*a^2*f^2*cos(2*f*x + 2*e) + 3*a
^2*f^2*cos(f*x + e) + a^2*f^2)*cos(3*f*x + 3*e) + 6*(3*a^2*f^2*cos(f*x + e)
 + a^2*f^2)*cos(2*f*x + 2*e) + 6*(a^2*f^2*sin(2*f*x + 2*e) + a^2*f^2*sin(f*
x + e))*sin(3*f*x + 3*e)) - 6*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*
e) + cos(f*x + e))*cos(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f
*x + e) + 1)*cos(2*f*x + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (
2*(3*cos(2*f*x + 2*e) + 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x +
3*e)^2 + 6*(3*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9
*cos(f*x + e)^2 + 6*(sin(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + si
n(3*f*x + 3*e)^2 + 9*sin(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e)
+ 9*sin(f*x + e)^2 + 6*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^
2 + 2*cos(f*x + e) + 1) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x
 + 2*e) - sin(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e
) + e - 2*sin(f*x + e))*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x
 + e)^2 + 2*cos(f*x + e))*c^2*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f
*x + 2*e)^2 + 9*a^2*f*cos(f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 9*a^2*f*s
in(2*f*x + 2*e)^2 + 18*a^2*f*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f*sin(f*
x + e)^2 + 6*a^2*f*cos(f*x + e) + a^2*f + 2*(3*a^2*f*cos(2*f*x + 2*e) + 3*a
^2*f*cos(f*x + e) + a^2*f)*cos(3*f*x + 3*e) + 6*(3*a^2*f*cos(f*x + e) + a^2
*f)*cos(2*f*x + 2*e) + 6*(a^2*f*sin(2*f*x + 2*e) + a^2*f*sin(f*x + e))*sin(
3*f*x + 3*e)) + c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/a^2 + 3*c*d^2*e^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + s
in(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*f^2) - 3*c^2*d*e*(3*sin(f*x + e)/(
cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*f) - 6*(2*d^3
*e^3 + 12*d^3*e - 12*c*d^2*f - 6*((f*x + e)^2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^
3*e - c*d^2*f)*(f*x + e) + ((f*x + e)^2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e -
c*d^2*f)*(f*x + e))*cos(3*f*x + 3*e) + 3*((f*x + e)^2*d^3 + d^3*e^2 + 2*d^3
 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(2*f*x + 2*e) + 3*((f*x + e)^2*d^3 + d
^3*e^2 + 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x + e) - (-I*(f*x + e
)^2*d^3 - I*d^3*e^2 - 2*I*d^3 + 2*(I*d^3*e - I*c*d^2*f)*(f*x + e))*sin(3*f*
x + 3*e) - 3*(-I*(f*x + e)^2*d^3 - I*d^3*e^2 - 2*I*d^3 + 2*(I*d^3*e - I*c*d
^2*f)*(f*x + e))*sin(2*f*x + 2*e) - 3*(-I*(f*x + e)^2*d^3 - I*d^3*e^2 - 2*I
*d^3 + 2*(I*d^3*e - I*c*d^2*f)*(f*x + e))*sin(f*x + e))*arctan2(sin(f*x + e
), cos(f*x + e) + 1) + 2*((f*x + e)^3*d^3 - 3*(d^3*e - c*d^2*f)*(f*x + e)^2
```

```
 + 3*(d^3*e^2 + 2*d^3)*(f*x + e))*cos(3*f*x + 3*e) + 6*((f*x + e)^3*d^3 - I
*d^3*e^2 + 2*d^3*e - 2*c*d^2*f - (3*d^3*e - 3*c*d^2*f + I*d^3)*(f*x + e)^2
 + (3*d^3*e^2 + 2*I*d^3*e - 2*I*c*d^2*f + 4*d^3)*(f*x + e))*cos(2*f*x + 2*e)
 + 6*(d^3*e^3 - I*(f*x + e)^2*d^3 - I*d^3*e^2 + 4*d^3*e - 4*c*d^2*f + 2*(I*
d^3*e - I*c*d^2*f + d^3)*(f*x + e))*cos(f*x + e) + 12*((f*x + e)*d^3 - d^3*
e + c*d^2*f + ((f*x + e)*d^3 - d^3*e + c*d^2*f)*cos(3*f*x + 3*e) + 3*((f*x
 + e)*d^3 - d^3*e + c*d^2*f)*cos(2*f*x + 2*e) + 3*((f*x + e)*d^3 - d^3*e + c
*d^2*f)*cos(f*x + e) + (I*(f*x + e)*d^3 - I*d^3*e + I*c*d^2*f)*sin(3*f*x +
3*e) + 3*(I*(f*x + e)*d^3 - I*d^3*e + I*c*d^2*f)*sin(2*f*x + 2*e) + 3*(I*(f
*x + e)*d^3 - I*d^3*e + I*c*d^2*f)*sin(f*x + e))*dilog(-e^(I*f*x + I*e)) +
3*(I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*I*d^3 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x
 + e) + (I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*I*d^3 + 2*(-I*d^3*e + I*c*d^2*f)*
(f*x + e))*cos(3*f*x + 3*e) + 3*(I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*I*d^3 +
2*(-I*d^3*e + I*c*d^2*f)*(f*x + e))*cos(2*f*x + 2*e) + 3*(I*(f*x + e)^2*d^3
 + I*d^3*e^2 + 2*I*d^3 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x + e))*cos(f*x + e) -
 ((f*x + e)^2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(3*
f*x + 3*e) - 3*((f*x + e)^2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e - c*d^2*f)*(f*
x + e))*sin(2*f*x + 2*e) - 3*((f*x + e)^2*d^3 + d^3*e^2 + 2*d^3 - 2*(d^3*e
 - c*d^2*f)*(f*x + e))*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2
*cos(f*x + e) + 1) + 12*(I*d^3*cos(3*f*x + 3*e) + 3*I*d^3*cos(2*f*x + 2*e)
+ 3*I*d^3*cos(f*x + e) - d^3*sin(3*f*x + 3*e) - 3*d^3*sin(2*f*x + 2*e) - 3*
d^3*sin(f*x + e) + I*d^3)*polylog(3, -e^(I*f*x + I*e)) + 2*(I*(f*x + e)^3*d
^3 + 3*(-I*d^3*e + I*c*d^2*f)*(f*x + e)^2 + 3*(I*d^3*e^2 + 2*I*d^3)*(f*x +
e))*sin(3*f*x + 3*e) + 6*(I*(f*x + e)^3*d^3 + d^3*e^2 + 2*I*d^3*e - 2*I*c*d
^2*f + (-3*I*d^3*e + 3*I*c*d^2*f + d^3)*(f*x + e)^2 + (3*I*d^3*e^2 - 2*d^3*
e + 2*c*d^2*f + 4*I*d^3)*(f*x + e))*sin(2*f*x + 2*e) + 6*(I*d^3*e^3 + (f*x
+ e)^2*d^3 + d^3*e^2 + 4*I*d^3*e - 4*I*c*d^2*f - 2*(d^3*e - c*d^2*f - I*d^3
)*(f*x + e))*sin(f*x + e))/(-3*I*a^2*f^3*cos(3*f*x + 3*e) - 9*I*a^2*f^3*cos
(2*f*x + 2*e) - 9*I*a^2*f^3*cos(f*x + e) + 3*a^2*f^3*sin(3*f*x + 3*e) + 9*a
^2*f^3*sin(2*f*x + 2*e) + 9*a^2*f^3*sin(f*x + e) - 3*I*a^2*f^3))/f
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \int \frac{(dx+c)^3}{(a\cos(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cos(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \text{Hanged}$$

```
[In] int((c + d*x)^3/(a + a*cos(e + f*x))^2,x)
[Out] \text{Hanged}
```

3.134 $\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$

Optimal result . . . 841
Rubi [A] (verified) . . . 841
Mathematica [A] (verified) . . . 844
Maple [B] (verified) . . . 844
Fricas [B] (verification not implemented) . . . 845
Sympy [F] . . . 845
Maxima [B] (verification not implemented) . . . 846
Giac [F] . . . 846
Mupad [F(-1)] . . . 847

Optimal result

Integrand size = 20, antiderivative size = 212

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{4id^2\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{3a^2f^3} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

```
[Out] -1/3*I*(d*x+c)^2/a^2/f+4/3*d*(d*x+c)*ln(1+exp(I*(f*x+e)))/a^2/f^2-4/3*I*d^2
*polylog(2,-exp(I*(f*x+e)))/a^2/f^3-1/3*d*(d*x+c)*sec(1/2*f*x+1/2*e)^2/a^2/
f^2+2/3*d^2*tan(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^2*tan(1/2*f*x+1/2*e)/a^2
/f+1/6*(d*x+c)^2*sec(1/2*f*x+1/2*e)^2*tan(1/2*f*x+1/2*e)/a^2/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3399, 4271, 3852, 8, 4269, 3800, 2221, 2317, 2438}

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} - \frac{i(c+dx)^2}{3a^2f} - \frac{4id^2\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{3a^2f^3} + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3}$$

```
[In] Int[(c + d*x)^2/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] ((-1/3*I)*(c + d*x)^2)/(a^2*f) + (4*d*(c + d*x)*Log[1 + E^(I*(e + f*x))])/(
3*a^2*f^2) - (((4*I)/3)*d^2*PolyLog[2, -E^(I*(e + f*x))])/(a^2*f^3) - (d*(c
 + d*x)*Sec[e/2 + (f*x)/2]^2)/(3*a^2*f^2) + (2*d^2*Tan[e/2 + (f*x)/2])/(3*a
^2*f^3) + ((c + d*x)^2*Tan[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)^2*Sec[e/2
 + (f*x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{\int (c+dx)^2 \csc^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{4a^2}$$

$$= -\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{6a^2} + \frac{d^2\int \sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{3a^2f^2}$$

$$= -\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} - \frac{(2d^2)\,\text{Subst}\left(\int 1\,dx, x, -\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^3} - \frac{(2d)\int (c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{3a^2f}$$

$$= -\frac{i(c+dx)^2}{3a^2f} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{(4id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{3a^2f}$$

$$= -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} - \frac{(4d^2)\int \log\left(1+e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\,dx}{3a^2f^2}$$

$$
\begin{aligned}
&= -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} \\
&\quad + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} \\
&\quad + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{(4id^2)\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{3a^2f^3} \\
&= -\frac{i(c+dx)^2}{3a^2f} + \frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2} - \frac{4id^2\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{3a^2f^3} \\
&\quad - \frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} \\
&\quad + \frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$
\begin{aligned}
&\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx \\
&= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-2df(c+dx)\cos\left(\frac{1}{2}(e+fx)\right) - 2if(c+dx)\cos^3\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)+4id\log\left(1+\right.\right.\right.}{}
\end{aligned}
$$

```
[In] Integrate[(c + d*x)^2/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-2*d*f*(c + d*x)*Cos[(e + f*x)/2] - (2*I)*f*(c + d*x)*
Cos[(e + f*x)/2]^3*(f*(c + d*x) + (4*I)*d*Log[1 + E^(I*(e + f*x))]) - (8*I)
*d^2*Cos[(e + f*x)/2]^3*PolyLog[2, -E^(I*(e + f*x))] + (2*(c^2*f^2 + 2*c*d*
f^2*x + d^2*(1 + f^2*x^2)) + (c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Co
s[e + f*x])*Sin[(e + f*x)/2]))/(3*a^2*f^3*(1 + Cos[e + f*x])^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(172) = 344.

Time = 2.55 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.69

method	result
risch	$\frac{2i\left(2id^2fx\,e^{2i(fx+e)}+3d^2f^2x^2e^{i(fx+e)}+2icdf\,e^{2i(fx+e)}+2id^2fx\,e^{i(fx+e)}+6cdf^2x\,e^{i(fx+e)}+d^2x^2f^2+2icdf\,e^{i(fx+e)}+3c^2f^2e^{i(fx+e)}+\right.}{3f^3a^2\left(e^{i(fx+e)}+1\right)^3}$

```
[In] int((d*x+c)^2/(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*(2*I*d^2*f*x*exp(2*I*(f*x+e))+3*d^2*f^2*x^2*exp(I*(f*x+e))+2*I*c*d*f*
exp(2*I*(f*x+e))+2*I*d^2*f*x*exp(I*(f*x+e))+6*c*d*f^2*x*exp(I*(f*x+e))+d^2*
x^2*f^2+2*I*c*d*f*exp(I*(f*x+e))+3*c^2*f^2*exp(I*(f*x+e))+2*c*d*f^2*x+c^2*f
^2+2*d^2*exp(2*I*(f*x+e))+4*d^2*exp(I*(f*x+e))+2*d^2)/f^3/a^2/(exp(I*(f*x+e
))+1)^3-4/3/a^2*d/f^2*c*ln(exp(I*(f*x+e)))+4/3/a^2*d/f^2*c*ln(exp(I*(f*x+e)
)+1)-2/3*I/a^2*d^2/f*x^2-4/3*I/a^2*d^2/f^2*e*x-2/3*I/a^2*d^2/f^3*e^2+4/3/a^
2*d^2/f^2*ln(exp(I*(f*x+e))+1)*x-4/3*I*d^2*polylog(2,-exp(I*(f*x+e)))/a^2/f
^3+4/3/a^2*d^2/f^3*e*ln(exp(I*(f*x+e)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(169) = 338$.

Time $= 0.27$ (sec) , antiderivative size $= 390$, normalized size of antiderivative $= 1.84$

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = -\frac{2\,d^2fx + 2\,cdf + 2\left(d^2fx + cdf\right)\cos\left(fx+e\right) + 2\left(-i\,d^2\cos\left(fx+e\right)^2 - 2i\,d^2\cos\left(fx+e\right) - i\,d^2\right)\mathrm{Li}_2(-$$

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*d^2*f*x + 2*c*d*f + 2*(d^2*f*x + c*d*f)*cos(f*x + e) + 2*(-I*d^2*co
s(f*x + e)^2 - 2*I*d^2*cos(f*x + e) - I*d^2)*dilog(-cos(f*x + e) + I*sin(f*
x + e)) + 2*(I*d^2*cos(f*x + e)^2 + 2*I*d^2*cos(f*x + e) + I*d^2)*dilog(-co
s(f*x + e) - I*sin(f*x + e)) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f
*x + e)^2 + 2*(d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) + I*sin(f*x
+ e) + 1) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e)^2 + 2*(d^2*
f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) - (2*d^2*
f^2*x^2 + 4*c*d*f^2*x + 2*c^2*f^2 + 2*d^2 + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^
2*f^2 + 2*d^2)*cos(f*x + e))*sin(f*x + e))/(a^2*f^3*cos(f*x + e)^2 + 2*a^2*
f^3*cos(f*x + e) + a^2*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = \frac{\int \frac{c^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{2cdx}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx}{a^2}$$

```
[In] integrate((d*x+c)**2/(a+a*cos(f*x+e))**2,x)
```

```
[Out] (Integral(c**2/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(d**2*x
**2/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(2*c*d*x/(cos(e +
f*x)**2 + 2*cos(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(169) = 338$.

Time = 0.61 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.66

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$$
$$= \frac{2\left(c^2f^2+2d^2+2\left(d^2fx+cdf+\left(d^2fx+cdf\right)\cos\left(3fx+3e\right)+3\left(d^2fx+cdf\right)\cos\left(2fx+2e\right)+3\left(d^2fx-\right.\right.\right.}{}$$

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*(c^2*f^2 + 2*d^2 + 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(3*f*x + 3*e
) + 3*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e) + 3*(d^2*f*x + c*d*f)*cos(f*x + e)
 - (-I*d^2*f*x - I*c*d*f)*sin(3*f*x + 3*e) - 3*(-I*d^2*f*x - I*c*d*f)*sin(2
*f*x + 2*e) - 3*(-I*d^2*f*x - I*c*d*f)*sin(f*x + e))*arctan2(sin(f*x + e),
cos(f*x + e) + 1) - (d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(3*f*x + 3*e) - (3*d^2*f
^2*x^2 - 2*I*c*d*f - 2*d^2 + 2*(3*c*d*f^2 - I*d^2*f)*x)*cos(2*f*x + 2*e) +
(3*c^2*f^2 + 2*I*d^2*f*x + 2*I*c*d*f + 4*d^2)*cos(f*x + e) - 2*(d^2*cos(3*f
*x + 3*e) + 3*d^2*cos(2*f*x + 2*e) + 3*d^2*cos(f*x + e) + I*d^2*sin(3*f*x +
 3*e) + 3*I*d^2*sin(2*f*x + 2*e) + 3*I*d^2*sin(f*x + e) + d^2)*dilog(-e^(I*
f*x + I*e)) - (I*d^2*f*x + I*c*d*f + (I*d^2*f*x + I*c*d*f)*cos(3*f*x + 3*e)
 + 3*(I*d^2*f*x + I*c*d*f)*cos(2*f*x + 2*e) + 3*(I*d^2*f*x + I*c*d*f)*cos(f
*x + e) - (d^2*f*x + c*d*f)*sin(3*f*x + 3*e) - 3*(d^2*f*x + c*d*f)*sin(2*f*
x + 2*e) - 3*(d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x +
 e)^2 + 2*cos(f*x + e) + 1) - (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin(3*f*x + 3
*e) - (3*I*d^2*f^2*x^2 + 2*c*d*f - 2*I*d^2 + 2*(3*I*c*d*f^2 + d^2*f)*x)*sin
(2*f*x + 2*e) - (-3*I*c^2*f^2 + 2*d^2*f*x + 2*c*d*f - 4*I*d^2)*sin(f*x + e)
)/(-3*I*a^2*f^3*cos(3*f*x + 3*e) - 9*I*a^2*f^3*cos(2*f*x + 2*e) - 9*I*a^2*f
^3*cos(f*x + e) + 3*a^2*f^3*sin(3*f*x + 3*e) + 9*a^2*f^3*sin(2*f*x + 2*e) +
 9*a^2*f^3*sin(f*x + e) - 3*I*a^2*f^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(a\cos(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cos(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = \text{Hanged}$$

```
[In] int((c + d*x)^2/(a + a*cos(e + f*x))^2,x)
[Out] \text{Hanged}
```

3.135 $\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx$

Optimal result . . . 848
Rubi [A] (verified) . . . 848
Mathematica [A] (verified) . . . 850
Maple [A] (verified) . . . 850
Fricas [A] (verification not implemented) . . . 851
Sympy [A] (verification not implemented) . . . 851
Maxima [B] (verification not implemented) . . . 851
Giac [B] (verification not implemented) . . . 852
Mupad [B] (verification not implemented) . . . 853

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2} - \frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

```
[Out] 2/3*d*ln(cos(1/2*f*x+1/2*e))/a^2/f^2-1/6*d*sec(1/2*f*x+1/2*e)^2/a^2/f^2+1/3
*(d*x+c)*tan(1/2*f*x+1/2*e)/a^2/f+1/6*(d*x+c)*sec(1/2*f*x+1/2*e)^2*tan(1/2*
f*x+1/2*e)/a^2/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3399, 4270, 4269, 3556}

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} - \frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}$$

```
[In] Int[(c + d*x)/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (2*d*Log[Cos[e/2 + (f*x)/2]])/(3*a^2*f^2) - (d*Sec[e/2 + (f*x)/2]^2)/(6*a^2
*f^2) + ((c + d*x)*Tan[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)*Sec[e/2 + (f*
x)/2]^2*Tan[e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (c+dx)\csc^4\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)\,dx}{4a^2} \\
&= -\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{\int (c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{6a^2} \\
&= -\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} \\
&\quad +\frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}-\frac{d\int \tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{3a^2f} \\
&= \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}-\frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} \\
&\quad +\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(2d\cos\left(\frac{3}{2}(e+fx)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+2d\cos\left(\frac{1}{2}(e+fx)\right)\left(-1+3\log\left(\cos\left(\frac{1}{2}(e+fx\right)\right.\right.\right.}{3a^2f^2(1+\cos(e+fx))^2}$$

```
[In] Integrate[(c + d*x)/(a + a*Cos[e + f*x])^2,x]
```

```
[Out] (Cos[(e + f*x)/2]*(2*d*Cos[(3*(e + f*x))/2]*Log[Cos[(e + f*x)/2]] + 2*d*Cos
[(e + f*x)/2]*(-1 + 3*Log[Cos[(e + f*x)/2]]) + f*(c + d*x)*(3*Sin[(e + f*x)
/2] + Sin[(3*(e + f*x))/2])))/(3*a^2*f^2*(1 + Cos[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

method	result	size
parallelrisch	$\frac{-2d\ln\left(\sec^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(f(dx+c)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3(dx+c)f\right)}{6a^2f^2}$	74
default	$\frac{\frac{c\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}\right)}{f}-\frac{d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f^2}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}+\frac{dx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f^2}}{a^2}$	109
norman	$\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}-\frac{d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af^2}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{dx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}}{a}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^2f^2}$	128
risch	$-\frac{2idx}{3a^2f}-\frac{2ide}{3a^2f^2}-\frac{2\left(-3idfx\,e^{i(fx+e)}-3icf\,e^{i(fx+e)}-idfx-icf+e^{2i(fx+e)}d+d\,e^{i(fx+e)}\right)}{3f^2a^2\left(e^{i(fx+e)}+1\right)^3}+\frac{2d\ln\left(e^{i(fx+e)}+1\right)}{3a^2f^2}$	129

```
[In] int((d*x+c)/(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-2*d*ln(sec(1/2*f*x+1/2*e)^2)+tan(1/2*f*x+1/2*e)*(f*(d*x+c)*tan(1/2*f*
x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)+3*(d*x+c)*f))/a^2/f^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = -\frac{d\cos(fx+e)-\left(d\cos(fx+e)^2+2\,d\cos(fx+e)+d\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)-(2\,dfx+2\,cf+(df}{3\left(a^2f^2\cos(fx+e)^2+2\,a^2f^2\cos(fx+e)+a^2f^2\right)}$$

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(d*cos(f*x + e) - (d*cos(f*x + e)^2 + 2*d*cos(f*x + e) + d)*log(1/2*co
s(f*x + e) + 1/2) - (2*d*f*x + 2*c*f + (d*f*x + c*f)*cos(f*x + e))*sin(f*x
+ e) + d)/(a^2*f^2*cos(f*x + e)^2 + 2*a^2*f^2*cos(f*x + e) + a^2*f^2)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.19

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \begin{cases} \frac{c\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f}+\frac{dx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{dx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f}-\frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{3a^2f^2}-\frac{d\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} & \text{for } f\neq \\ \frac{cx+\frac{dx^2}{2}}{(a\cos(e)+a)^2} & \text{otherwi} \end{cases}$$

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))**2,x)
```

```
[Out] Piecewise((c*tan(e/2 + f*x/2)**3/(6*a**2*f) + c*tan(e/2 + f*x/2)/(2*a**2*f)
 + d*x*tan(e/2 + f*x/2)**3/(6*a**2*f) + d*x*tan(e/2 + f*x/2)/(2*a**2*f) - d
*log(tan(e/2 + f*x/2)**2 + 1)/(3*a**2*f**2) - d*tan(e/2 + f*x/2)**2/(6*a**2
*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cos(e) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(95) = 190$.

Time = 0.35 (sec) , antiderivative size = 763, normalized size of antiderivative = 6.20

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = -\frac{2\left(2\left(3\left(fx+e\right)\sin(fx+e)+\cos(2\,fx+2\,e)+\cos(fx+e)\right)\cos(3\,fx+3\,e)+2\left(9\left(fx+e\right)\sin(fx+e)+6\,\cos(fx+e)+1\right)\cos(2\,fx+2\,e)+6\,\cos(2\,fx+2\,e\right.}{}$$

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*e) + cos(f*x + e))*cos
(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f*x + e) + 1)*cos(2*f*x
 + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (2*(3*cos(2*f*x + 2*e)
+ 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x + 3*e)^2 + 6*(3*cos(f*x
+ e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9*cos(f*x + e)^2 + 6*(s
in(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + sin(3*f*x + 3*e)^2 + 9*s
in(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e) + 9*sin(f*x + e)^2 + 6
*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1
) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x + 2*e) - sin(f*x + e)
)*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e) + e - 2*sin(f*x + e)
)*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x + e)^2 + 2*cos(f*x +
e))*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f*x + 2*e)^2 + 9*a^2*f*cos(
f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 9*a^2*f*sin(2*f*x + 2*e)^2 + 18*a^2
*f*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f*sin(f*x + e)^2 + 6*a^2*f*cos(f*x
 + e) + a^2*f + 2*(3*a^2*f*cos(2*f*x + 2*e) + 3*a^2*f*cos(f*x + e) + a^2*f)
*cos(3*f*x + 3*e) + 6*(3*a^2*f*cos(f*x + e) + a^2*f)*cos(2*f*x + 2*e) + 6*(
a^2*f*sin(2*f*x + 2*e) + a^2*f*sin(f*x + e))*sin(3*f*x + 3*e)) - c*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 + d*e
*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/
(a^2*f))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(95) = 190$.

Time $= 0.49$ (sec) , antiderivative size $= 661$, normalized size of antiderivative $= 5.37$

$$\int \frac{c + dx}{(a + a\cos(e + fx))^2}\, dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*d*f*x*tan(1/2*f*x)^3*tan(1/2*e)^2 + 3*d*f*x*tan(1/2*f*x)^2*tan(1/2*
e)^3 - 2*d*log(4*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e) +
 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(
1/2*f*x)^3*tan(1/2*e)^3 + 3*c*f*tan(1/2*f*x)^3*tan(1/2*e)^2 + 3*c*f*tan(1/2
*f*x)^2*tan(1/2*e)^3 + d*tan(1/2*f*x)^3*tan(1/2*e)^3 + d*f*x*tan(1/2*f*x)^3
 - 3*d*f*x*tan(1/2*f*x)^2*tan(1/2*e) - 3*d*f*x*tan(1/2*f*x)*tan(1/2*e)^2 +
6*d*log(4*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(ta
n(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x
)^2*tan(1/2*e)^2 + d*f*x*tan(1/2*e)^3 + c*f*tan(1/2*f*x)^3 - 3*c*f*tan(1/2*
f*x)^2*tan(1/2*e) + d*tan(1/2*f*x)^3*tan(1/2*e) - 3*c*f*tan(1/2*f*x)*tan(1/
2*e)^2 - d*tan(1/2*f*x)^2*tan(1/2*e)^2 + c*f*tan(1/2*e)^3 + d*tan(1/2*f*x)*
tan(1/2*e)^3 + 3*d*f*x*tan(1/2*f*x) + 3*d*f*x*tan(1/2*e) - 6*d*log(4*(tan(1
```

```
/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan
(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x)*tan(1/2*e) + 3
*c*f*tan(1/2*f*x) - d*tan(1/2*f*x)^2 + 3*c*f*tan(1/2*e) + d*tan(1/2*f*x)*ta
n(1/2*e) - d*tan(1/2*e)^2 + 2*d*log(4*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(
1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + ta
n(1/2*e)^2 + 1)) - d)/(a^2*f^2*tan(1/2*f*x)^3*tan(1/2*e)^3 - 3*a^2*f^2*tan(
1/2*f*x)^2*tan(1/2*e)^2 + 3*a^2*f^2*tan(1/2*f*x)*tan(1/2*e) - a^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\right)}{3\,a^2\,f^2} + \frac{(c\,f+d\,f\,x-d\,1\mathrm{i})\;2\mathrm{i}}{3\,a^2\,f^2\;\left(2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} - \frac{d\,x\,2\mathrm{i}}{3\,a^2\,f} - \frac{2\,d}{3\,a^2\,f^2\;\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)} + \frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,(c+d\,x)\;4\mathrm{i}}{3\,a^2\,f\;\left(3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+3\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}+1\right)}$$

```
[In] int((c + d*x)/(a + a*cos(e + f*x))^2,x)
```

```
[Out] (2*d*log(exp(e*1i)*exp(f*x*1i) + 1))/(3*a^2*f^2) + ((c*f - d*1i + d*f*x)*2i
)/(3*a^2*f^2*(2*exp(e*1i + f*x*1i) + exp(e*2i + f*x*2i) + 1)) - (d*x*2i)/(3
*a^2*f) - (2*d)/(3*a^2*f^2*(exp(e*1i + f*x*1i) + 1)) + (exp(e*1i + f*x*1i)*
(c + d*x)*4i)/(3*a^2*f*(3*exp(e*1i + f*x*1i) + 3*exp(e*2i + f*x*2i) + exp(e
*3i + f*x*3i) + 1))
```

3.136 $\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$

Optimal result . . . 854
Rubi [N/A] . . . 854
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Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+a\cos(e+fx))^2}, x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+a*cos(f*x+e))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

```
[In] Int[1/((c + d*x)*(a + a*Cos[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + a*Cos[e + f*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

```
[In] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+\cos\left(fx+e\right)a\right)^2}dx$$

```
[In] int(1/(d*x+c)/(a+cos(f*x+e)*a)^2,x)
[Out] int(1/(d*x+c)/(a+cos(f*x+e)*a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\cos\left(fx+e\right)+a)^2}\,dx$$

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*d*x + a^2*c + (a^2*d*x + a^2*c)*cos(f*x + e)^2 + 2*(a^2*d*x
 + a^2*c)*cos(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \frac{\int \frac{1}{c\cos^2(e+fx)+2c\cos(e+fx)+c+dx\cos^2(e+fx)+2dx\cos(e+fx)+dx}\,dx}{a^2}$$

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))**2,x)
```

```
[Out] Integral(1/(c*cos(e + f*x)**2 + 2*c*cos(e + f*x) + c + d*x*cos(e + f*x)**2
+ 2*d*x*cos(e + f*x) + d*x), x)/a**2
```

Maxima [N/A]

Not integrable

Time = 10.73 (sec) , antiderivative size = 2913, normalized size of antiderivative = 145.65

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + 6*(d^2*f*x + c*d*f)*cos(f*x +
 e)^2 + 6*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 4*d^2*sin(f*x + e) + 6*(d^
2*f*x + c*d*f)*sin(f*x + e)^2 - 2*(2*d^2*sin(2*f*x + 2*e) - (d^2*f*x + c*d*
f)*cos(2*f*x + 2*e) - (d^2*f*x + c*d*f)*cos(f*x + e) + (3*d^2*f^2*x^2 + 6*c
*d*f^2*x + 3*c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) + 2*(d^2*f*x +
 c*d*f + 6*(d^2*f*x + c*d*f)*cos(f*x + e) - 3*(3*d^2*f^2*x^2 + 6*c*d*f^2*x
+ 3*c^2*f^2 + 2*d^2)*sin(f*x + e))*cos(2*f*x + 2*e) + 2*(d^2*f*x + c*d*f)*c
os(f*x + e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x
+ a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x
+ a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^
2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^
3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)^2 +
 (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*
sin(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d
*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^
2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)*sin(f*x + e)
+ 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^
3)*sin(f*x + e)^2 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*
f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*
```

```
d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*
f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) +
 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3
 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f
^3)*cos(f*x + e))*cos(2*f*x + 2*e) + 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x
^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e) + 6*((a^2*d^3*f^3*x^3 +
3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e) + (
a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*si
n(f*x + e))*sin(3*f*x + 3*e))*integrate(2/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x +
c^2*d*f^2 + 6*d^3)*sin(f*x + e)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*
a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 +
4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f
^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2
*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^2*d^4*f^3
*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^
2*c^4*f^3)*cos(f*x + e)), x) + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d
^2*cos(2*f*x + 2*e) + 2*d^2 + (3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 4*
d^2)*cos(f*x + e) + (d^2*f*x + c*d*f)*sin(2*f*x + 2*e) + (d^2*f*x + c*d*f)*
sin(f*x + e))*sin(3*f*x + 3*e) + 2*(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2
 + 4*d^2 + 3*(3*d^2*f^2*x^2 + 6*c*d*f^2*x + 3*c^2*f^2 + 2*d^2)*cos(f*x + e)
 + 6*(d^2*f*x + c*d*f)*sin(f*x + e))*sin(2*f*x + 2*e))/(a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3
*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 +
9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)
*cos(2*f*x + 2*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*
d*f^3*x + a^2*c^3*f^3)*cos(f*x + e)^2 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*
x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*
x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*
e)^2 + 18*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*
c^3*f^3)*sin(2*f*x + 2*e)*sin(f*x + e) + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f
^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e)^2 + 2*(a^2*d^3*f^3*x
^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3
*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2
*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^
3*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) + 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^
3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*
f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(f*x + e))*cos(2*f*x + 2*e) +
 6*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3
)*cos(f*x + e) + 6*((a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^
3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e) + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^
2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(f*x + e))*sin(3*f*x + 3*e))
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/((d*x + c)*(a*cos(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 14.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(a+a\,\cos(e+f\,x))^2\,(c+d\,x)}\,dx$$

```
[In] int(1/((a + a*cos(e + f*x))^2*(c + d*x)),x)
[Out] int(1/((a + a*cos(e + f*x))^2*(c + d*x)), x)
```

3.137 $\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$

Optimal result . . . 859
Rubi [N/A] . . . 859
Mathematica [N/A] . . . 860
Maple [N/A] (verified) . . . 860
Fricas [N/A] . . . 860
Sympy [N/A] . . . 861
Maxima [N/A] . . . 861
Giac [N/A] . . . 863
Mupad [N/A] . . . 863

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}, x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

```
[In] Int[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cos[e + f*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(dx+c\right)^2\left(a+\cos\left(fx+e\right)a\right)^2}dx$$

```
[In] int(1/(d*x+c)^2/(a+cos(f*x+e)*a)^2,x)
[Out] int(1/(d*x+c)^2/(a+cos(f*x+e)*a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.95

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{\left(dx+c\right)^2(a\cos\left(fx+e\right)+a)^2}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (a^2*d^2*x^2 + 2*a^2*c*d*
x + a^2*c^2)*cos(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*cos(f
*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.25

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$
$$= \frac{\int \frac{1}{c^2\cos^2(e+fx)+2c^2\cos(e+fx)+c^2+2cdx\cos^2(e+fx)+4cdx\cos(e+fx)+2cdx+d^2x^2\cos^2(e+fx)+2d^2x^2\cos(e+fx)+d^2x^2}\,dx}{a^2}$$

```
[In] integrate(1/(d*x+c)**2/(a+a*cos(f*x+e))**2,x)
```

```
[Out] Integral(1/(c**2*cos(e + f*x)**2 + 2*c**2*cos(e + f*x) + c**2 + 2*c*d*x*cos
(e + f*x)**2 + 4*c*d*x*cos(e + f*x) + 2*c*d*x + d**2*x**2*cos(e + f*x)**2 +
 2*d**2*x**2*cos(e + f*x) + d**2*x**2), x)/a**2
```

Maxima [N/A]

Not integrable

Time = 19.79 (sec) , antiderivative size = 3521, normalized size of antiderivative = 176.05

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(12*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + 12*(d^2*f*x + c*d*f)*cos(f*x
 + e)^2 + 12*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 12*d^2*sin(f*x + e) + 1
2*(d^2*f*x + c*d*f)*sin(f*x + e)^2 - 2*(6*d^2*sin(2*f*x + 2*e) - 2*(d^2*f*x
 + c*d*f)*cos(2*f*x + 2*e) - 2*(d^2*f*x + c*d*f)*cos(f*x + e) + 3*(d^2*f^2*
x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) + 2*(2*
d^2*f*x + 2*c*d*f + 12*(d^2*f*x + c*d*f)*cos(f*x + e) - 9*(d^2*f^2*x^2 + 2*
c*d*f^2*x + c^2*f^2 + 2*d^2)*sin(f*x + e))*cos(2*f*x + 2*e) + 4*(d^2*f*x +
c*d*f)*cos(f*x + e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*
d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*
d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(
3*f*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f
^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^4*f
^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x +
a^2*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^
2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x + 3*e)^2 + 9
*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3
*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^4*f^3*x^4 + 4*a^2*c*
d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(
```

```
2*f*x + 2*e)*sin(f*x + e) + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^
2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^
2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f
^3*x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d
^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^4
*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x
+ a^2*c^4*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) + 6*(a^2*d^4*f^3*x^4 + 4*a^2*
c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3
*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3
*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e))*cos(2*f*x + 2*e) + 6*(a^2*d^4*f^3*x^4
 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^
4*f^3)*cos(f*x + e) + 6*((a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2
*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e) + (a^2*d^4
*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x
+ a^2*c^4*f^3)*sin(f*x + e))*sin(3*f*x + 3*e))*integrate(4/3*(d^3*f^2*x^2 +
 2*c*d^2*f^2*x + c^2*d*f^2 + 12*d^3)*sin(f*x + e)/(a^2*d^5*f^3*x^5 + 5*a^2*
c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4
*d*f^3*x + a^2*c^5*f^3 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^
2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*c
os(f*x + e)^2 + (a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3
*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*sin(f*x +
e)^2 + 2*(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 +
10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*cos(f*x + e)), x)
 + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 6*d^2*cos(2*f*x + 2*e) + 6*d^2
+ 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*cos(f*x + e) + 2*(d^2*f*x
 + c*d*f)*sin(2*f*x + 2*e) + 2*(d^2*f*x + c*d*f)*sin(f*x + e))*sin(3*f*x +
3*e) + 6*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2 + 3*(d^2*f^2*x^2 + 2*
c*d*f^2*x + c^2*f^2 + 2*d^2)*cos(f*x + e) + 4*(d^2*f*x + c*d*f)*sin(f*x + e
))*sin(2*f*x + 2*e))/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2
*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3
*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(3*f
*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*
x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2*d^4*f^3*
x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2
*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f*x + 3*e)^2 + 9*(a
^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*
f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3
*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f
*x + 2*e)*sin(f*x + e) + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c
^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(f*x + e)^2 + 2*(a^2*d
^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*
x + a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*
f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^4*f^
3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a
```

```
^2*c^4*f^3)*cos(f*x + e))*cos(3*f*x + 3*e) + 6*(a^2*d^4*f^3*x^4 + 4*a^2*c*d
^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + 3*(a
^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*
f^3*x + a^2*c^4*f^3)*cos(f*x + e))*cos(2*f*x + 2*e) + 6*(a^2*d^4*f^3*x^4 +
4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f
^3)*cos(f*x + e) + 6*((a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^
2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e) + (a^2*d^4*f^
3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a
^2*c^4*f^3)*sin(f*x + e))*sin(3*f*x + 3*e))
```

Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*cos(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 14.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(a+a\,\cos(e+f\,x))^2\,(c+d\,x)^2}\,dx$$

```
[In] int(1/((a + a*cos(e + f*x))^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*cos(e + f*x))^2*(c + d*x)^2), x)
```

3.138 $\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx$

Optimal result . . . 864
Rubi [A] (verified) . . . 864
Mathematica [A] (verified) . . . 867
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Fricas [B] (verification not implemented) . . . 868
Sympy [F] . . . 868
Maxima [B] (verification not implemented) . . . 869
Giac [F] . . . 869
Mupad [F(-1)] . . . 870

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3} + \frac{12d^3\,\mathrm{PolyLog}\left(3,e^{i(e+fx)}\right)}{af^4}$$

```
[Out] -I*(d*x+c)^3/a/f-(d*x+c)^3*cot(1/2*f*x+1/2*e)/a/f+6*d*(d*x+c)^2*ln(1-exp(I*
(f*x+e)))/a/f^2-12*I*d^2*(d*x+c)*polylog(2,exp(I*(f*x+e)))/a/f^3+12*d^3*pol
ylog(3,exp(I*(f*x+e)))/a/f^4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3399, 4269, 3798, 2221, 2611, 2320, 6724}

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = -\frac{12id^2(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3} + \frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{i(c+dx)^3}{af} + \frac{12d^3\,\mathrm{PolyLog}\left(3,e^{i(e+fx)}\right)}{af^4}$$

```
[In] Int[(c + d*x)^3/(a - a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^3)/(a*f) - ((c + d*x)^3*Cot[e/2 + (f*x)/2])/(a*f) + (6*d*(c
 + d*x)^2*Log[1 - E^(I*(e + f*x))])/(a*f^2) - ((12*I)*d^2*(c + d*x)*PolyLog
[2, E^(I*(e + f*x))])/(a*f^3) + (12*d^3*PolyLog[3, E^(I*(e + f*x))])/(a*f^4
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (c+dx)^3 \csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{2a} \\
&= -\frac{(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(3d)\int (c+dx)^2 \cot\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af} \\
&= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(6id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af} \\
&= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2 \log\left(1-e^{i(e+fx)}\right)}{af^2} \\
&\quad - \frac{(12d^2)\int (c+dx)\log\left(1-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\,dx}{af^2} \\
&= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2 \log\left(1-e^{i(e+fx)}\right)}{af^2} \\
&\quad - \frac{12id^2(c+dx)\,\text{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3} + \frac{(12id^3)\int \text{PolyLog}\left(2,e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\,dx}{af^3} \\
&= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2 \log\left(1-e^{i(e+fx)}\right)}{af^2} \\
&\quad - \frac{12id^2(c+dx)\,\text{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3} \\
&\quad + \frac{(12d^3)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{af^4} \\
&= -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3 \cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2 \log\left(1-e^{i(e+fx)}\right)}{af^2} \\
&\quad - \frac{12id^2(c+dx)\,\text{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3} + \frac{12d^3\,\text{PolyLog}\left(3,e^{i(e+fx)}\right)}{af^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx$$
$$= \frac{2\sin\left(\frac{1}{2}(e+fx)\right)\left(f^3(c+dx)^3\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+2\left(-\frac{if^3(c+dx)^3}{-1+e^{ie}}+3df^2(c+dx)^2\log\left(1-e^{-i(e+fx)}\right)+6id^2\right.\right.}{f^4(a-a\cos(e+fx))}$$

```
[In] Integrate[(c + d*x)^3/(a - a*Cos[e + f*x]),x]
```

```
[Out] (2*Sin[(e + f*x)/2]*(f^3*(c + d*x)^3*Csc[e/2]*Sin[(f*x)/2] + 2*(((-I)*f^3*(
c + d*x)^3)/(-1 + E^(I*e)) + 3*d*f^2*(c + d*x)^2*Log[1 - E^((-I)*(e + f*x))
] + (6*I)*d^2*f*(c + d*x)*PolyLog[2, E^((-I)*(e + f*x))] + 6*d^3*PolyLog[3,
 E^((-I)*(e + f*x))])*Sin[(e + f*x)/2]))/(f^4*(a - a*Cos[e + f*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(121) = 242.

Time = 1.34 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.52

method	result
risch	$-\frac{6id^2cx^2}{af}-\frac{6d^3\ln(1-e^{i(fx+e)})e^2}{af^4}-\frac{6id^2ce^2}{af^3}+\frac{4id^3e^3}{af^4}-\frac{6d^3e^2\ln(e^{i(fx+e)})}{af^4}+\frac{6d^3e^2\ln(e^{i(fx+e)}-1)}{af^4}-\frac{12d^2ce\ln(e^{i(fx+}$

```
[In] int((d*x+c)^3/(a-cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

```
[Out] -6*I*d^2/a/f*c*x^2-6*d^3/a/f^4*ln(1-exp(I*(f*x+e)))*e^2-6*I*d^2/a/f^3*c*e^2
+4*I*d^3/a/f^4*e^3-6*d^3/a/f^4*e^2*ln(exp(I*(f*x+e)))+6*d^3/a/f^4*e^2*ln(ex
p(I*(f*x+e))-1)-12*d^2/a/f^3*c*e*ln(exp(I*(f*x+e))-1)+12*d^2/a/f^3*c*e*ln(e
xp(I*(f*x+e)))-12*I*d^2/a/f^3*c*polylog(2,exp(I*(f*x+e)))-2*I*(d^3*x^3+3*c*
d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))-1)+12*d^2/a/f^3*c*ln(1-exp(I*(f*
x+e)))*e-2*I*d^3/a/f*x^3-12*I*d^3/a/f^3*polylog(2,exp(I*(f*x+e)))*x-12*I*d^
2/a/f^2*c*e*x+6*I*d^3/a/f^3*e^2*x+12*d^2/a/f^2*c*ln(1-exp(I*(f*x+e)))*x+6*d
^3/a/f^2*ln(1-exp(I*(f*x+e)))*x^2+12*d^3*polylog(3,exp(I*(f*x+e)))/a/f^4-6*
d/a/f^2*c^2*ln(exp(I*(f*x+e)))+6*d/a/f^2*c^2*ln(exp(I*(f*x+e))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(118) = 236$.

Time $= 0.30$ (sec) , antiderivative size $= 467$, normalized size of antiderivative $= 3.51$

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx =$$
$$-\frac{d^3f^3x^3+3\,cd^2f^3x^2+3\,c^2df^3x+c^3f^3-6\,d^3\mathrm{polylog}(3,\cos(fx+e)+i\,\sin(fx+e))\sin(fx+e)-6\,d^3\mathrm{p}}{}$$

```
[In] integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 - 6*d^3*polylog(3
, cos(f*x + e) + I*sin(f*x + e))*sin(f*x + e) - 6*d^3*polylog(3, cos(f*x +
e) - I*sin(f*x + e))*sin(f*x + e) + 6*(I*d^3*f*x + I*c*d^2*f)*dilog(cos(f*x
 + e) + I*sin(f*x + e))*sin(f*x + e) + 6*(-I*d^3*f*x - I*c*d^2*f)*dilog(cos
(f*x + e) - I*sin(f*x + e))*sin(f*x + e) - 3*(d^3*e^2 - 2*c*d^2*e*f + c^2*d
*f^2)*log(-1/2*cos(f*x + e) + 1/2*I*sin(f*x + e) + 1/2)*sin(f*x + e) - 3*(d
^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*log(-1/2*cos(f*x + e) - 1/2*I*sin(f*x + e
) + 1/2)*sin(f*x + e) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*
e*f)*log(-cos(f*x + e) + I*sin(f*x + e) + 1)*sin(f*x + e) - 3*(d^3*f^2*x^2
+ 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*log(-cos(f*x + e) - I*sin(f*x + e)
 + 1)*sin(f*x + e) + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f
^3)*cos(f*x + e))/(a*f^4*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx$$
$$= -\frac{\int \frac{c^3}{\cos(e+fx)-1}\,dx + \int \frac{d^3x^3}{\cos(e+fx)-1}\,dx + \int \frac{3cd^2x^2}{\cos(e+fx)-1}\,dx + \int \frac{3c^2dx}{\cos(e+fx)-1}\,dx}{a}$$

```
[In] integrate((d*x+c)**3/(a-a*cos(f*x+e)),x)
```

```
[Out] -(Integral(c**3/(cos(e + f*x) - 1), x) + Integral(d**3*x**3/(cos(e + f*x) -
 1), x) + Integral(3*c*d**2*x**2/(cos(e + f*x) - 1), x) + Integral(3*c**2*d
*x/(cos(e + f*x) - 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(118) = 236$.

Time = 0.45 (sec) , antiderivative size = 967, normalized size of antiderivative = 7.27

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f*x + e
)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e))*c*d^
2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 - 2*a*f^2*cos(f*x + e) + a
*f^2) - 3*((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f
*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e)
)*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*cos(f*x + e) + a*f
) + c^3*(cos(f*x + e) + 1)/(a*sin(f*x + e)) + 3*c*d^2*e^2*(cos(f*x + e) + 1
)/(a*f^2*sin(f*x + e)) - 3*c^2*d*e*(cos(f*x + e) + 1)/(a*f*sin(f*x + e)) -
(2*d^3*e^3 + 6*(d^3*e^2*cos(f*x + e) + I*d^3*e^2*sin(f*x + e) - d^3*e^2)*ar
ctan2(sin(f*x + e), cos(f*x + e) - 1) + 6*((f*x + e)^2*d^3 - 2*(d^3*e - c*d
^2*f)*(f*x + e) - ((f*x + e)^2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(f*x
 + e) - (I*(f*x + e)^2*d^3 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x + e))*sin(f*x +
e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - 2*((f*x + e)^3*d^3 + 3*(f*x
+ e)*d^3*e^2 - 3*(d^3*e - c*d^2*f)*(f*x + e)^2)*cos(f*x + e) + 12*((f*x + e
)*d^3 - d^3*e + c*d^2*f - ((f*x + e)*d^3 - d^3*e + c*d^2*f)*cos(f*x + e) -
(I*(f*x + e)*d^3 - I*d^3*e + I*c*d^2*f)*sin(f*x + e))*dilog(e^(I*f*x + I*e)
) - 3*(-I*(f*x + e)^2*d^3 - I*d^3*e^2 + 2*(I*d^3*e - I*c*d^2*f)*(f*x + e) +
 (I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x + e))*cos(f
*x + e) - ((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*sin(f
*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 12*(I*
d^3*cos(f*x + e) - d^3*sin(f*x + e) - I*d^3)*polylog(3, e^(I*f*x + I*e)) -
2*(I*(f*x + e)^3*d^3 + 3*I*(f*x + e)*d^3*e^2 + 3*(-I*d^3*e + I*c*d^2*f)*(f*
x + e)^2)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + I*a*f
^3))/f
```

Giac [F]

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = \int -\frac{(dx+c)^3}{a\cos\left(fx+e\right)-a}\,dx$$

```
[In] integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^3/(a*cos(f*x + e) - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a-a\,\cos\left(e+f\,x\right)}\,dx$$

```
[In] int((c + d*x)^3/(a - a*cos(e + f*x)),x)
[Out] int((c + d*x)^3/(a - a*cos(e + f*x)), x)
```

3.139 $\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx$

Optimal result . . . 871
Rubi [A] (verified) . . . 871
Mathematica [B] (verified) . . . 873
Maple [B] (verified) . . . 874
Fricas [B] (verification not implemented) . . . 874
Sympy [F] . . . 875
Maxima [B] (verification not implemented) . . . 875
Giac [F] . . . 875
Mupad [F(-1)] . . . 876

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = -\frac{i(c+dx)^2}{af} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{4id^2\,\mathrm{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3}$$

```
[Out] -I*(d*x+c)^2/a/f-(d*x+c)^2*cot(1/2*f*x+1/2*e)/a/f+4*d*(d*x+c)*ln(1-exp(I*(f
*x+e)))/a/f^2-4*I*d^2*polylog(2,exp(I*(f*x+e)))/a/f^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3399, 4269, 3798, 2221, 2317, 2438}

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = \frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{i(c+dx)^2}{af} - \frac{4id^2\,\mathrm{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3}$$

```
[In] Int[(c + d*x)^2/(a - a*Cos[e + f*x]),x]
```

```
[Out] ((-I)*(c + d*x)^2)/(a*f) - ((c + d*x)^2*Cot[e/2 + (f*x)/2])/(a*f) + (4*d*(c
 + d*x)*Log[1 - E^(I*(e + f*x))])/(a*f^2) - ((4*I)*d^2*PolyLog[2, E^(I*(e +
 f*x))])/(a*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (c+dx)^2 \csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{2a} \\ &= -\frac{(c+dx)^2 \cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(2d)\int (c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{i(c+dx)^2}{af} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(4id)\int \frac{e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af} \\
&= -\frac{i(c+dx)^2}{af} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} \\
&\quad + \frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{(4d^2)\int \log\left(1-e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\,dx}{af^2} \\
&= -\frac{i(c+dx)^2}{af} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2} \\
&\quad + \frac{(4id^2)\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{af^3} \\
&= -\frac{i(c+dx)^2}{af} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} \\
&\quad + \frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{4id^2\,\mathrm{PolyLog}\left(2, e^{i(e+fx)}\right)}{af^3}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 292 vs. $2(102) = 204$.

Time = 6.46 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.86

$$
\begin{aligned}
&\int \frac{(c+dx)^2}{a - a\cos(e+fx)}\,dx \\
&= \frac{2\csc\left(\frac{e}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(f^2(c+dx)^2\sin\left(\frac{fx}{2}\right) - 2cdf\left(fx\cos\left(\frac{e}{2}\right) - 2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{e}{2}\right)\right)\sin\left(\frac{1}{2}\right.\right.}{}
\end{aligned}
$$

```
[In] Integrate[(c + d*x)^2/(a - a*Cos[e + f*x]),x]

[Out] (2*Csc[e/2]*Sin[(e + f*x)/2]*(f^2*(c + d*x)^2*Sin[(f*x)/2] - 2*c*d*f*(f*x*C
os[e/2] - 2*Log[Sin[(e + f*x)/2]]*Sin[e/2])*Sin[(e + f*x)/2] + d^2*(-(E^(I*
ArcTan[Tan[e/2]])*f^2*x^2*Cos[e/2]*Sqrt[Sec[e/2]^2]) - 4*((-1/2*I)*f*x*(Pi
 - 2*ArcTan[Tan[e/2]]) - Pi*Log[1 + E^((-I)*f*x)] - (f*x + 2*ArcTan[Tan[e/2]
])*Log[1 - E^(I*(f*x + 2*ArcTan[Tan[e/2]]))] + Pi*Log[Cos[(f*x)/2]] + 2*Arc
Tan[Tan[e/2]]*Log[Sin[(f*x)/2 + ArcTan[Tan[e/2]]]] + I*PolyLog[2, E^(I*(f*x
 + 2*ArcTan[Tan[e/2]]))])*Sin[e/2])*Sin[(e + f*x)/2]))/(f^3*(a - a*Cos[e +
f*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(92) = 184$.

Time = 1.24 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{2i(x^2d^2+2cdx+c^2)}{fa\left(e^{i(fx+e)}-1\right)}-\frac{4dc\ln\left(e^{i(fx+e)}\right)}{af^2}+\frac{4dc\ln\left(e^{i(fx+e)}-1\right)}{af^2}-\frac{2id^2x^2}{af}-\frac{4id^2ex}{af^2}-\frac{2id^2e^2}{af^3}+\frac{4d^2\ln\left(1-e^{i(fx+e)}\right)x}{af^2}+\frac{4d}{}$

```
[In] int((d*x+c)^2/(a-cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))-1)-4/a/f^2*d*c*ln(exp(I*(f*x
+e)))+4*d/a/f^2*c*ln(exp(I*(f*x+e))-1)-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-2*
I/a/f^3*d^2*e^2+4*d^2/a/f^2*ln(1-exp(I*(f*x+e)))*x+4*d^2/a/f^3*ln(1-exp(I*(
f*x+e)))*e-4*I*d^2*polylog(2,exp(I*(f*x+e)))/a/f^3+4/a/f^3*d^2*e*ln(exp(I*(
f*x+e)))-4*d^2/a/f^3*e*ln(exp(I*(f*x+e))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(89) = 178$.

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.77

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx =$$
$$-\frac{d^2f^2x^2+2\,cdf^2x+c^2f^2+2i\,d^2\mathrm{Li}_2(\cos(fx+e)+i\,\sin(fx+e))\sin(fx+e)-2i\,d^2\mathrm{Li}_2(\cos(fx+e)-}{}$$

```
[In] integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*I*d^2*dilog(cos(f*x + e) + I*sin(
f*x + e))*sin(f*x + e) - 2*I*d^2*dilog(cos(f*x + e) - I*sin(f*x + e))*sin(f
*x + e) + 2*(d^2*e - c*d*f)*log(-1/2*cos(f*x + e) + 1/2*I*sin(f*x + e) + 1/
2)*sin(f*x + e) + 2*(d^2*e - c*d*f)*log(-1/2*cos(f*x + e) - 1/2*I*sin(f*x +
 e) + 1/2)*sin(f*x + e) - 2*(d^2*f*x + d^2*e)*log(-cos(f*x + e) + I*sin(f*x
 + e) + 1)*sin(f*x + e) - 2*(d^2*f*x + d^2*e)*log(-cos(f*x + e) - I*sin(f*x
 + e) + 1)*sin(f*x + e) + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*cos(f*x + e
))/(a*f^3*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = -\frac{\int \frac{c^2}{\cos(e+fx)-1}\,dx + \int \frac{d^2x^2}{\cos(e+fx)-1}\,dx + \int \frac{2cdx}{\cos(e+fx)-1}\,dx}{a}$$

```
[In] integrate((d*x+c)**2/(a-a*cos(f*x+e)),x)
```

```
[Out] -(Integral(c**2/(cos(e + f*x) - 1), x) + Integral(d**2*x**2/(cos(e + f*x) -
 1), x) + Integral(2*c*d*x/(cos(e + f*x) - 1), x))/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(89) = 178$.

Time $= 0.39$ (sec) , antiderivative size $= 310$, normalized size of antiderivative $= 3.04$

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx =$$
$$-\frac{2\left(c^2f^2 - 2\left(cdf\cos\left(fx+e\right) + i\,cdf\sin\left(fx+e\right) - cdf\right)\arctan\left(\sin\left(fx+e\right), \cos\left(fx+e\right)-1\right) + 2\left(d^2f\right.}{}$$

```
[In] integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(c^2*f^2 - 2*(c*d*f*cos(f*x + e) + I*c*d*f*sin(f*x + e) - c*d*f)*arctan2
(sin(f*x + e), cos(f*x + e) - 1) + 2*(d^2*f*x*cos(f*x + e) + I*d^2*f*x*sin(
f*x + e) - d^2*f*x)*arctan2(sin(f*x + e), -cos(f*x + e) + 1) + (d^2*f^2*x^2
 + 2*c*d*f^2*x)*cos(f*x + e) + 2*(d^2*cos(f*x + e) + I*d^2*sin(f*x + e) - d
^2)*dilog(e^(I*f*x + I*e)) + (-I*d^2*f*x - I*c*d*f + (I*d^2*f*x + I*c*d*f)*
cos(f*x + e) - (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x
 + e)^2 - 2*cos(f*x + e) + 1) + (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin(f*x + e
))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + I*a*f^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = \int -\frac{(dx+c)^2}{a\cos\left(fx+e\right)-a}\,dx$$

```
[In] integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x + c)^2/(a*cos(f*x + e) - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a-a\,\cos\left(e+f\,x\right)}\,dx$$

```
[In] int((c + d*x)^2/(a - a*cos(e + f*x)),x)
[Out] int((c + d*x)^2/(a - a*cos(e + f*x)), x)
```

3.140 $\int \frac{c+dx}{a-a\cos(e+fx)}\,dx$

Optimal result . . . 877
Rubi [A] (verified) . . . 877
Mathematica [A] (verified) . . . 878
Maple [A] (verified) . . . 878
Fricas [A] (verification not implemented) . . . 879
Sympy [B] (verification not implemented) . . . 879
Maxima [B] (verification not implemented) . . . 880
Giac [B] (verification not implemented) . . . 880
Mupad [B] (verification not implemented) . . . 881

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

```
[Out] -(d*x+c)*cot(1/2*f*x+1/2*e)/a/f+2*d*ln(sin(1/2*f*x+1/2*e))/a/f^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3399, 4269, 3556}

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = \frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2} - \frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[In] Int[(c + d*x)/(a - a*Cos[e + f*x]),x]
```

```
[Out] -(((c + d*x)*Cot[e/2 + (f*x)/2])/(a*f)) + (2*d*Log[Sin[e/2 + (f*x)/2]])/(a*
f^2)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{2a} \\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{d\int \cot\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af} \\ &= -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\begin{aligned}&\int \frac{c+dx}{a-a\cos(e+fx)}\,dx \\ &= \frac{-4d\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right) + f(c+dx)\sin(e+fx)}{af^2(-1+\cos(e+fx))}\end{aligned}$$

```
[In] Integrate[(c + d*x)/(a - a*Cos[e + f*x]),x]
```

```
[Out] (-4*d*Log[Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^2 + f*(c + d*x)*Sin[e + f*x])/
(a*f^2*(-1 + Cos[e + f*x]))
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-d\ln\left(\sec^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)f(dx+c)}{f^2a}$	54
risch	$-\frac{2idx}{af}-\frac{2ide}{af^2}-\frac{2i(dx+c)}{fa\left(e^{i(fx+e)}-1\right)}+\frac{2d\ln\left(e^{i(fx+e)}-1\right)}{af^2}$	72
norman	$\frac{-\frac{c}{af}-\frac{dx}{af}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af^2}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af^2}$	76

```
[In] int((d*x+c)/(a-cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*ln(sec(1/2*f*x+1/2*e)^2)+2*d*ln(tan(1/2*f*x+1/2*e))-cot(1/2*f*x+1/2*e)*
f*(d*x+c))/f^2/a
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = -\frac{dfx-d\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+cf+\left(dfx+cf\right)\cos\left(fx+e\right)}{af^2\sin\left(fx+e\right)}$$

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*f*x - d*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + c*f + (d*f*x + c*f)
*cos(f*x + e))/(a*f^2*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = \begin{cases} -\frac{c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}-\frac{dx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}-\frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2}+\frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2} & \text{for } f\neq 0 \\ \frac{cx+\frac{dx^2}{2}}{-a\cos(e)+a} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x)
```

```
[Out] Piecewise((-c/(a*f*tan(e/2 + f*x/2)) - d*x/(a*f*tan(e/2 + f*x/2)) - d*log(t
an(e/2 + f*x/2)**2 + 1)/(a*f**2) + 2*d*log(tan(e/2 + f*x/2))/(a*f**2), Ne(f
, 0)), ((c*x + d*x**2/2)/(-a*cos(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.20

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx$$
$$= \frac{\frac{\left(\left(\cos(fx+e)^2+\sin(fx+e)^2-2\cos(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2-2\cos(fx+e)+1\right)-2(fx+e)\sin(fx+e)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2-2af\cos(fx+e)+af} - \frac{c(\cos(fx+e)+1)}{a\sin(fx+e)} + \frac{d}{f}}$$

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] (((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f*x + e)^2
 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e))*d/(a*f*
cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*cos(f*x + e) + a*f) - c*(cos(f*
x + e) + 1)/(a*sin(f*x + e)) + d*e*(cos(f*x + e) + 1)/(a*f*sin(f*x + e)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(42) = 84$.

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.78

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx$$
$$= \frac{dfx\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+cf\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)-dfx+d\log\left(\frac{4\left(\tan\left(\frac{1}{2}fx\right)^2+2\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+\tan\left(\frac{1}{2}e\right)^2\right)}{\tan\left(\frac{1}{2}fx\right)^2\tan\left(\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx\right)^2+\tan\left(\frac{1}{2}e\right)^2+1}\right)\tan\left(\frac{1}{2}fx\right)+\cdots}{af^2\tan\left(\frac{1}{2}fx\right)+af^2\tan\left(\frac{1}{2}e\right)}$$

```
[In] integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*f*x*tan(1/2*f*x)*tan(1/2*e) + c*f*tan(1/2*f*x)*tan(1/2*e) - d*f*x + d*lo
g(4*(tan(1/2*f*x)^2 + 2*tan(1/2*f*x)*tan(1/2*e) + tan(1/2*e)^2)/(tan(1/2*f*
x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x) + d*lo
g(4*(tan(1/2*f*x)^2 + 2*tan(1/2*f*x)*tan(1/2*e) + tan(1/2*e)^2)/(tan(1/2*f*
x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*e) - c*f)/(
a*f^2*tan(1/2*f*x) + a*f^2*tan(1/2*e))
```

Mupad [B] (verification not implemented)

Time = 13.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = \frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}-1\right)}{a\,f^2} - \frac{(c+d\,x)\;2\mathrm{i}}{a\,f\;\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)} - \frac{d\,x\,2\mathrm{i}}{a\,f}$$

```
[In] int((c + d*x)/(a - a*cos(e + f*x)),x)
```

```
[Out] (2*d*log(exp(e*1i)*exp(f*x*1i) - 1))/(a*f^2) - ((c + d*x)*2i)/(a*f*(exp(e*1
i + f*x*1i) - 1)) - (d*x*2i)/(a*f)
```

3.141 $\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$

Optimal result . . . 882
Rubi [N/A] . . . 882
Mathematica [N/A] . . . 883
Maple [N/A] (verified) . . . 883
Fricas [N/A] . . . 883
Sympy [N/A] . . . 883
Maxima [N/A] . . . 884
Giac [N/A] . . . 884
Mupad [N/A] . . . 884

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a-a\cos(e+fx))}, x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

```
[In] Int[1/((c + d*x)*(a - a*Cos[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a - a*Cos[e + f*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

```
[In] Integrate[1/((c + d*x)*(a - a*Cos[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a - a*Cos[e + f*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a-\cos(fx+e)\,a)}dx$$

```
[In] int(1/(d*x+c)/(a-cos(f*x+e)*a),x)
[Out] int(1/(d*x+c)/(a-cos(f*x+e)*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)(a\cos(fx+e)-a)}\,dx$$

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="fricas")
[Out] integral(1/(a*d*x + a*c - (a*d*x + a*c)*cos(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = -\frac{\int \frac{1}{c\cos(e+fx)-c+dx\cos(e+fx)-dx}\,dx}{a}$$

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x)
[Out] -Integral(1/(c*cos(e + f*x) - c + d*x*cos(e + f*x) - d*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 285, normalized size of antiderivative = 13.57

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)(a\cos(fx+e)-a)}\,dx$$

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f*
x + a*c*d*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e))*integra
te(sin(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*a*c
*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*si
n(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)), x) +
sin(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (a*d*f*
x + a*c*f)*sin(f*x + e)^2 - 2*(a*d*f*x + a*c*f)*cos(f*x + e))
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)(a\cos(fx+e)-a)}\,dx$$

```
[In] integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x + c)*(a*cos(f*x + e) - a)), x)
```

Mupad [N/A]

Not integrable

Time = 13.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int \frac{1}{(a-a\,\cos(e+f\,x))\,(c+d\,x)}\,dx$$

```
[In] int(1/((a - a*cos(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a - a*cos(e + f*x))*(c + d*x)), x)
```

3.142 $\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$

Optimal result . . . 885
Rubi [N/A] . . . 885
Mathematica [N/A] . . . 886
Maple [N/A] (verified) . . . 886
Fricas [N/A] . . . 886
Sympy [N/A] . . . 887
Maxima [N/A] . . . 887
Giac [N/A] . . . 887
Mupad [N/A] . . . 888

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a-a\cos(e+fx))}, x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a-a*cos(f*x+e)),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

```
[In] Int[1/((c + d*x)^2*(a - a*Cos[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a - a*Cos[e + f*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

```
[In] Integrate[1/((c + d*x)^2*(a - a*Cos[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a - a*Cos[e + f*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\,(a-\cos(fx+e)\,a)}dx$$

```
[In] int(1/(d*x+c)^2/(a-cos(f*x+e)*a),x)
[Out] int(1/(d*x+c)^2/(a-cos(f*x+e)*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)^2(a\cos(fx+e)-a)}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="fricas")
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*cos(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.86

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = -\frac{\int \frac{1}{c^2\cos(e+fx)-c^2+2cdx\cos(e+fx)-2cdx+d^2x^2\cos(e+fx)-d^2x^2}\,dx}{a}$$

```
[In] integrate(1/(d*x+c)**2/(a-a*cos(f*x+e)),x)
```

```
[Out] -Integral(1/(c**2*cos(e + f*x) - c**2 + 2*c*d*x*cos(e + f*x) - 2*c*d*x + d*
**2*x**2*cos(e + f*x) - d**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 442, normalized size of antiderivative = 21.05

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)^2(a\cos(fx+e)-a)}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*f
*x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*
sin(f*x + e)^2 - 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*cos(f*x + e))*
integrate(sin(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c
^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x +
e)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f*x +
e)^2 - 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*x
+ e)), x) + sin(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x
^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a
*c^2*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x +
e))
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)^2(a\cos(fx+e)-a)}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x + c)^2*(a*cos(f*x + e) - a)), x)
```

Mupad [N/A]

Not integrable

Time = 13.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int \frac{1}{(a-a\,\cos(e+f\,x))\,(c+d\,x)^2}\,dx$$

```
[In] int(1/((a - a*cos(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a - a*cos(e + f*x))*(c + d*x)^2), x)
```

3.143 $\int x^3 \sqrt{a + a\cos(c + dx)}\, dx$

Optimal result . . . 889
Rubi [A] (verified) . . . 889
Mathematica [A] (verified) . . . 891
Maple [C] (verified) . . . 891
Fricas [F(-2)] . . . 892
Sympy [F] . . . 892
Maxima [B] (verification not implemented) . . . 892
Giac [A] (verification not implemented) . . . 893
Mupad [B] (verification not implemented) . . . 893

Optimal result

Integrand size = 18, antiderivative size = 110

$$\int x^3 \sqrt{a + a\cos(c + dx)}\, dx = -\frac{96\sqrt{a + a\cos(c + dx)}}{d^4} + \frac{12x^2\sqrt{a + a\cos(c + dx)}}{d^2} - \frac{48x\sqrt{a + a\cos(c + dx)}\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^3} + \frac{2x^3\sqrt{a + a\cos(c + dx)}\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

```
[Out] -96*(a+a*cos(d*x+c))^(1/2)/d^4+12*x^2*(a+a*cos(d*x+c))^(1/2)/d^2-48*x*(a+a*
cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)/d^3+2*x^3*(a+a*cos(d*x+c))^(1/2)*tan(1
/2*d*x+1/2*c)/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3400, 3377, 2718}

$$\int x^3 \sqrt{a + a\cos(c + dx)}\, dx = -\frac{96\sqrt{a\cos(c + dx) + a}}{d^4} - \frac{48x\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{a\cos(c + dx) + a}}{d^3} + \frac{12x^2\sqrt{a\cos(c + dx) + a}}{d^2} + \frac{2x^3\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{a\cos(c + dx) + a}}{d}$$

```
[In] Int[x^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-96*Sqrt[a + a*Cos[c + d*x]])/d^4 + (12*x^2*Sqrt[a + a*Cos[c + d*x]])/d^2
- (48*x*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d^3 + (2*x^3*Sqrt[a +
a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^3\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)dx$$

$$= \frac{2x^3\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(6\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}$$

$$= \frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2} + \frac{2x^3\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(24\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d^2}$$

$$
\begin{aligned}
&= \frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2} - \frac{48x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} \\
&\quad + \frac{2x^3\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} \\
&\quad + \frac{\left(48\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,dx}{d^3} \\
&= -\frac{96\sqrt{a+a\cos(c+dx)}}{d^4} + \frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2} \\
&\quad - \frac{48x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^3\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.48

$$
\begin{aligned}
&\int x^3\sqrt{a+a\cos(c+dx)}\,dx \\
&= \frac{2\sqrt{a(1+\cos(c+dx))}\left(6(-8+d^2x^2)+dx(-24+d^2x^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d^4}
\end{aligned}
$$

```
[In] Integrate[x^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(6*(-8 + d^2*x^2) + d*x*(-24 + d^2*x^2)*Tan[(
c + d*x)/2]))/d^4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{i\sqrt{2}\sqrt{a\left(e^{i(dx+c)}+1\right)^2e^{-i(dx+c)}}\left(d^3x^3e^{i(dx+c)}+6id^2x^2e^{i(dx+c)}-d^3x^3+6id^2x^2-24dx\,e^{i(dx+c)}-48ie^{i(dx+c)}+24dx-48i\right)}{\left(e^{i(dx+c)}+1\right)d^4}$	132

```
[In] int(x^3*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+1
)*(d^3*x^3*exp(I*(d*x+c))+6*I*d^2*x^2*exp(I*(d*x+c))-d^3*x^3+6*I*d^2*x^2-24
*d*x*exp(I*(d*x+c))-48*I*exp(I*(d*x+c))+24*d*x-48*I)/d^4
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \sqrt{a + a\cos(c + dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3 \sqrt{a + a\cos(c + dx)}\, dx = \int x^3 \sqrt{a\left(\cos\left(c + dx\right) + 1\right)}\, dx$$

```
[In] integrate(x**3*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(94) = 188$.

Time = 0.50 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int x^3 \sqrt{a + a\cos(c + dx)}\, dx =$$

$$-\frac{2\left(\sqrt{2}\sqrt{a}c^3 \sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 3\left(\sqrt{2}(dx + c)\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 2\sqrt{2}\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sqrt{a}c^2 + 3\left(\sqrt{2}(dx +\right.}{}$$

```
[In] integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(2)*sqrt(a)*c^3*sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*(d*x + c)*sin(1/2
*d*x + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a)*c^2 + 3*(sqrt(2)*(d
*x + c)^2*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*(d*x + c)*cos(1/2*d*x + 1/2*c) -
 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)*c - (sqrt(2)*(d*x + c)^3*sin(1/2*d
*x + 1/2*c) + 6*sqrt(2)*(d*x + c)^2*cos(1/2*d*x + 1/2*c) - 24*sqrt(2)*(d*x
+ c)*sin(1/2*d*x + 1/2*c) - 48*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a))/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\int x^3 \sqrt{a + a\cos(c + dx)}\, dx$$

$$= 2\sqrt{2}\sqrt{a}\left(\frac{6\left(d^2x^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right) - 8\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\right)\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{d^4} + \frac{\left(d^3x^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx\right.\right.\right.\right.$$

```
[In] integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*(6*(d^2*x^2*sgn(cos(1/2*d*x + 1/2*c)) - 8*sgn(cos(1/2*d*x
 + 1/2*c)))*cos(1/2*d*x + 1/2*c)/d^4 + (d^3*x^3*sgn(cos(1/2*d*x + 1/2*c)) -
 24*d*x*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/d^4)
```

Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int x^3 \sqrt{a + a\cos(c + dx)}\, dx =$$

$$-\frac{2\sqrt{a\,(\cos(c + d\,x) + 1)}\,(48\,\cos(c + d\,x) - 6\,d^2\,x^2 - 6\,d^2\,x^2\,\cos(c + d\,x) - d^3\,x^3\,\sin(c + d\,x) + 24\,d\,x$$

$$\frac{}{d^4\,(\cos(c + d\,x) + 1)}$$

```
[In] int(x^3*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*(a*(cos(c + d*x) + 1))^(1/2)*(48*cos(c + d*x) - 6*d^2*x^2 - 6*d^2*x^2*c
os(c + d*x) - d^3*x^3*sin(c + d*x) + 24*d*x*sin(c + d*x) + 48))/(d^4*(cos(c
 + d*x) + 1))
```

3.144 $\int x^2\sqrt{a+a\cos(c+dx)}\,dx$

Optimal result . . . 894
Rubi [A] (verified) . . . 894
Mathematica [A] (verified) . . . 896
Maple [C] (verified) . . . 896
Fricas [F(-2)] . . . 896
Sympy [F] . . . 897
Maxima [A] (verification not implemented) . . . 897
Giac [A] (verification not implemented) . . . 897
Mupad [B] (verification not implemented) . . . 898

Optimal result

Integrand size = 18, antiderivative size = 88

$$\int x^2\sqrt{a+a\cos(c+dx)}\,dx = \frac{8x\sqrt{a+a\cos(c+dx)}}{d^2} - \frac{16\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^2\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

```
[Out] 8*x*(a+a*cos(d*x+c))^(1/2)/d^2-16*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)
/d^3+2*x^2*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3400, 3377, 2717}

$$\int x^2\sqrt{a+a\cos(c+dx)}\,dx = -\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d^3} + \frac{8x\sqrt{a\cos(c+dx)+a}}{d^2} + \frac{2x^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d}$$

```
[In] Int[x^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (8*x*Sqrt[a + a*Cos[c + d*x]])/d^2 - (16*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 +
 (d*x)/2])/d^3 + (2*x^2*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 + (d*x)/2])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)dx$$

$$= \frac{2x^2\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(4\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}$$

$$= \frac{8x\sqrt{a+a\cos(c+dx)}}{d^2} + \frac{2x^2\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(8\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\cos\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d^2}$$

$$= \frac{8x\sqrt{a+a\cos(c+dx)}}{d^2} - \frac{16\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^2\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.50

$$\int x^2 \sqrt{a + a\cos(c + dx)}\, dx = \frac{2\sqrt{a(1+\cos(c+dx))}\left(4dx + (-8 + d^2x^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d^3}$$

```
[In] Integrate[x^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(4*d*x + (-8 + d^2*x^2)*Tan[(c + d*x)/2]))/d^
3
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{i\sqrt{2}\sqrt{a\left(e^{i(dx+c)}+1\right)^2 e^{-i(dx+c)}}\left(d^2x^2e^{i(dx+c)}+4idx\,e^{i(dx+c)}-x^2d^2+4idx-8\,e^{i(dx+c)}+8\right)}{\left(e^{i(dx+c)}+1\right)d^3}$	105

```
[In] int(x^2*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+1
)*(d^2*x^2*exp(I*(d*x+c))+4*I*d*x*exp(I*(d*x+c))-x^2*d^2+4*I*d*x-8*exp(I*(d
*x+c))+8)/d^3
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{a + a\cos(c + dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2 \sqrt{a + a\cos(c + dx)}\, dx = \int x^2 \sqrt{a\left(\cos\left(c + dx\right) + 1\right)}\, dx$$

```
[In] integrate(x**2*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

$$\int x^2 \sqrt{a + a\cos(c + dx)}\, dx$$
$$= \frac{2\left(\sqrt{2}\sqrt{a}c^2 \sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 2\left(\sqrt{2}(dx + c)\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 2\sqrt{2}\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\sqrt{a}c + \left(\sqrt{2}(dx + c)^{2}\right.\right.}{d^3}$$

```
[In] integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*sqrt(a)*c^2*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*(d*x + c)*sin(1/2*
d*x + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a)*c + (sqrt(2)*(d*x +
c)^2*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*(d*x + c)*cos(1/2*d*x + 1/2*c) - 8*sq
rt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a))/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int x^2 \sqrt{a + a\cos(c + dx)}\, dx$$
$$= 2\sqrt{2}\sqrt{a}\left(\frac{4\,x\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{d^2} + \frac{\left(d^2x^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right) - 8\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right.\right.}{d^3}\right.$$

```
[In] integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*(4*x*cos(1/2*d*x + 1/2*c)*sgn(cos(1/2*d*x + 1/2*c))/d^2 +
 (d^2*x^2*sgn(cos(1/2*d*x + 1/2*c)) - 8*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*
d*x + 1/2*c)/d^3)
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int x^2 \sqrt{a + a\cos(c + dx)}\, dx$$
$$= \frac{2\sqrt{a\,(\cos(c+d\,x)+1)}\,(4\,d\,x - 8\,\sin(c+d\,x) + d^2\,x^2\,\sin(c+d\,x) + 4\,d\,x\,\cos(c+d\,x))}{d^3\,(\cos(c+d\,x)+1)}$$

```
[In] int(x^2*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(a*(cos(c + d*x) + 1))^(1/2)*(4*d*x - 8*sin(c + d*x) + d^2*x^2*sin(c + d
*x) + 4*d*x*cos(c + d*x)))/(d^3*(cos(c + d*x) + 1))
```

3.145 $\int x\sqrt{a+a\cos(c+dx)}\,dx$

Optimal result . . . 899
Rubi [A] (verified) . . . 899
Mathematica [A] (verified) . . . 900
Maple [C] (verified) . . . 900
Fricas [F(-2)] . . . 901
Sympy [F] . . . 901
Maxima [A] (verification not implemented) . . . 901
Giac [A] (verification not implemented) . . . 902
Mupad [B] (verification not implemented) . . . 902

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int x\sqrt{a+a\cos(c+dx)}\,dx = \frac{4\sqrt{a+a\cos(c+dx)}}{d^2} + \frac{2x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

```
[Out] 4*(a+a*cos(d*x+c))^(1/2)/d^2+2*x*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)/
d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3400, 3377, 2718}

$$\int x\sqrt{a+a\cos(c+dx)}\,dx = \frac{4\sqrt{a\cos(c+dx)+a}}{d^2} + \frac{2x\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{d}$$

```
[In] Int[x*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + a*Cos[c + d*x]])/d^2 + (2*x*Sqrt[a + a*Cos[c + d*x]]*Tan[c/2 +
(d*x)/2])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx \\
&= \frac{2x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} \\
&\quad -\frac{\left(2\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,dx}{d} \\
&= \frac{4\sqrt{a+a\cos(c+dx)}}{d^2}+\frac{2x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int x\sqrt{a+a\cos(c+dx)}\,dx = \frac{2\sqrt{a(1+\cos(c+dx))}\left(2+dx\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d^2}$$

```
[In] Integrate[x*Sqrt[a + a*Cos[c + d*x]],x]
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(2 + d*x*Tan[(c + d*x)/2]))/d^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

method	result	size
risch	$-\frac{i\sqrt{2}\sqrt{a\left(e^{i(dx+c)}+1\right)^2e^{-i(dx+c)}}\left(dx\,e^{i(dx+c)}+2ie^{i(dx+c)}-dx+2i\right)}{\left(e^{i(dx+c)}+1\right)d^2}$	80

```
[In] int(x*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+1
)*(d*x*exp(I*(d*x+c))+2*I*exp(I*(d*x+c))-d*x+2*I)/d^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+a\cos(c+dx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{a+a\cos(c+dx)}\,dx = \int x\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,dx$$

```
[In] integrate(x*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int x\sqrt{a+a\cos(c+dx)}\,dx$$
$$= -\frac{2\left(\sqrt{2}\sqrt{a}c\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\left(\sqrt{2}(dx+c)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\right)}{d^2}$$

```
[In] integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(2)*sqrt(a)*c*sin(1/2*d*x + 1/2*c) - (sqrt(2)*(d*x + c)*sin(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a))/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int x\sqrt{a+a\cos(c+dx)}\,dx$$
$$=2\sqrt{2}\left(\frac{x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}+\frac{2\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d^2}\right)\sqrt{a}$$

```
[In] integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*(x*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d + 2*cos(1/2*d
*x + 1/2*c)*sgn(cos(1/2*d*x + 1/2*c))/d^2)*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int x\sqrt{a+a\cos(c+dx)}\,dx=\frac{2\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\left(2\cos\left(c+d\,x\right)+d\,x\,\sin\left(c+d\,x\right)+2\right)}{d^2\,\left(\cos\left(c+d\,x\right)+1\right)}$$

```
[In] int(x*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(a*(cos(c + d*x) + 1))^(1/2)*(2*cos(c + d*x) + d*x*sin(c + d*x) + 2))/(d
^2*(cos(c + d*x) + 1))
```

3.146 $\int \sqrt{a + a\cos(c + dx)}\, dx$

Optimal result . . . 903
Rubi [A] (verified) . . . 903
Mathematica [A] (verified) . . . 904
Maple [A] (verified) . . . 904
Fricas [A] (verification not implemented) . . . 904
Sympy [F] . . . 905
Maxima [A] (verification not implemented) . . . 905
Giac [A] (verification not implemented) . . . 905
Mupad [B] (verification not implemented) . . . 905

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \sqrt{a + a\cos(c + dx)}\, dx = \frac{2a\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}}$$

```
[Out] 2*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2725}

$$\int \sqrt{a + a\cos(c + dx)}\, dx = \frac{2a\sin(c + dx)}{d\sqrt{a\cos(c + dx) + a}}$$

```
[In] Int[Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2a\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{a + a\cos(c + dx)}\, dx = \frac{2\sqrt{a(1 + \cos(c + dx))}\tan\left(\frac{1}{2}(c + dx)\right)}{d}$$

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]],x]
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	43
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{i(dx+c)}+1\right)^2 e^{-i(dx+c)}}\,\left(e^{i(dx+c)}-1\right)}{\left(e^{i(dx+c)}+1\right)d}$	60

```
[In] int((a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \sqrt{a + a\cos(c + dx)}\, dx = \frac{2\sqrt{a\cos(dx + c) + a}\sin(dx + c)}{d\cos(dx + c) + d}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \sqrt{a + a\cos(c + dx)}\,dx = \int \sqrt{a\cos(c + dx) + a}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*cos(c + d*x) + a), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sqrt{a + a\cos(c + dx)}\,dx = \frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{d}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \sqrt{a + a\cos(c + dx)}\,dx = \frac{2\sqrt{2}\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{d}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d
```

Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \sqrt{a + a\cos(c + dx)}\,dx = \frac{2\,\sin(c + d\,x)\,\sqrt{a\,(\cos(c + d\,x) + 1)}}{d\,(\cos(c + d\,x) + 1)}$$

```
[In] int((a + a*cos(c + d*x))^(1/2),x)
[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*(cos(c + d*x) + 1))
```

3.147 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx$

Optimal result . . . 906
Rubi [A] (verified) . . . 906
Mathematica [A] (verified) . . . 907
Maple [F] . . . 908
Fricas [F(-2)] . . . 908
Sympy [F] . . . 908
Maxima [C] (verification not implemented) . . . 908
Giac [C] (verification not implemented) . . . 909
Mupad [F(-1)] . . . 909

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right) - \sqrt{a+a\cos(c+dx)}\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)$$

```
[Out] Ci(1/2*d*x)*cos(1/2*c)*sec(1/2*d*x+1/2*c)*(a+a*cos(d*x+c))^(1/2)-sec(1/2*d*
x+1/2*c)*Si(1/2*d*x)*sin(1/2*c)*(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3400, 3384, 3380, 3383}

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a} - \sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}$$

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/x,x]
```

```
[Out] Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]*CosIntegral[(d*x)/2]*Sec[c/2 + (d*x)/2] -
 Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)}{x}\,dx \\
&= \left(\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\frac{\cos\left(\frac{dx}{2}\right)}{x}\,dx \\
&\quad -\left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)\right)\int\frac{\sin\left(\frac{dx}{2}\right)}{x}\,dx \\
&= \cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right) \\
&\quad -\sqrt{a+a\cos(c+dx)}\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int\frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)$$

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/x,x]
```

```
[Out] Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Cos[c/2]*CosIntegral[(d*x)/2]
- Sin[c/2]*SinIntegral[(d*x)/2])
```

Maple [F]

$$\int \frac{\sqrt{a+\cos\left(dx+c\right)a}}{x}dx$$

```
[In] int((a+cos(d*x+c)*a)^(1/2)/x,x)
```

```
[Out] int((a+cos(d*x+c)*a)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{x}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx =$$
$$-\frac{1}{2}\left(\left(\sqrt{2}E_1\left(\frac{1}{2}i\,dx\right)+\sqrt{2}E_1\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}\,c\right)-\left(i\sqrt{2}E_1\left(\frac{1}{2}i\,dx\right)-i\sqrt{2}E_1\left(-\frac{1}{2}i\,dx\right)\right)\sin\left(\frac{1}{2}\,c\right)\right.$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*exp_integral_e(1, 1/2*I*d*x) + sqrt(2)*exp_integral_e(1, -1/
2*I*d*x))*cos(1/2*c) - (I*sqrt(2)*exp_integral_e(1, 1/2*I*d*x) - I*sqrt(2)*
exp_integral_e(1, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x}\,dx =$$
$$-\frac{\sqrt{2}\Big(\Re\big(\mathrm{Ci}\left(\frac{1}{2}\,dx\right)\big)\,\mathrm{sgn}\big(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\big)\tan\left(\frac{1}{4}\,c\right)^2 + \Re\big(\mathrm{Ci}\left(-\frac{1}{2}\,dx\right)\big)\,\mathrm{sgn}\big(\cos\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\big)\tan\left(\frac{1}{4}\,c\right)^2 +$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*ta
n(1/4*c)^2 + real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*ta
n(1/4*c)^2 + 2*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/4*c) - 2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/4*c) + 4*sgn(cos(1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*c) -
real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)) - real_part(cos_
integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)))*sqrt(a)/(tan(1/4*c)^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x}\,dx = \int \frac{\sqrt{a + a\,\cos\left(c + d\,x\right)}}{x}\,dx$$

```
[In] int((a + a*cos(c + d*x))^(1/2)/x,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/x, x)
```

3.148 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx$

Optimal result . . . 910
Rubi [A] (verified) . . . 910
Mathematica [A] (verified) . . . 912
Maple [F] . . . 912
Fricas [F(-2)] . . . 913
Sympy [F] . . . 913
Maxima [C] (verification not implemented) . . . 913
Giac [C] (verification not implemented) . . . 914
Mupad [F(-1)] . . . 914

Optimal result

Integrand size = 18, antiderivative size = 110

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = -\frac{\sqrt{a+a\cos(c+dx)}}{x} - \frac{1}{2}d\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right) - \frac{1}{2}d\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Si}\left(\frac{dx}{2}\right)$$

```
[Out] -(a+a*cos(d*x+c))^(1/2)/x-1/2*d*cos(1/2*c)*sec(1/2*d*x+1/2*c)*Si(1/2*d*x)*(
a+a*cos(d*x+c))^(1/2)-1/2*d*Ci(1/2*d*x)*sec(1/2*d*x+1/2*c)*sin(1/2*c)*(a+a*
cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3378, 3384, 3380, 3383}

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = -\frac{1}{2}d\sin\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a} - \frac{1}{2}d\cos\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a} - \frac{\sqrt{a\cos(c+dx)+a}}{x}$$

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Cos[c + d*x]]/x) - (d*Sqrt[a + a*Cos[c + d*x]]*CosIntegral[(d*
x)/2]*Sec[c/2 + (d*x)/2]*Sin[c/2])/2 - (d*Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]
*Sec[c/2 + (d*x)/2]*SinIntegral[(d*x)/2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \left(\sqrt{a + a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)}{x^2}\,dx$$

$$= -\frac{\sqrt{a + a\cos(c+dx)}}{x}$$

$$-\frac{1}{2}\left(d\sqrt{a + a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\,dx$$

$$
\begin{aligned}
&= -\frac{\sqrt{a + a\cos(c + dx)}}{x} \\
&\quad - \frac{1}{2}\left(d\cos\left(\frac{c}{2}\right)\sqrt{a + a\cos(c + dx)}\csc\left(\frac{1}{2}\left(c + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{dx}{2}\right)\right)\int \frac{\sin\left(\frac{dx}{2}\right)}{x}\,dx \\
&\quad - \frac{1}{2}\left(d\sqrt{a + a\cos(c + dx)}\csc\left(\frac{1}{2}\left(c + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)\right)\int \frac{\cos\left(\frac{dx}{2}\right)}{x}\,dx \\
&= -\frac{\sqrt{a + a\cos(c + dx)}}{x} \\
&\quad - \frac{1}{2}d\sqrt{a + a\cos(c + dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2} + \frac{dx}{2}\right)\sin\left(\frac{c}{2}\right) \\
&\quad - \frac{1}{2}d\cos\left(\frac{c}{2}\right)\sqrt{a + a\cos(c + dx)}\sec\left(\frac{c}{2} + \frac{dx}{2}\right)\text{Si}\left(\frac{dx}{2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$
\int \frac{\sqrt{a + a\cos(c + dx)}}{x^2}\,dx = -\frac{\sqrt{a(1 + \cos(c + dx))}\left(2 + dx\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c + dx)\right)\sin\left(\frac{c}{2}\right) + dx\cos\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c + dx)\right)\text{Si}\left(\frac{dx}{2}\right)\right.}{2x}
$$

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/x^2,x]
```

```
[Out] -1/2*(Sqrt[a*(1 + Cos[c + d*x])]*(2 + d*x*CosIntegral[(d*x)/2]*Sec[(c + d*x
)/2]*Sin[c/2] + d*x*Cos[c/2]*Sec[(c + d*x)/2]*SinIntegral[(d*x)/2]))/x
```

Maple [F]

$$
\int \frac{\sqrt{a + \cos(dx + c)\,a}}{x^2}dx
$$

```
[In] int((a+cos(d*x+c)*a)^(1/2)/x^2,x)
```

```
[Out] int((a+cos(d*x+c)*a)^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = \int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{x^2}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = -\frac{\left(\left(E_2\left(\frac{1}{2}i\,dx\right)+E_2\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}\,c\right)^3+\left(E_2\left(\frac{1}{2}i\,dx\right)+E_2\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}\,c\right)\sin\left(\frac{1}{2}\,c\right)^2+\left(-i\,E_2\left(\frac{1}{2}i\,dx\right)\right.\right.}{2\left(\left(\sqrt{2}\cos\right.\right.}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*((exp_integral_e(2, 1/2*I*d*x) + exp_integral_e(2, -1/2*I*d*x))*cos(1/
2*c)^3 + (exp_integral_e(2, 1/2*I*d*x) + exp_integral_e(2, -1/2*I*d*x))*cos
(1/2*c)*sin(1/2*c)^2 + (-I*exp_integral_e(2, 1/2*I*d*x) + I*exp_integral_e(
2, -1/2*I*d*x))*sin(1/2*c)^3 + (exp_integral_e(2, 1/2*I*d*x) + exp_integral
_e(2, -1/2*I*d*x))*cos(1/2*c) + ((-I*exp_integral_e(2, 1/2*I*d*x) + I*exp_i
ntegral_e(2, -1/2*I*d*x))*cos(1/2*c)^2 - I*exp_integral_e(2, 1/2*I*d*x) + I
*exp_integral_e(2, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)*d/((sqrt(2)*cos(1/2*c)^
2 + sqrt(2)*sin(1/2*c)^2)*(d*x + c) - (sqrt(2)*cos(1/2*c)^2 + sqrt(2)*sin(1
/2*c)^2)*c)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 560, normalized size of antiderivative = 5.09

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/4*d*x)^2*tan(1/4*c)^2 - d*x*imag_part(cos_integral(-1/2*d*x))*sgn(co
s(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d*x*sgn(cos(1/2*d*x + 1
/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c)^2 - 2*d*x*real_part(
cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)
- 2*d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4
*d*x)^2*tan(1/4*c) - d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x +
 1/2*c))*tan(1/4*d*x)^2 + d*x*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2
*d*x + 1/2*c))*tan(1/4*d*x)^2 - 2*d*x*sgn(cos(1/2*d*x + 1/2*c))*sin_integra
l(1/2*d*x)*tan(1/4*d*x)^2 + d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*c)^2 - d*x*imag_part(cos_integral(-1/2*d*x))*sgn(co
s(1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 2*d*x*sgn(cos(1/2*d*x + 1/2*c))*sin_inte
gral(1/2*d*x)*tan(1/4*c)^2 - 2*d*x*real_part(cos_integral(1/2*d*x))*sgn(cos
(1/2*d*x + 1/2*c))*tan(1/4*c) - 2*d*x*real_part(cos_integral(-1/2*d*x))*sgn
(cos(1/2*d*x + 1/2*c))*tan(1/4*c) - 4*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x
)^2*tan(1/4*c)^2 - d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1
/2*c)) + d*x*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)) -
2*d*x*sgn(cos(1/2*d*x + 1/2*c))*sin_integral(1/2*d*x) + 4*sgn(cos(1/2*d*x +
 1/2*c))*tan(1/4*d*x)^2 + 16*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/4
*c) + 4*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 - 4*sgn(cos(1/2*d*x + 1/2*c)
))*sqrt(a)/(x*tan(1/4*d*x)^2*tan(1/4*c)^2 + x*tan(1/4*d*x)^2 + x*tan(1/4*c)
^2 + x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x^2}\,dx = \int \frac{\sqrt{a + a\,\cos\left(c + d\,x\right)}}{x^2}\,dx$$

```
[In] int((a + a*cos(c + d*x))^(1/2)/x^2,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/x^2, x)
```

3.149 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx$

Optimal result . . . 915
Rubi [A] (verified) . . . 915
Mathematica [A] (verified) . . . 918
Maple [F] . . . 918
Fricas [F(-2)] . . . 918
Sympy [F] . . . 918
Maxima [C] (verification not implemented) . . . 919
Giac [C] (verification not implemented) . . . 919
Mupad [F(-1)] . . . 920

Optimal result

Integrand size = 18, antiderivative size = 151

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx = -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} - \frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{1}{8}d^2\sqrt{a+a\cos(c+dx)}\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right) + \frac{d\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x}$$

```
[Out] -1/2*(a+a*cos(d*x+c))^(1/2)/x^2-1/8*d^2*Ci(1/2*d*x)*cos(1/2*c)*sec(1/2*d*x+
1/2*c)*(a+a*cos(d*x+c))^(1/2)+1/8*d^2*sec(1/2*d*x+1/2*c)*Si(1/2*d*x)*sin(1/
2*c)*(a+a*cos(d*x+c))^(1/2)+1/4*d*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)
/x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used

$= \{3400, 3378, 3384, 3380, 3383\}$

$$\int \frac{\sqrt{a + a\cos(c+dx)}}{x^3}\,dx = -\frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\operatorname{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a} + \frac{1}{8}d^2\sin\left(\frac{c}{2}\right)\operatorname{Si}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a} - \frac{\sqrt{a\cos(c+dx)+a}}{2x^2} + \frac{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}}{4x}$$

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + a*Cos[c + d*x]]/x^2 - (d^2*Cos[c/2]*Sqrt[a + a*Cos[c + d*x]]*
CosIntegral[(d*x)/2]*Sec[c/2 + (d*x)/2])/8 + (d^2*Sqrt[a + a*Cos[c + d*x]]*
Sec[c/2 + (d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2])/8 + (d*Sqrt[a + a*Cos[c +
 d*x]]*Tan[c/2 + (d*x)/2])/(4*x)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
```

```
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)}{x^3}\,dx \\
&= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} \\
&\quad -\frac{1}{4}\left(d\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^2}\,dx \\
&= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2}+\frac{d\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x} \\
&\quad -\frac{1}{8}\left(d^2\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\,dx \\
&= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2}+\frac{d\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x} \\
&\quad -\frac{1}{8}\left(d^2\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\frac{\cos\left(\frac{dx}{2}\right)}{x}\,dx \\
&\quad +\frac{1}{8}\left(d^2\sqrt{a+a\cos(c+dx)}\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)\right)\int\frac{\sin\left(\frac{dx}{2}\right)}{x}\,dx \\
&= -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} \\
&\quad -\frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right) \\
&\quad +\frac{1}{8}d^2\sqrt{a+a\cos(c+dx)}\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right) \\
&\quad +\frac{d\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx$$
$$= \frac{\sqrt{a(1+\cos(c+dx))}\left(-4-d^2x^2\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+d^2x^2\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}\right)\text{Si}(\right.}{8x^2}$$

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/x^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(-4 - d^2*x^2*Cos[c/2]*CosIntegral[(d*x)/2]*Sec
[(c + d*x)/2] + d^2*x^2*Sec[(c + d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2] + 2*
d*x*Tan[(c + d*x)/2]))/(8*x^2)
```

Maple [F]

$$\int \frac{\sqrt{a+\cos(dx+c)\,a}}{x^3}\,dx$$

```
[In] int((a+cos(d*x+c)*a)^(1/2)/x^3,x)
```

```
[Out] int((a+cos(d*x+c)*a)^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx = \int \frac{\sqrt{a\left(\cos(c+dx)+1\right)}}{x^3}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x^3}\,dx = -\frac{\left(\left(E_3\left(\frac{1}{2}i\,dx\right) + E_3\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}\,c\right)^3 + \left(E_3\left(\frac{1}{2}i\,dx\right) + E_3\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}\,c\right)\sin\left(\frac{1}{2}\,c\right)^2 + \left(-i\,E_3\left(\frac{1}{2}i\,dx\right)\right.\right.}{2\left(\left(\sqrt{2}\cos\left(\frac{1}{2}\,c\right)^2 + \sqrt{2}\sin\left(\frac{1}{2}\,c\right)^2\right)($$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((exp_integral_e(3, 1/2*I*d*x) + exp_integral_e(3, -1/2*I*d*x))*cos(1/
2*c)^3 + (exp_integral_e(3, 1/2*I*d*x) + exp_integral_e(3, -1/2*I*d*x))*cos
(1/2*c)*sin(1/2*c)^2 + (-I*exp_integral_e(3, 1/2*I*d*x) + I*exp_integral_e(
3, -1/2*I*d*x))*sin(1/2*c)^3 + (exp_integral_e(3, 1/2*I*d*x) + exp_integral
_e(3, -1/2*I*d*x))*cos(1/2*c) + ((-I*exp_integral_e(3, 1/2*I*d*x) + I*exp_i
ntegral_e(3, -1/2*I*d*x))*cos(1/2*c)^2 - I*exp_integral_e(3, 1/2*I*d*x) + I
*exp_integral_e(3, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)*d^2/((sqrt(2)*cos(1/2*c
)^2 + sqrt(2)*sin(1/2*c)^2)*(d*x + c)^2 - 2*(sqrt(2)*cos(1/2*c)^2 + sqrt(2)
*sin(1/2*c)^2)*(d*x + c)*c + (sqrt(2)*cos(1/2*c)^2 + sqrt(2)*sin(1/2*c)^2)*
c^2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 662, normalized size of antiderivative = 4.38

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/
2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + d^2*x^2*real_part(cos_integral(-1/2*d*x
))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d^2*x^2*imag_p
art(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4
*c) - 2*d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/4*d*x)^2*tan(1/4*c) + 4*d^2*x^2*sgn(cos(1/2*d*x + 1/2*c))*sin_integr
al(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c) - d^2*x^2*real_part(cos_integral(1/2*
d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - d^2*x^2*real_part(cos_inte
gral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 + d^2*x^2*real_par
t(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 + d^2*x^2*r
```

```
eal_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 2
*d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4
*c) - 2*d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/4*c) + 4*d^2*x^2*sgn(cos(1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan
(1/4*c) - d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)
) - d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)) - 8
*d*x*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) - 8*d*x*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/4*c)^2 - 8*sgn(cos(1/2*d*x + 1/2*c))*tan
(1/4*d*x)^2*tan(1/4*c)^2 + 8*d*x*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x) + 8
*d*x*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c) + 8*sgn(cos(1/2*d*x + 1/2*c))*tan
(1/4*d*x)^2 + 32*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/4*c) + 8*sgn(
cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 - 8*sgn(cos(1/2*d*x + 1/2*c)))*sqrt(a)/(
x^2*tan(1/4*d*x)^2*tan(1/4*c)^2 + x^2*tan(1/4*d*x)^2 + x^2*tan(1/4*c)^2 + x
^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x^3}\,dx = \int \frac{\sqrt{a + a\,\cos\left(c + d\,x\right)}}{x^3}\,dx$$

```
[In] int((a + a*cos(c + d*x))^(1/2)/x^3,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/x^3, x)
```

3.150 $\int x^3\sqrt{a+a\cos(x)}\,dx$

Optimal result . . . 921
Rubi [A] (verified) . . . 921
Mathematica [A] (verified) . . . 922
Maple [C] (verified) . . . 923
Fricas [F(-2)] . . . 923
Sympy [F] . . . 923
Maxima [A] (verification not implemented) . . . 923
Giac [A] (verification not implemented) . . . 924
Mupad [B] (verification not implemented) . . . 924

Optimal result

Integrand size = 14, antiderivative size = 68

$$\int x^3\sqrt{a+a\cos(x)}\,dx = -96\sqrt{a+a\cos(x)} + 12x^2\sqrt{a+a\cos(x)} - 48x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + 2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

```
[Out] -96*(a+a*cos(x))^(1/2)+12*x^2*(a+a*cos(x))^(1/2)-48*x*(a+a*cos(x))^(1/2)*ta
n(1/2*x)+2*x^3*(a+a*cos(x))^(1/2)*tan(1/2*x)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3377, 2718}

$$\int x^3\sqrt{a+a\cos(x)}\,dx = 2x^3\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + 12x^2\sqrt{a\cos(x)+a} - 96\sqrt{a\cos(x)+a} - 48x\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[In] Int[x^3*Sqrt[a + a*Cos[x]],x]
```

```
[Out] -96*Sqrt[a + a*Cos[x]] + 12*x^2*Sqrt[a + a*Cos[x]] - 48*x*Sqrt[a + a*Cos[x]
]*Tan[x/2] + 2*x^3*Sqrt[a + a*Cos[x]]*Tan[x/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^3\cos\left(\frac{x}{2}\right)\,dx \\
&= 2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \left(6\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\sin\left(\frac{x}{2}\right)\,dx \\
&= 12x^2\sqrt{a+a\cos(x)} + 2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) \\
&\quad - \left(24\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx \\
&= 12x^2\sqrt{a+a\cos(x)} - 48x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) \\
&\quad + 2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + \left(48\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \sin\left(\frac{x}{2}\right)\,dx \\
&= -96\sqrt{a+a\cos(x)} + 12x^2\sqrt{a+a\cos(x)} \\
&\quad - 48x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + 2x^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int x^3\sqrt{a+a\cos(x)}\,dx = 2\sqrt{a(1+\cos(x))}\left(6\left(-8+x^2\right)+x\left(-24+x^2\right)\tan\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[x^3*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 2*Sqrt[a*(1 + Cos[x])]*(6*(-8 + x^2) + x*(-24 + x^2)*Tan[x/2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

method	result	size
risch	$-\frac{i\sqrt{2}\sqrt{a(\mathrm{e}^{ix}+1)^2\mathrm{e}^{-ix}}\,(6ix^2\mathrm{e}^{ix}+x^3\mathrm{e}^{ix}+6ix^2-x^3-48i\mathrm{e}^{ix}-24x\,\mathrm{e}^{ix}-48i+24x)}{\mathrm{e}^{ix}+1}$	87

```
[In] int(x^3*(a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(6*I*x^2*exp(I*x
)+x^3*exp(I*x)+6*I*x^2-x^3-48*I*exp(I*x)-24*x*exp(I*x)-48*I+24*x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3\sqrt{a+a\cos(x)}\,dx = \int x^3\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

```
[In] integrate(x**3*(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cos(x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{a+a\cos(x)}\,dx$$
$$= 2\left(\sqrt{2}x^3\sin\left(\frac{1}{2}\,x\right)+6\sqrt{2}x^2\cos\left(\frac{1}{2}\,x\right)-24\sqrt{2}x\sin\left(\frac{1}{2}\,x\right)-48\sqrt{2}\cos\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

```
[In] integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*x^3*sin(1/2*x) + 6*sqrt(2)*x^2*cos(1/2*x) - 24*sqrt(2)*x*sin(1/2
*x) - 48*sqrt(2)*cos(1/2*x))*sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int x^3 \sqrt{a + a\cos(x)}\, dx$$

$$= 2\sqrt{2}\left(6\left(x^2 \mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right) - 8\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\cos\left(\frac{1}{2}x\right) + \left(x^3 \mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right) - 24\,x\mathrm{sgn}\left(\cos\left(\right.\right.\right.\right.$$

```
[In] integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*(6*(x^2*sgn(cos(1/2*x)) - 8*sgn(cos(1/2*x)))*cos(1/2*x) + (x^3*sg
n(cos(1/2*x)) - 24*x*sgn(cos(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.34

$$\int x^3 \sqrt{a + a\cos(x)}\, dx$$

$$= \frac{2\sqrt{a}\,\sqrt{\cos(x)+1}\,(24\,x - \cos(x)\ 48\mathrm{i} + 48\,\sin(x) + x^2\,\cos(x)\ 6\mathrm{i} + x^3\,\cos(x) - 6\,x^2\,\sin(x) + x^3\,\sin(x)\ 1}{\cos(x)\ 1\mathrm{i} - \sin(x) + 1\mathrm{i}}$$

```
[In] int(x^3*(a + a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(cos(x) + 1)^(1/2)*(24*x - cos(x)*48i + 48*sin(x) + x^2*cos(x)*6
i + x^3*cos(x) - 6*x^2*sin(x) + x^3*sin(x)*1i - 24*x*cos(x) - x*sin(x)*24i
+ x^2*6i - x^3 - 48i))/(cos(x)*1i - sin(x) + 1i)
```

3.151 $\int x^2\sqrt{a+a\cos(x)}\,dx$

Optimal result . . . 925
Rubi [A] (verified) . . . 925
Mathematica [A] (verified) . . . 926
Maple [C] (verified) . . . 927
Fricas [F(-2)] . . . 927
Sympy [F] . . . 927
Maxima [A] (verification not implemented) . . . 927
Giac [A] (verification not implemented) . . . 928
Mupad [B] (verification not implemented) . . . 928

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int x^2\sqrt{a+a\cos(x)}\,dx = 8x\sqrt{a+a\cos(x)} - 16\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + 2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

```
[Out] 8*x*(a+a*cos(x))^(1/2)-16*(a+a*cos(x))^(1/2)*tan(1/2*x)+2*x^2*(a+a*cos(x))^
(1/2)*tan(1/2*x)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3377, 2717}

$$\int x^2\sqrt{a+a\cos(x)}\,dx = 2x^2\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + 8x\sqrt{a\cos(x)+a} - 16\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[In] Int[x^2*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 8*x*Sqrt[a + a*Cos[x]] - 16*Sqrt[a + a*Cos[x]]*Tan[x/2] + 2*x^2*Sqrt[a + a*
Cos[x]]*Tan[x/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\cos\left(\frac{x}{2}\right)\,dx \\
&= 2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \left(4\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\sin\left(\frac{x}{2}\right)\,dx \\
&= 8x\sqrt{a+a\cos(x)} + 2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \left(8\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \cos\left(\frac{x}{2}\right)\,dx \\
&= 8x\sqrt{a+a\cos(x)} - 16\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + 2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.10$ (sec) , antiderivative size $= 29$, normalized size of antiderivative $= 0.55$

$$\int x^2\sqrt{a+a\cos(x)}\,dx = 8\sqrt{a(1+\cos(x))}\left(x + \frac{1}{4}(-8+x^2)\tan\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[x^2*Sqrt[a + a*Cos[x]],x]
[Out] 8*Sqrt[a*(1 + Cos[x])]*(x + ((-8 + x^2)*Tan[x/2])/4)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{i\sqrt{2}\sqrt{a(e^{ix}+1)^2e^{-ix}}\,(4ix\,e^{ix}+x^2e^{ix}+4ix-x^2-8\,e^{ix}+8)}{e^{ix}+1}$	70

```
[In] int(x^2*(a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(4*I*x*exp(I*x)+
x^2*exp(I*x)+4*I*x-x^2-8*exp(I*x)+8)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2\sqrt{a+a\cos(x)}\,dx = \int x^2\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

```
[In] integrate(x**2*(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cos(x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{a+a\cos(x)}\,dx = 2\left(\sqrt{2}x^2\sin\left(\frac{1}{2}\,x\right)+4\sqrt{2}x\cos\left(\frac{1}{2}\,x\right)-8\sqrt{2}\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

```
[In] integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*x^2*sin(1/2*x) + 4*sqrt(2)*x*cos(1/2*x) - 8*sqrt(2)*sin(1/2*x))*
sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int x^2 \sqrt{a + a\cos(x)}\, dx$$
$$= 2\sqrt{2}\left(4\, x \cos\left(\frac{1}{2}\, x\right) \operatorname{sgn}\left(\cos\left(\frac{1}{2}\, x\right)\right) + \left(x^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2}\, x\right)\right) - 8 \operatorname{sgn}\left(\cos\left(\frac{1}{2}\, x\right)\right)\right) \sin\left(\frac{1}{2}\, x\right)\right)\sqrt{a}$$

```
[In] integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*(4*x*cos(1/2*x)*sgn(cos(1/2*x)) + (x^2*sgn(cos(1/2*x)) - 8*sgn(co
s(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int x^2 \sqrt{a + a\cos(x)}\, dx$$
$$= \frac{2\sqrt{a}\,\sqrt{\cos(x)+1}\,(x^2\,\cos(x) - 8\,\cos(x) - 4\,x\,\sin(x) - x^2 + 8 + x\,4\mathrm{i} - \sin(x)\,8\mathrm{i} + x^2\,\sin(x)\,1\mathrm{i} + x\,\cos($$
$$\cos(x)\,1\mathrm{i} - \sin(x) + 1\mathrm{i}$$

```
[In] int(x^2*(a + a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(cos(x) + 1)^(1/2)*(x*4i - 8*cos(x) - sin(x)*8i + x^2*cos(x) + x
^2*sin(x)*1i + x*cos(x)*4i - 4*x*sin(x) - x^2 + 8))/(cos(x)*1i - sin(x) + 1
i)
```

3.152 $\int x\sqrt{a+a\cos(x)}\,dx$

Optimal result . . . 929
Rubi [A] (verified) . . . 929
Mathematica [A] (verified) . . . 930
Maple [C] (verified) . . . 930
Fricas [F(-2)] . . . 931
Sympy [F] . . . 931
Maxima [A] (verification not implemented) . . . 931
Giac [A] (verification not implemented) . . . 931
Mupad [B] (verification not implemented) . . . 932

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int x\sqrt{a+a\cos(x)}\,dx = 4\sqrt{a+a\cos(x)} + 2x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

```
[Out] 4*(a+a*cos(x))^(1/2)+2*x*(a+a*cos(x))^(1/2)*tan(1/2*x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3400, 3377, 2718}

$$\int x\sqrt{a+a\cos(x)}\,dx = 4\sqrt{a\cos(x)+a} + 2x\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[In] Int[x*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 4*Sqrt[a + a*Cos[x]] + 2*x*Sqrt[a + a*Cos[x]]*Tan[x/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx \\ &= 2x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \left(2\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \sin\left(\frac{x}{2}\right)\,dx \\ &= 4\sqrt{a+a\cos(x)} + 2x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int x\sqrt{a+a\cos(x)}\,dx = 2\sqrt{a(1+\cos(x))}\left(2+x\tan\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[x*Sqrt[a + a*Cos[x]],x]
```

```
[Out] 2*Sqrt[a*(1 + Cos[x])]*(2 + x*Tan[x/2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

method	result	size
risch	$-\frac{i\sqrt{2}\sqrt{a\left(\mathrm{e}^{ix}+1\right)^2\mathrm{e}^{-ix}}\left(2i\mathrm{e}^{ix}+x\,\mathrm{e}^{ix}+2i-x\right)}{\mathrm{e}^{ix}+1}$	55

```
[In] int(x*(a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(2*I*exp(I*x)+x*
exp(I*x)+2*I-x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+a*cos(x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{a+a\cos(x)}\,dx = \int x\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

```
[In] integrate(x*(a+a*cos(x))**(1/2),x)
[Out] Integral(x*sqrt(a*(cos(x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int x\sqrt{a+a\cos(x)}\,dx = 2\left(\sqrt{2}x\sin\left(\frac{1}{2}\,x\right)+2\sqrt{2}\cos\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

```
[In] integrate(x*(a+a*cos(x))^(1/2),x, algorithm="maxima")
[Out] 2*(sqrt(2)*x*sin(1/2*x) + 2*sqrt(2)*cos(1/2*x))*sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int x\sqrt{a+a\cos(x)}\,dx\\&= 2\sqrt{2}\left(x\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)+2\cos\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}\end{aligned}$$

```
[In] integrate(x*(a+a*cos(x))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*(x*sgn(cos(1/2*x))*sin(1/2*x) + 2*cos(1/2*x)*sgn(cos(1/2*x)))*sqr
t(a)
```

Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56

$$\int x\sqrt{a+a\cos(x)}\,dx$$
$$= \frac{2\sqrt{a}\,\sqrt{\cos(x)+1}\,(x\,\cos(x)+\cos(x)\,2\mathrm{i}-2\,\sin(x)-x+x\,\sin(x)\,1\mathrm{i}+2\mathrm{i})}{\cos(x)\,1\mathrm{i}-\sin(x)+1\mathrm{i}}$$

```
[In] int(x*(a + a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(cos(x) + 1)^(1/2)*(cos(x)*2i - x - 2*sin(x) + x*cos(x) + x*sin(
x)*1i + 2i))/(cos(x)*1i - sin(x) + 1i)
```

3.153 $\int \sqrt{a + a\cos(x)}\, dx$

Optimal result . . . 933
Rubi [A] (verified) . . . 933
Mathematica [A] (verified) . . . 934
Maple [A] (verified) . . . 934
Fricas [A] (verification not implemented) . . . 934
Sympy [F] . . . 935
Maxima [A] (verification not implemented) . . . 935
Giac [A] (verification not implemented) . . . 935
Mupad [B] (verification not implemented) . . . 935

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \sqrt{a + a\cos(x)}\, dx = \frac{2a\sin(x)}{\sqrt{a + a\cos(x)}}$$

```
[Out] 2*a*sin(x)/(a+a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2725}

$$\int \sqrt{a + a\cos(x)}\, dx = \frac{2a\sin(x)}{\sqrt{a\cos(x) + a}}$$

```
[In] Int[Sqrt[a + a*Cos[x]],x]
```

```
[Out] (2*a*Sin[x])/Sqrt[a + a*Cos[x]]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2a\sin(x)}{\sqrt{a + a\cos(x)}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \sqrt{a + a\cos(x)}\, dx = 2\sqrt{a(1+\cos(x))}\tan\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sqrt[a + a*Cos[x]],x]
[Out] 2*Sqrt[a*(1 + Cos[x])]*Tan[x/2]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{2a\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^2\left(\frac{x}{2}\right)\right)}}$	25
risch	$-\frac{i\sqrt{2}\sqrt{a\left(e^{ix}+1\right)^2 e^{-ix}}\left(e^{ix}-1\right)}{e^{ix}+1}$	41

```
[In] int((a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(a*cos(1/2*x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \sqrt{a + a\cos(x)}\, dx = \frac{2\sqrt{a\cos(x)+a}\sin(x)}{\cos(x)+1}$$

```
[In] integrate((a+a*cos(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*cos(x) + a)*sin(x)/(cos(x) + 1)
```

Sympy [F]

$$\int \sqrt{a + a\cos(x)}\,dx = \int \sqrt{a\cos(x) + a}\,dx$$

```
[In] integrate((a+a*cos(x))**(1/2),x)
[Out] Integral(sqrt(a*cos(x) + a), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \sqrt{a + a\cos(x)}\,dx = 2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}x\right)$$

```
[In] integrate((a+a*cos(x))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sqrt{a + a\cos(x)}\,dx = 2\sqrt{2}\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)$$

```
[In] integrate((a+a*cos(x))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(1/2*x))*sin(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \sqrt{a + a\cos(x)}\,dx = \frac{2\sqrt{a}\,\sqrt{\cos(x)+1}\,(\cos(x) - 1 + \sin(x)\ 1\mathrm{i})}{\cos(x)\ 1\mathrm{i} - \sin(x) + 1\mathrm{i}}$$

```
[In] int((a + a*cos(x))^(1/2),x)
[Out] (2*a^(1/2)*(cos(x) + 1)^(1/2)*(cos(x) + sin(x)*1i - 1))/(cos(x)*1i - sin(x)
 + 1i)
```

3.154 $\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx$

Optimal result . . . 936
Rubi [A] (verified) . . . 936
Mathematica [A] (verified) . . . 937
Maple [F] . . . 937
Fricas [F(-2)] . . . 937
Sympy [F] . . . 938
Maxima [C] (verification not implemented) . . . 938
Giac [A] (verification not implemented) . . . 938
Mupad [F(-1)] . . . 938

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)$$

```
[Out] Ci(1/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3400, 3383}

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[In] Int[Sqrt[a + a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
```

```
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{a + a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx \\ &= \sqrt{a + a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + a\cos(x)}}{x}\,dx = \sqrt{a(1 + \cos(x))}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sqrt[a + a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a*(1 + Cos[x])]*CosIntegral[x/2]*Sec[x/2]
```

Maple [F]

$$\int \frac{\sqrt{a + \cos(x)\,a}}{x}dx$$

```
[In] int((a+cos(x)*a)^(1/2)/x,x)
```

```
[Out] int((a+cos(x)*a)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + a\cos(x)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \int \frac{\sqrt{a\left(\cos\left(x\right)+1\right)}}{x}\,dx$$

```
[In] integrate((a+a*cos(x))**(1/2)/x,x)
[Out] Integral(sqrt(a*(cos(x) + 1))/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \frac{1}{2}\sqrt{2}\sqrt{a}\left(\mathrm{Ei}\left(\frac{1}{2}i\,x\right)+\mathrm{Ei}\left(-\frac{1}{2}i\,x\right)\right)$$

```
[In] integrate((a+a*cos(x))^(1/2)/x,x, algorithm="maxima")
[Out] 1/2*sqrt(2)*sqrt(a)*(Ei(1/2*I*x) + Ei(-1/2*I*x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \sqrt{2}\sqrt{a}\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)$$

```
[In] integrate((a+a*cos(x))^(1/2)/x,x, algorithm="giac")
[Out] sqrt(2)*sqrt(a)*cos_integral(1/2*x)*sgn(cos(1/2*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \int \frac{\sqrt{a+a\,\cos\left(x\right)}}{x}\,dx$$

```
[In] int((a + a*cos(x))^(1/2)/x,x)
[Out] int((a + a*cos(x))^(1/2)/x, x)
```

3.155 $\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx$

Optimal result . . . 939
Rubi [A] (verified) . . . 939
Mathematica [A] (verified) . . . 940
Maple [F] . . . 940
Fricas [F(-2)] . . . 941
Sympy [F] . . . 941
Maxima [C] (verification not implemented) . . . 941
Giac [A] (verification not implemented) . . . 941
Mupad [F(-1)] . . . 942

Optimal result

Integrand size = 14, antiderivative size = 42

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = -\frac{\sqrt{a+a\cos(x)}}{x} - \frac{1}{2}\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)$$

```
[Out] -(a+a*cos(x))^(1/2)/x-1/2*sec(1/2*x)*Si(1/2*x)*(a+a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3378, 3380}

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = -\frac{1}{2}\operatorname{Si}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} - \frac{\sqrt{a\cos(x)+a}}{x}$$

```
[In] Int[Sqrt[a + a*Cos[x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Cos[x]]/x) - (Sqrt[a + a*Cos[x]]*Sec[x/2]*SinIntegral[x/2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\frac{\cos\left(\frac{x}{2}\right)}{x^2}\,dx \\ &= -\frac{\sqrt{a+a\cos(x)}}{x} - \frac{1}{2}\left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\frac{\sin\left(\frac{x}{2}\right)}{x}\,dx \\ &= -\frac{\sqrt{a+a\cos(x)}}{x} - \frac{1}{2}\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int\frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = -\frac{\sqrt{a(1+\cos(x))}\left(2+x\sec\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)\right)}{2x}$$

```
[In] Integrate[Sqrt[a + a*Cos[x]]/x^2,x]
[Out] -1/2*(Sqrt[a*(1 + Cos[x])]*(2 + x*Sec[x/2]*SinIntegral[x/2]))/x
```

Maple [F]

$$\int\frac{\sqrt{a+\cos(x)\,a}}{x^2}dx$$

```
[In] int((a+cos(x)*a)^(1/2)/x^2,x)
[Out] int((a+cos(x)*a)^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{a\left(\cos\left(x\right)+1\right)}}{x^2}\,dx$$

```
[In] integrate((a+a*cos(x))**(1/2)/x**2,x)
[Out] Integral(sqrt(a*(cos(x) + 1))/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = -\frac{1}{4}\sqrt{2}\sqrt{a}\left(i\,\Gamma\left(-1,\frac{1}{2}i\,x\right) - i\,\Gamma\left(-1,-\frac{1}{2}i\,x\right)\right)$$

```
[In] integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="maxima")
[Out] -1/4*sqrt(2)*sqrt(a)*(I*gamma(-1, 1/2*I*x) - I*gamma(-1, -1/2*I*x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = -\frac{\sqrt{2}\left(x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\mathrm{Si}\left(\frac{1}{2}\,x\right) + 2\,\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}}{2\,x}$$

```
[In] integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="giac")
[Out] -1/2*sqrt(2)*(x*sgn(cos(1/2*x))*sin_integral(1/2*x) + 2*cos(1/2*x)*sgn(cos(
1/2*x)))*sqrt(a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{a+a\,\cos(x)}}{x^2}\,dx$$

```
[In] int((a + a*cos(x))^(1/2)/x^2,x)
[Out] int((a + a*cos(x))^(1/2)/x^2, x)
```

3.156 $\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx$

Optimal result . . . 943
Rubi [A] (verified) . . . 943
Mathematica [A] (verified) . . . 944
Maple [F] . . . 945
Fricas [F(-2)] . . . 945
Sympy [F] . . . 945
Maxima [C] (verification not implemented) . . . 945
Giac [A] (verification not implemented) . . . 946
Mupad [F(-1)] . . . 946

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{a+a\cos(x)}}{2x^2} - \frac{1}{8}\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)}{4x}$$

```
[Out] -1/2*(a+a*cos(x))^(1/2)/x^2-1/8*Ci(1/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)+1/4
*(a+a*cos(x))^(1/2)*tan(1/2*x)/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3378, 3383}

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = -\frac{1}{8}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} - \frac{\sqrt{a\cos(x)+a}}{2x^2} + \frac{\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{4x}$$

```
[In] Int[Sqrt[a + a*Cos[x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + a*Cos[x]]/x^2 - (Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2]
)/8 + (Sqrt[a + a*Cos[x]]*Tan[x/2])/(4*x)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x^3}\,dx \\
&= -\frac{\sqrt{a+a\cos(x)}}{2x^2} - \frac{1}{4}\left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x^2}\,dx \\
&= -\frac{\sqrt{a+a\cos(x)}}{2x^2} + \frac{\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)}{4x} - \frac{1}{8}\left(\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx \\
&= -\frac{\sqrt{a+a\cos(x)}}{2x^2} - \frac{1}{8}\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)}{4x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{a(1+\cos(x))}\left(4 + x^2\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) - 2x\tan\left(\frac{x}{2}\right)\right)}{8x^2}$$

```
[In] Integrate[Sqrt[a + a*Cos[x]]/x^3,x]
```

```
[Out] -1/8*(Sqrt[a*(1 + Cos[x])]*(4 + x^2*CosIntegral[x/2]*Sec[x/2] - 2*x*Tan[x/2
]))/x^2
```

Maple [F]

$$\int \frac{\sqrt{a+\cos(x)\,a}}{x^3}dx$$

```
[In] int((a+cos(x)*a)^(1/2)/x^3,x)
[Out] int((a+cos(x)*a)^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{a\,(\cos(x)+1)}}{x^3}\,dx$$

```
[In] integrate((a+a*cos(x))**(1/2)/x**3,x)
[Out] Integral(sqrt(a*(cos(x) + 1))/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = \frac{1}{8}\sqrt{2}\sqrt{a}\left(\Gamma\left(-2,\frac{1}{2}i\,x\right)+\Gamma\left(-2,-\frac{1}{2}i\,x\right)\right)$$

```
[In] integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="maxima")
[Out] 1/8*sqrt(2)*sqrt(a)*(gamma(-2, 1/2*I*x) + gamma(-2, -1/2*I*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{2}\left(x^2\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right) - 2\,x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right) + 4\,\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}}{8\,x^2}$$

```
[In] integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(x^2*cos_integral(1/2*x)*sgn(cos(1/2*x)) - 2*x*sgn(cos(1/2*x))
*sin(1/2*x) + 4*cos(1/2*x)*sgn(cos(1/2*x)))*sqrt(a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{a+a\,\cos(x)}}{x^3}\,dx$$

```
[In] int((a + a*cos(x))^(1/2)/x^3,x)
```

```
[Out] int((a + a*cos(x))^(1/2)/x^3, x)
```

3.157 $\int x^3\sqrt{a-a\cos(x)}\,dx$

Optimal result . . . 947
Rubi [A] (verified) . . . 947
Mathematica [A] (verified) . . . 948
Maple [C] (verified) . . . 949
Fricas [F(-2)] . . . 949
Sympy [F] . . . 949
Maxima [B] (verification not implemented) . . . 949
Giac [A] (verification not implemented) . . . 950
Mupad [B] (verification not implemented) . . . 950

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int x^3\sqrt{a-a\cos(x)}\,dx = -96\sqrt{a-a\cos(x)} + 12x^2\sqrt{a-a\cos(x)} + 48x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right) - 2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)$$

```
[Out] -96*(a-a*cos(x))^(1/2)+12*x^2*(a-a*cos(x))^(1/2)+48*x*cot(1/2*x)*(a-a*cos(x
))^(1/2)-2*x^3*cot(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3400, 3377, 2717}

$$\int x^3\sqrt{a-a\cos(x)}\,dx = -2x^3\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)} + 12x^2\sqrt{a-a\cos(x)} - 96\sqrt{a-a\cos(x)} + 48x\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

```
[In] Int[x^3*Sqrt[a - a*Cos[x]],x]
```

```
[Out] -96*Sqrt[a - a*Cos[x]] + 12*x^2*Sqrt[a - a*Cos[x]] + 48*x*Sqrt[a - a*Cos[x]
]*Cot[x/2] - 2*x^3*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x^3\sin\left(\frac{x}{2}\right)\,dx \\
&= -2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)+\left(6\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x^2\cos\left(\frac{x}{2}\right)\,dx \\
&= 12x^2\sqrt{a-a\cos(x)}-2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right) \\
&\quad -\left(24\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x\sin\left(\frac{x}{2}\right)\,dx \\
&= 12x^2\sqrt{a-a\cos(x)}+48x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right) \\
&\quad -2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-\left(48\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int\cos\left(\frac{x}{2}\right)\,dx \\
&= -96\sqrt{a-a\cos(x)}+12x^2\sqrt{a-a\cos(x)} \\
&\quad +48x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.47

$$\int x^3\sqrt{a-a\cos(x)}\,dx=-2\sqrt{a-a\cos(x)}\left(-6\left(-8+x^2\right)+x\left(-24+x^2\right)\cot\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[x^3*Sqrt[a - a*Cos[x]],x]
[Out] -2*Sqrt[a - a*Cos[x]]*(-6*(-8 + x^2) + x*(-24 + x^2)*Cot[x/2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{-a(e^{ix}-1)^2e^{-ix}}\,(6ix^2e^{ix}+x^3e^{ix}-6ix^2+x^3-48ie^{ix}-24x\,e^{ix}+48i-24x)}{e^{ix}-1}$	86

```
[In] int(x^3*(a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(6*I*x^2*exp(I*
x)+x^3*exp(I*x)-6*I*x^2+x^3-48*I*exp(I*x)-24*x*exp(I*x)+48*I-24*x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a-a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3\sqrt{a-a\cos(x)}\,dx = \int x^3\sqrt{-a\left(\cos\left(x\right)-1\right)}\,dx$$

```
[In] integrate(x**3*(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-a*(cos(x) - 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(60) = 120$.

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.79

$$\int x^3\sqrt{a-a\cos(x)}\,dx =$$

$$-\left(\left(6\sqrt{2}x^2-6\left(\sqrt{2}x^2-8\sqrt{2}\right)\cos\left(x\right)-\left(\sqrt{2}x^3-24\sqrt{2}x\right)\sin\left(x\right)-48\sqrt{2}\right)\cos\left(\frac{1}{2}\pi+\frac{1}{2}\arctan\left(\sin\right.\right.\right.$$

```
[In] integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -((6*sqrt(2)*x^2 - 6*(sqrt(2)*x^2 - 8*sqrt(2))*cos(x) - (sqrt(2)*x^3 - 24*s
qrt(2)*x)*sin(x) - 48*sqrt(2))*cos(1/2*pi + 1/2*arctan2(sin(x), cos(x))) +
(sqrt(2)*x^3 + (sqrt(2)*x^3 - 24*sqrt(2)*x)*cos(x) - 6*(sqrt(2)*x^2 - 8*sqr
t(2))*sin(x) - 24*sqrt(2)*x)*sin(1/2*pi + 1/2*arctan2(sin(x), cos(x))))*sqr
t(a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int x^3 \sqrt{a - a\cos(x)}\, dx =$$

$$-2\sqrt{2}\left(\left(x^3 \mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right) - 24\, x \mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\right)\cos\left(\frac{1}{2}x\right) - 6\left(x^2 \mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right) - 8\,\mathrm{sgn}\left(\sin\right.\right.\right.$$

```
[In] integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*((x^3*sgn(sin(1/2*x)) - 24*x*sgn(sin(1/2*x)))*cos(1/2*x) - 6*(x^
2*sgn(sin(1/2*x)) - 8*sgn(sin(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 14.84 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int x^3 \sqrt{a - a\cos(x)}\, dx$$

$$= \frac{2\sqrt{a}\,\sqrt{1-\cos(x)}\,(24\,x + \cos(x)\ 48\mathrm{i} - 48\,\sin(x) - x^2\,\cos(x)\ 6\mathrm{i} - x^3\,\cos(x) + 6\,x^2\,\sin(x) - x^3\,\sin(x)\ 1}{\sin(x) - \cos(x)\ 1\mathrm{i} + 1\mathrm{i}}$$

```
[In] int(x^3*(a - a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(1 - cos(x))^(1/2)*(24*x + cos(x)*48i - 48*sin(x) - x^2*cos(x)*6
i - x^3*cos(x) + 6*x^2*sin(x) - x^3*sin(x)*1i + 24*x*cos(x) + x*sin(x)*24i
+ x^2*6i - x^3 - 48i))/(sin(x) - cos(x)*1i + 1i)
```

3.158 $\int x^2\sqrt{a-a\cos(x)}\,dx$

Optimal result . . . 951
Rubi [A] (verified) . . . 951
Mathematica [A] (verified) . . . 952
Maple [C] (verified) . . . 953
Fricas [F(-2)] . . . 953
Sympy [F] . . . 953
Maxima [B] (verification not implemented) . . . 953
Giac [A] (verification not implemented) . . . 954
Mupad [B] (verification not implemented) . . . 954

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int x^2\sqrt{a-a\cos(x)}\,dx = 8x\sqrt{a-a\cos(x)} + 16\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right) - 2x^2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)$$

```
[Out] 8*x*(a-a*cos(x))^(1/2)+16*cot(1/2*x)*(a-a*cos(x))^(1/2)-2*x^2*cot(1/2*x)*(a
-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3400, 3377, 2718}

$$\int x^2\sqrt{a-a\cos(x)}\,dx = -2x^2\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)} + 8x\sqrt{a-a\cos(x)} + 16\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

```
[In] Int[x^2*Sqrt[a - a*Cos[x]],x]
```

```
[Out] 8*x*Sqrt[a - a*Cos[x]] + 16*Sqrt[a - a*Cos[x]]*Cot[x/2] - 2*x^2*Sqrt[a - a*
Cos[x]]*Cot[x/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x^2\sin\left(\frac{x}{2}\right)\,dx \\
&= -2x^2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)+\left(4\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx \\
&= 8x\sqrt{a-a\cos(x)}-2x^2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-\left(8\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \sin\left(\frac{x}{2}\right)\,dx \\
&= 8x\sqrt{a-a\cos(x)}+16\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-2x^2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54

$$\int x^2\sqrt{a-a\cos(x)}\,dx = 8\sqrt{a-a\cos(x)}\left(x-\frac{1}{4}(-8+x^2)\cot\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[x^2*Sqrt[a - a*Cos[x]],x]
[Out] 8*Sqrt[a - a*Cos[x]]*(x - ((-8 + x^2)*Cot[x/2])/4)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

method	result	size
risch	$-\frac{i\sqrt{2}\sqrt{-a(e^{ix}-1)^2e^{-ix}}\,(4ix\,e^{ix}+x^2e^{ix}-4ix+x^2-8\,e^{ix}-8)}{e^{ix}-1}$	69

```
[In] int(x^2*(a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(4*I*x*exp(I*x)
+x^2*exp(I*x)-4*I*x+x^2-8*exp(I*x)-8)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a-a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2\sqrt{a-a\cos(x)}\,dx = \int x^2\sqrt{-a\left(\cos\left(x\right)-1\right)}\,dx$$

```
[In] integrate(x**2*(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a*(cos(x) - 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(46) = 92$.

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.79

$$\int x^2\sqrt{a-a\cos(x)}\,dx$$

$$= \left(\left(4\sqrt{2}x\cos\left(x\right)+\left(\sqrt{2}x^2-8\sqrt{2}\right)\sin\left(x\right)-4\sqrt{2}x\right)\cos\left(\frac{1}{2}\pi+\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)-\left(\sqrt{2}x^2-\right.$$

```
[In] integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] ((4*sqrt(2)*x*cos(x) + (sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 4*sqrt(2)*x)*cos(
1/2*pi + 1/2*arctan2(sin(x), cos(x))) - (sqrt(2)*x^2 - 4*sqrt(2)*x*sin(x) +
 (sqrt(2)*x^2 - 8*sqrt(2))*cos(x) - 8*sqrt(2))*sin(1/2*pi + 1/2*arctan2(sin
(x), cos(x))))*sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int x^2 \sqrt{a - a\cos(x)}\, dx$$
$$= 2\sqrt{2}\left(4\, x \mathrm{sgn}\left(\sin\left(\frac{1}{2}\, x\right)\right)\sin\left(\frac{1}{2}\, x\right) - \left(x^2 \mathrm{sgn}\left(\sin\left(\frac{1}{2}\, x\right)\right) - 8\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}\, x\right)\right)\right)\cos\left(\frac{1}{2}\, x\right) - 8\,\mathrm{sgn}\left(\mathrm{s}\right.$$

```
[In] integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*(4*x*sgn(sin(1/2*x))*sin(1/2*x) - (x^2*sgn(sin(1/2*x)) - 8*sgn(si
n(1/2*x)))*cos(1/2*x) - 8*sgn(sin(1/2*x)))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int x^2 \sqrt{a - a\cos(x)}\, dx$$
$$= \frac{2\sqrt{a}\,\sqrt{1-\cos(x)}\,(8\,\cos(x) - x^2\,\cos(x) + 4\,x\,\sin(x) - x^2 + 8 + x\,4\mathrm{i} + \sin(x)\,8\mathrm{i} - x^2\,\sin(x)\,1\mathrm{i} - x\,\cos($$
$$\sin(x) - \cos(x)\,1\mathrm{i} + 1\mathrm{i}$$

```
[In] int(x^2*(a - a*cos(x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*(1 - cos(x))^(1/2)*(x*4i + 8*cos(x) + sin(x)*8i - x^2*cos(x) - x
^2*sin(x)*1i - x*cos(x)*4i + 4*x*sin(x) - x^2 + 8))/(sin(x) - cos(x)*1i + 1
i)
```

3.159 $\int x\sqrt{a-a\cos(x)}\,dx$

Optimal result . . . 955
Rubi [A] (verified) . . . 955
Mathematica [A] (verified) . . . 956
Maple [C] (verified) . . . 956
Fricas [F(-2)] . . . 957
Sympy [F] . . . 957
Maxima [B] (verification not implemented) . . . 957
Giac [A] (verification not implemented) . . . 958
Mupad [B] (verification not implemented) . . . 958

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int x\sqrt{a-a\cos(x)}\,dx = 4\sqrt{a-a\cos(x)} - 2x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)$$

```
[Out] 4*(a-a*cos(x))^(1/2)-2*x*cot(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3400, 3377, 2717}

$$\int x\sqrt{a-a\cos(x)}\,dx = 4\sqrt{a-a\cos(x)} - 2x\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

```
[In] Int[x*Sqrt[a - a*Cos[x]],x]
```

```
[Out] 4*Sqrt[a - a*Cos[x]] - 2*x*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int x\sin\left(\frac{x}{2}\right)\,dx \\ &= -2x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)+\left(2\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int\cos\left(\frac{x}{2}\right)\,dx \\ &= 4\sqrt{a-a\cos(x)}-2x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int x\sqrt{a-a\cos(x)}\,dx = -2\sqrt{a-a\cos(x)}\left(-2+x\cot\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[x*Sqrt[a - a*Cos[x]],x]
```

```
[Out] -2*Sqrt[a - a*Cos[x]]*(-2 + x*Cot[x/2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

method	result	size
risch	$-\frac{i\sqrt{2}\sqrt{-a\left(\mathrm{e}^{ix}-1\right)^2\mathrm{e}^{-ix}}\left(2i\mathrm{e}^{ix}+x\,\mathrm{e}^{ix}-2i+x\right)}{\mathrm{e}^{ix}-1}$	54

```
[In] int(x*(a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(2*I*exp(I*x)+x
*exp(I*x)-2*I+x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a-a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{a-a\cos(x)}\,dx = \int x\sqrt{-a\left(\cos\left(x\right)-1\right)}\,dx$$

```
[In] integrate(x*(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a*(cos(x) - 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(28) = 56$.

Time $= 0.34$ (sec) , antiderivative size $= 72$, normalized size of antiderivative $= 2.12$

$$\int x\sqrt{a-a\cos(x)}\,dx$$
$$= \left(\left(\sqrt{2}x\sin\left(x\right)+2\sqrt{2}\cos\left(x\right)-2\sqrt{2}\right)\cos\left(\frac{1}{2}\pi+\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)-\left(\sqrt{2}x\cos\left(x\right)+\sqrt{2}x-2\right.$$

```
[In] integrate(x*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] ((sqrt(2)*x*sin(x) + 2*sqrt(2)*cos(x) - 2*sqrt(2))*cos(1/2*pi + 1/2*arctan2
(sin(x), cos(x))) - (sqrt(2)*x*cos(x) + sqrt(2)*x - 2*sqrt(2)*sin(x))*sin(1
/2*pi + 1/2*arctan2(sin(x), cos(x))))*sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int x\sqrt{a-a\cos(x)}\,dx$$
$$= -2\sqrt{2}\left(x\cos\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right) - 2\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

```
[In] integrate(x*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*(x*cos(1/2*x)*sgn(sin(1/2*x)) - 2*sgn(sin(1/2*x))*sin(1/2*x))*sq
rt(a)
```

Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int x\sqrt{a-a\cos(x)}\,dx$$
$$= -\frac{2\sqrt{a}\,\sqrt{1-\cos(x)}\,(x+\cos(x)\ 2\mathrm{i}-2\ \sin(x)+x\ \cos(x)+x\ \sin(x)\ 1\mathrm{i}-2\mathrm{i})}{\sin(x)-\cos(x)\ 1\mathrm{i}+1\mathrm{i}}$$

```
[In] int(x*(a - a*cos(x))^(1/2),x)
```

```
[Out] -(2*a^(1/2)*(1 - cos(x))^(1/2)*(x + cos(x)*2i - 2*sin(x) + x*cos(x) + x*sin
(x)*1i - 2i))/(sin(x) - cos(x)*1i + 1i)
```

3.160 $\int \sqrt{a - a\cos(x)}\, dx$

Optimal result . . . 959
Rubi [A] (verified) . . . 959
Mathematica [A] (verified) . . . 960
Maple [A] (verified) . . . 960
Fricas [A] (verification not implemented) . . . 960
Sympy [F] . . . 961
Maxima [A] (verification not implemented) . . . 961
Giac [A] (verification not implemented) . . . 961
Mupad [B] (verification not implemented) . . . 961

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \sqrt{a - a\cos(x)}\, dx = -\frac{2a\sin(x)}{\sqrt{a - a\cos(x)}}$$

```
[Out] -2*a*sin(x)/(a-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2725}

$$\int \sqrt{a - a\cos(x)}\, dx = -\frac{2a\sin(x)}{\sqrt{a - a\cos(x)}}$$

```
[In] Int[Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-2*a*Sin[x])/Sqrt[a - a*Cos[x]]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2a\sin(x)}{\sqrt{a - a\cos(x)}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \sqrt{a - a\cos(x)}\, dx = -2\sqrt{a - a\cos(x)}\cot\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sqrt[a - a*Cos[x]],x]
[Out] -2*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

method	result	size
default	$-\frac{2\sin\left(\frac{x}{2}\right)a\cos\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}}$	25
risch	$-\frac{i\sqrt{2}\sqrt{-a\left(e^{ix}-1\right)^2e^{-ix}}\left(e^{ix}+1\right)}{e^{ix}-1}$	42

```
[In] int((a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*sin(1/2*x)*a*cos(1/2*x)*2^(1/2)/(a*sin(1/2*x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \sqrt{a - a\cos(x)}\, dx = -\frac{2\sqrt{-a\cos(x) + a}(\cos(x) + 1)}{\sin(x)}$$

```
[In] integrate((a-a*cos(x))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*cos(x) + a)*(cos(x) + 1)/sin(x)
```

Sympy [F]

$$\int \sqrt{a - a\cos(x)}\,dx = \int \sqrt{-a\cos(x) + a}\,dx$$

```
[In] integrate((a-a*cos(x))**(1/2),x)
[Out] Integral(sqrt(-a*cos(x) + a), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \sqrt{a - a\cos(x)}\,dx = -\frac{2\sqrt{2}\sqrt{a}}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1}}$$

```
[In] integrate((a-a*cos(x))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(2)*sqrt(a)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \sqrt{a - a\cos(x)}\,dx = -2\sqrt{2}\left(\cos\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right) - \operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}$$

```
[In] integrate((a-a*cos(x))^(1/2),x, algorithm="giac")
[Out] -2*sqrt(2)*(cos(1/2*x)*sgn(sin(1/2*x)) - sgn(sin(1/2*x)))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 14.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \sqrt{a - a\cos(x)}\,dx = -\frac{2\sqrt{a}\,\sqrt{1-\cos(x)}\,(\cos(x) + 1 + \sin(x)\ 1\mathrm{i})}{\sin(x) - \cos(x)\ 1\mathrm{i} + 1\mathrm{i}}$$

```
[In] int((a - a*cos(x))^(1/2),x)
[Out] -(2*a^(1/2)*(1 - cos(x))^(1/2)*(cos(x) + sin(x)*1i + 1))/(sin(x) - cos(x)*1
i + 1i)
```

3.161 $\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx$

Optimal result . . . 962
Rubi [A] (verified) . . . 962
Mathematica [A] (verified) . . . 963
Maple [F] . . . 963
Fricas [F(-2)] . . . 963
Sympy [F] . . . 964
Maxima [F] . . . 964
Giac [A] (verification not implemented) . . . 964
Mupad [F(-1)] . . . 964

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx = \sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)$$

```
[Out] csc(1/2*x)*Si(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3400, 3380}

$$\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx = \operatorname{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

```
[In] Int[Sqrt[a - a*Cos[x]]/x,x]
```

```
[Out] Sqrt[a - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
```

```
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{a - a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x}\,dx \\ &= \sqrt{a - a\cos(x)}\csc\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a - a\cos(x)}}{x}\,dx = \sqrt{a - a\cos(x)}\csc\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)$$

```
[In] Integrate[Sqrt[a - a*Cos[x]]/x,x]
[Out] Sqrt[a - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2]
```

Maple [F]

$$\int \frac{\sqrt{a - \cos(x)\,a}}{x}dx$$

```
[In] int((a-cos(x)*a)^(1/2)/x,x)
[Out] int((a-cos(x)*a)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a - a\cos(x)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a-a*cos(x))^(1/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a - a\cos(x)}}{x}\,dx = \int \frac{\sqrt{-a\left(\cos\left(x\right) - 1\right)}}{x}\,dx$$

```
[In] integrate((a-a*cos(x))**(1/2)/x,x)
[Out] Integral(sqrt(-a*(cos(x) - 1))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a - a\cos(x)}}{x}\,dx = \int \frac{\sqrt{-a\cos\left(x\right) + a}}{x}\,dx$$

```
[In] integrate((a-a*cos(x))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(-a*cos(x) + a)/x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a - a\cos(x)}}{x}\,dx = \sqrt{2}\sqrt{a}\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\,\mathrm{Si}\left(\frac{1}{2}\,x\right)$$

```
[In] integrate((a-a*cos(x))^(1/2)/x,x, algorithm="giac")
[Out] sqrt(2)*sqrt(a)*sgn(sin(1/2*x))*sin_integral(1/2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a - a\cos(x)}}{x}\,dx = \int \frac{\sqrt{a - a\,\cos\left(x\right)}}{x}\,dx$$

```
[In] int((a - a*cos(x))^(1/2)/x,x)
[Out] int((a - a*cos(x))^(1/2)/x, x)
```

3.162 $\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx$

Optimal result . . . 965
Rubi [A] (verified) . . . 965
Mathematica [A] (verified) . . . 966
Maple [F] . . . 966
Fricas [F(-2)] . . . 967
Sympy [F] . . . 967
Maxima [F] . . . 967
Giac [A] (verification not implemented) . . . 967
Mupad [F(-1)] . . . 968

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = -\frac{\sqrt{a-a\cos(x)}}{x} + \frac{1}{2}\sqrt{a-a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

```
[Out] -(a-a*cos(x))^(1/2)/x+1/2*Ci(1/2*x)*csc(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3400, 3378, 3383}

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = \frac{1}{2}\,\text{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)} - \frac{\sqrt{a-a\cos(x)}}{x}$$

```
[In] Int[Sqrt[a - a*Cos[x]]/x^2,x]
```

```
[Out] -(Sqrt[a - a*Cos[x]]/x) + (Sqrt[a - a*Cos[x]]*CosIntegral[x/2]*Csc[x/2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int\frac{\sin\left(\frac{x}{2}\right)}{x^2}\,dx \\ &= -\frac{\sqrt{a-a\cos(x)}}{x}+\frac{1}{2}\left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int\frac{\cos\left(\frac{x}{2}\right)}{x}\,dx \\ &= -\frac{\sqrt{a-a\cos(x)}}{x}+\frac{1}{2}\sqrt{a-a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int\frac{\sqrt{a-a\cos(x)}}{x^2}\,dx=\frac{\sqrt{a-a\cos(x)}\left(-2+x\,\text{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\right)}{2x}$$

```
[In] Integrate[Sqrt[a - a*Cos[x]]/x^2,x]
[Out] (Sqrt[a - a*Cos[x]]*(-2 + x*CosIntegral[x/2]*Csc[x/2]))/(2*x)
```

Maple [F]

$$\int\frac{\sqrt{a-\cos(x)\,a}}{x^2}dx$$

```
[In] int((a-cos(x)*a)^(1/2)/x^2,x)
[Out] int((a-cos(x)*a)^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{-a\left(\cos\left(x\right)-1\right)}}{x^2}\,dx$$

```
[In] integrate((a-a*cos(x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-a*(cos(x) - 1))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{-a\cos\left(x\right)+a}}{x^2}\,dx$$

```
[In] integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*cos(x) + a)/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = \frac{\sqrt{2}\left(x\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)-2\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}}{2\,x}$$

```
[In] integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(x*cos_integral(1/2*x)*sgn(sin(1/2*x)) - 2*sgn(sin(1/2*x))*sin(
1/2*x))*sqrt(a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a - a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{a - a\,\cos(x)}}{x^2}\,dx$$

```
[In] int((a - a*cos(x))^(1/2)/x^2,x)
[Out] int((a - a*cos(x))^(1/2)/x^2, x)
```

3.163 $\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx$

Optimal result . . . 969
Rubi [A] (verified) . . . 969
Mathematica [A] (verified) . . . 970
Maple [F] . . . 971
Fricas [F(-2)] . . . 971
Sympy [F] . . . 971
Maxima [F] . . . 971
Giac [A] (verification not implemented) . . . 972
Mupad [F(-1)] . . . 972

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{a-a\cos(x)}}{2x^2} - \frac{\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)}{4x} - \frac{1}{8}\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)$$

```
[Out] -1/2*(a-a*cos(x))^(1/2)/x^2-1/4*cot(1/2*x)*(a-a*cos(x))^(1/2)/x-1/8*csc(1/2
*x)*Si(1/2*x)*(a-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3400, 3378, 3380}

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = -\frac{1}{8}\operatorname{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)} - \frac{\sqrt{a-a\cos(x)}}{2x^2} - \frac{\cot\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}}{4x}$$

```
[In] Int[Sqrt[a - a*Cos[x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a - a*Cos[x]]/x^2 - (Sqrt[a - a*Cos[x]]*Cot[x/2])/(4*x) - (Sqrt[a
 - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x^3}\,dx \\
&= -\frac{\sqrt{a-a\cos(x)}}{2x^2} + \frac{1}{4}\left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x^2}\,dx \\
&= -\frac{\sqrt{a-a\cos(x)}}{2x^2} - \frac{\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)}{4x} - \frac{1}{8}\left(\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\right)\int \frac{\sin\left(\frac{x}{2}\right)}{x}\,dx \\
&= -\frac{\sqrt{a-a\cos(x)}}{2x^2} - \frac{\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)}{4x} - \frac{1}{8}\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{a-a\cos(x)}\left(4+2x\cot\left(\frac{x}{2}\right)+x^2\csc\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right)\right)}{8x^2}$$

```
[In] Integrate[Sqrt[a - a*Cos[x]]/x^3,x]
```

```
[Out] -1/8*(Sqrt[a - a*Cos[x]]*(4 + 2*x*Cot[x/2] + x^2*Csc[x/2]*SinIntegral[x/2])
)/x^2
```

Maple [F]

$$\int \frac{\sqrt{a-\cos(x)\,a}}{x^3}dx$$

```
[In] int((a-cos(x)*a)^(1/2)/x^3,x)
[Out] int((a-cos(x)*a)^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{-a\left(\cos(x)-1\right)}}{x^3}\,dx$$

```
[In] integrate((a-a*cos(x))**(1/2)/x**3,x)
[Out] Integral(sqrt(-a*(cos(x) - 1))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{-a\cos(x)+a}}{x^3}\,dx$$

```
[In] integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(-a*cos(x) + a)/x^3, x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{2}\left(x^2\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\operatorname{Si}\left(\frac{1}{2}\,x\right)+2\,x\cos\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)+4\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}}{8\,x^2}$$

```
[In] integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(x^2*sgn(sin(1/2*x))*sin_integral(1/2*x) + 2*x*cos(1/2*x)*sgn(
sin(1/2*x)) + 4*sgn(sin(1/2*x))*sin(1/2*x))*sqrt(a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{a-a\,\cos\left(x\right)}}{x^3}\,dx$$

```
[In] int((a - a*cos(x))^(1/2)/x^3,x)
```

```
[Out] int((a - a*cos(x))^(1/2)/x^3, x)
```

3.164 $\int x^3(a+a\cos(x))^{3/2}\,dx$

Optimal result . . . 973
Rubi [A] (verified) . . . 973
Mathematica [A] (verified) . . . 976
Maple [F] . . . 976
Fricas [F(-2)] . . . 976
Sympy [F] . . . 976
Maxima [A] (verification not implemented) . . . 977
Giac [A] (verification not implemented) . . . 977
Mupad [F(-1)] . . . 977

Optimal result

Integrand size = 14, antiderivative size = 185

$$\int x^3(a+a\cos(x))^{3/2}\,dx = -\frac{1280}{9}a\sqrt{a+a\cos(x)} + 16ax^2\sqrt{a+a\cos(x)} - \frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} - \frac{32}{9}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{4}{3}ax^3\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) - \frac{640}{9}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + \frac{8}{3}ax^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

```
[Out] -1280/9*a*(a+a*cos(x))^(1/2)+16*a*x^2*(a+a*cos(x))^(1/2)-64/27*a*cos(1/2*x)
^2*(a+a*cos(x))^(1/2)+8/3*a*x^2*cos(1/2*x)^2*(a+a*cos(x))^(1/2)-32/9*a*x*co
s(1/2*x)*sin(1/2*x)*(a+a*cos(x))^(1/2)+4/3*a*x^3*cos(1/2*x)*sin(1/2*x)*(a+a
*cos(x))^(1/2)-640/9*a*x*(a+a*cos(x))^(1/2)*tan(1/2*x)+8/3*a*x^3*(a+a*cos(x
))^(1/2)*tan(1/2*x)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3400, 3392, 3377, 2718, 3391}

$$\int x^3(a+a\cos(x))^{3/2}\,dx = \frac{4}{3}ax^3\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + \frac{8}{3}ax^3\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + \frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + 16ax^2\sqrt{a\cos(x)+a} - \frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} - \frac{1280}{9}a\sqrt{a\cos(x)+a} - \frac{32}{9}ax\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} - \frac{640}{9}ax\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[In] Int[x^3*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (-1280*a*Sqrt[a + a*Cos[x]])/9 + 16*a*x^2*Sqrt[a + a*Cos[x]] - (64*a*Cos[x/
2]^2*Sqrt[a + a*Cos[x]])/27 + (8*a*x^2*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/3 - (
32*a*x*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/9 + (4*a*x^3*Cos[x/2]*Sqrt[a +
 a*Cos[x]]*Sin[x/2])/3 - (640*a*x*Sqrt[a + a*Cos[x]]*Tan[x/2])/9 + (8*a*x^3
*Sqrt[a + a*Cos[x]]*Tan[x/2])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \left(2a\sqrt{a + a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^3\cos^3\left(\frac{x}{2}\right)\,dx$$

$$
\begin{aligned}
&= \frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax^3\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\
&\quad + \frac{1}{3}\left(4a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^3\cos\left(\frac{x}{2}\right)\,dx \\
&\quad - \frac{1}{3}\left(16a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos^3\left(\frac{x}{2}\right)\,dx \\
&= -\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} \\
&\quad - \frac{32}{9}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{4}{3}ax^3\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\
&\quad + \frac{8}{3}ax^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \frac{1}{9}\left(32a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx \\
&\quad - \left(8a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\sin\left(\frac{x}{2}\right)\,dx \\
&= 16ax^2\sqrt{a+a\cos(x)} - \frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} \\
&\quad + \frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} - \frac{32}{9}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\
&\quad + \frac{4}{3}ax^3\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) - \frac{64}{9}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) \\
&\quad + \frac{8}{3}ax^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + \frac{1}{9}\left(64a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \sin\left(\frac{x}{2}\right)\,dx \\
&\quad - \left(32a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx \\
&= -\frac{128}{9}a\sqrt{a+a\cos(x)} + 16ax^2\sqrt{a+a\cos(x)} - \frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} \\
&\quad + \frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} - \frac{32}{9}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\
&\quad + \frac{4}{3}ax^3\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) - \frac{640}{9}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) \\
&\quad + \frac{8}{3}ax^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + \left(64a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \sin\left(\frac{x}{2}\right)\,dx \\
&= -\frac{1280}{9}a\sqrt{a+a\cos(x)} + 16ax^2\sqrt{a+a\cos(x)} \\
&\quad - \frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} \\
&\quad - \frac{32}{9}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{4}{3}ax^3\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\
&\quad - \frac{640}{9}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + \frac{8}{3}ax^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.36

$$\int x^3(a+a\cos(x))^{3/2}\,dx = \frac{2}{27}a\sqrt{a(1+\cos(x))}\Big(-1936+234x^2 + 3x\left(-328+15x^2\right)\tan\left(\frac{x}{2}\right)+\cos(x)\left(2\left(-8+9x^2\right)+3x\left(-8+3x^2\right)\tan\left(\frac{x}{2}\right)\right)\Big)$$

```
[In] Integrate[x^3*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[x])]*(-1936 + 234*x^2 + 3*x*(-328 + 15*x^2)*Tan[x/2] +
 Cos[x]*(2*(-8 + 9*x^2) + 3*x*(-8 + 3*x^2)*Tan[x/2])))/27
```

Maple [F]

$$\int x^3(a+\cos(x)\,a)^{\frac{3}{2}}\,dx$$

```
[In] int(x^3*(a+cos(x)*a)^(3/2),x)
```

```
[Out] int(x^3*(a+cos(x)*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3(a+a\cos(x))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3(a+a\cos(x))^{3/2}\,dx = \int x^3(a(\cos(x)+1))^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**3*(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x**3*(a*(cos(x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.53

$$\int x^3(a + a\cos(x))^{3/2}\,dx = \frac{1}{27}\left(81\sqrt{2}ax^3\sin\left(\frac{1}{2}x\right) + 486\sqrt{2}ax^2\cos\left(\frac{1}{2}x\right) - 1944\sqrt{2}ax\sin\left(\frac{1}{2}x\right) - 3888\sqrt{2}a\cos\left(\frac{1}{2}x\right) + 2\left(9\sqrt{2}ax^2 - 8\sqrt{2}a\right)\cos\left(\frac{3}{2}x\right) + 3\left(3\sqrt{2}ax^3 - 8\sqrt{2}ax\right)\sin\left(\frac{3}{2}x\right)\right)\sqrt{a}$$

```
[In] integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/27*(81*sqrt(2)*a*x^3*sin(1/2*x) + 486*sqrt(2)*a*x^2*cos(1/2*x) - 1944*sqr
t(2)*a*x*sin(1/2*x) - 3888*sqrt(2)*a*cos(1/2*x) + 2*(9*sqrt(2)*a*x^2 - 8*sq
rt(2)*a)*cos(3/2*x) + 3*(3*sqrt(2)*a*x^3 - 8*sqrt(2)*a*x)*sin(3/2*x))*sqrt(
a)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.61

$$\int x^3(a + a\cos(x))^{3/2}\,dx = \frac{1}{27}\sqrt{2}\left(2\left(9\,ax^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right) - 8\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\cos\left(\frac{3}{2}x\right) + 486\left(ax^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right) - 8\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\cos\left(\frac{1}{2}x\right) + 3\left(3\,ax^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right) - 8\,ax\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\sin\left(\frac{3}{2}x\right) + 81\left(ax^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right) - 24\,ax\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\sin\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

```
[In] integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/27*sqrt(2)*(2*(9*a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*cos(3/2*x)
+ 486*(a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*cos(1/2*x) + 3*(3*a*x^3
*sgn(cos(1/2*x)) - 8*a*x*sgn(cos(1/2*x)))*sin(3/2*x) + 81*(a*x^3*sgn(cos(1/
2*x)) - 24*a*x*sgn(cos(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a + a\cos(x))^{3/2}\,dx = \int x^3\left(a + a\,\cos\left(x\right)\right)^{3/2}\,dx$$

```
[In] int(x^3*(a + a*cos(x))^(3/2),x)
```

```
[Out] int(x^3*(a + a*cos(x))^(3/2), x)
```

3.165 $\int x^2(a+a\cos(x))^{3/2}\,dx$

Optimal result . . . 978
Rubi [A] (verified) . . . 978
Mathematica [A] (verified) . . . 980
Maple [F] . . . 981
Fricas [F(-2)] . . . 981
Sympy [F] . . . 981
Maxima [A] (verification not implemented) . . . 981
Giac [A] (verification not implemented) . . . 982
Mupad [F(-1)] . . . 982

Optimal result

Integrand size = 14, antiderivative size = 145

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \frac{32}{3}ax\sqrt{a+a\cos(x)} + \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) - \frac{224}{9}a\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + \frac{8}{3}ax^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + \frac{32}{27}a\sqrt{a+a\cos(x)}\sin^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$$

```
[Out] 32/3*a*x*(a+a*cos(x))^(1/2)+16/9*a*x*cos(1/2*x)^2*(a+a*cos(x))^(1/2)+4/3*a*
x^2*cos(1/2*x)*sin(1/2*x)*(a+a*cos(x))^(1/2)-224/9*a*(a+a*cos(x))^(1/2)*tan
(1/2*x)+8/3*a*x^2*(a+a*cos(x))^(1/2)*tan(1/2*x)+32/27*a*sin(1/2*x)^2*(a+a*c
os(x))^(1/2)*tan(1/2*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3400, 3392, 3377, 2717, 2713}

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \frac{4}{3}ax^2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + \frac{8}{3}ax^2\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + \frac{32}{3}ax\sqrt{a\cos(x)+a} - \frac{224}{9}a\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + \frac{32}{27}a\sin^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[In] Int[x^2*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (32*a*x*Sqrt[a + a*Cos[x]])/3 + (16*a*x*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/9 +
(4*a*x^2*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/3 - (224*a*Sqrt[a + a*Cos[x]
```

```
]*Tan[x/2])/9 + (8*a*x^2*Sqrt[a + a*Cos[x]]*Tan[x/2])/3 + (32*a*Sqrt[a + a*
Cos[x]]*Sin[x/2]^2*Tan[x/2])/27
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\cos^3\left(\frac{x}{2}\right)\,dx \\
&= \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\
&\quad + \frac{1}{3}\left(4a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x^2\cos\left(\frac{x}{2}\right)\,dx \\
&\quad - \frac{1}{9}\left(16a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \cos^3\left(\frac{x}{2}\right)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\
&\quad + \frac{8}{3}ax^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) \\
&\quad + \frac{1}{9}\left(32a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int\left(1-x^2\right)\,dx, x, -\sin\left(\frac{x}{2}\right)\right) \\
&\quad - \frac{1}{3}\left(16a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\sin\left(\frac{x}{2}\right)\,dx \\
&= \frac{32}{3}ax\sqrt{a+a\cos(x)} + \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} \\
&\quad + \frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) - \frac{32}{9}a\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) \\
&\quad + \frac{8}{3}ax^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + \frac{32}{27}a\sqrt{a+a\cos(x)}\sin^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right) \\
&\quad - \frac{1}{3}\left(32a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\cos\left(\frac{x}{2}\right)\,dx \\
&= \frac{32}{3}ax\sqrt{a+a\cos(x)} + \frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} \\
&\quad + \frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) - \frac{224}{9}a\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) \\
&\quad + \frac{8}{3}ax^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) + \frac{32}{27}a\sqrt{a+a\cos(x)}\sin^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$
\begin{aligned}
&\int x^2(a+a\cos(x))^{3/2}\,dx = \frac{2}{27}a\sqrt{a(1+\cos(x))}\Big(156x \\
&\quad + \left(-328+45x^2\right)\tan\left(\frac{x}{2}\right) + \cos(x)\left(12x + \left(-8+9x^2\right)\tan\left(\frac{x}{2}\right)\right)\Big)
\end{aligned}
$$

```
[In] Integrate[x^2*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[x])]*(156*x + (-328 + 45*x^2)*Tan[x/2] + Cos[x]*(12*x
+ (-8 + 9*x^2)*Tan[x/2])))/27
```

Maple [F]

$$\int x^2(a+\cos(x)\,a)^{\frac{3}{2}}\,dx$$

```
[In] int(x^2*(a+cos(x)*a)^(3/2),x)
[Out] int(x^2*(a+cos(x)*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \int x^2(a(\cos(x)+1))^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(a+a*cos(x))**(3/2),x)
[Out] Integral(x**2*(a*(cos(x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \frac{1}{27}\left(81\sqrt{2}ax^2\sin\left(\frac{1}{2}x\right)+12\sqrt{2}ax\cos\left(\frac{3}{2}x\right)+324\sqrt{2}ax\cos\left(\frac{1}{2}x\right)-648\sqrt{2}a\sin\left(\frac{1}{2}\right.\right.$$

```
[In] integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="maxima")
[Out] 1/27*(81*sqrt(2)*a*x^2*sin(1/2*x) + 12*sqrt(2)*a*x*cos(3/2*x) + 324*sqrt(2)
*a*x*cos(1/2*x) - 648*sqrt(2)*a*sin(1/2*x) + (9*sqrt(2)*a*x^2 - 8*sqrt(2)*a
)*sin(3/2*x))*sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

$$\int x^2(a$$

$$+a\cos(x))^{3/2}\,dx = \frac{1}{27}\sqrt{2}\left(12\,ax\cos\left(\frac{3}{2}\,x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right) + 324\,ax\cos\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right) + \left(9\,ax^2\right.$$

```
[In] integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/27*sqrt(2)*(12*a*x*cos(3/2*x)*sgn(cos(1/2*x)) + 324*a*x*cos(1/2*x)*sgn(co
s(1/2*x)) + (9*a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*sin(3/2*x) + 81
*(a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*sin(1/2*x))*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a + a\cos(x))^{3/2}\,dx = \int x^2\,(a + a\,\cos(x))^{3/2}\,dx$$

```
[In] int(x^2*(a + a*cos(x))^(3/2),x)
```

```
[Out] int(x^2*(a + a*cos(x))^(3/2), x)
```

3.166 $\int x(a+a\cos(x))^{3/2}\,dx$

Optimal result . . . 983
Rubi [A] (verified) . . . 983
Mathematica [A] (verified) . . . 985
Maple [F] . . . 985
Fricas [F(-2)] . . . 985
Sympy [F] . . . 985
Maxima [A] (verification not implemented) . . . 986
Giac [A] (verification not implemented) . . . 986
Mupad [F(-1)] . . . 986

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x(a+a\cos(x))^{3/2}\,dx = \frac{16}{3}a\sqrt{a+a\cos(x)} + \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{8}{3}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

```
[Out] 16/3*a*(a+a*cos(x))^(1/2)+8/9*a*cos(1/2*x)^2*(a+a*cos(x))^(1/2)+4/3*a*x*cos
(1/2*x)*sin(1/2*x)*(a+a*cos(x))^(1/2)+8/3*a*x*(a+a*cos(x))^(1/2)*tan(1/2*x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3400, 3391, 3377, 2718}

$$\int x(a+a\cos(x))^{3/2}\,dx = \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + \frac{16}{3}a\sqrt{a\cos(x)+a} + \frac{4}{3}ax\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + \frac{8}{3}ax\tan\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[In] Int[x*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (16*a*Sqrt[a + a*Cos[x]])/3 + (8*a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/9 + (4*a*
x*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/3 + (8*a*x*Sqrt[a + a*Cos[x]]*Tan[x
/2])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos^3\left(\frac{x}{2}\right)\,dx \\
&= \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\
&\quad + \frac{1}{3}\left(4a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int x\cos\left(\frac{x}{2}\right)\,dx \\
&= \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\
&\quad + \frac{8}{3}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right) - \frac{1}{3}\left(8a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \sin\left(\frac{x}{2}\right)\,dx \\
&= \frac{16}{3}a\sqrt{a+a\cos(x)} + \frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} \\
&\quad + \frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) + \frac{8}{3}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int x(a+a\cos(x))^{3/2}\,dx = \frac{1}{9}a\sqrt{a(1+\cos(x))}\left(52 + 4\cos(x) + 3x\sec\left(\frac{x}{2}\right)\sin\left(\frac{3x}{2}\right) + 27x\tan\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[x*(a + a*Cos[x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[x])]*(52 + 4*Cos[x] + 3*x*Sec[x/2]*Sin[(3*x)/2] + 27*x*T
an[x/2]))/9
```

Maple [F]

$$\int x(a+\cos(x)\,a)^{\frac{3}{2}}\,dx$$

```
[In] int(x*(a+cos(x)*a)^(3/2),x)
```

```
[Out] int(x*(a+cos(x)*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+a\cos(x))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x(a+a\cos(x))^{3/2}\,dx = \int x(a(\cos(x)+1))^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x*(a*(cos(x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

$$\int x(a+a\cos(x))^{3/2}\,dx = \frac{1}{9}\left(3\sqrt{2}ax\sin\left(\frac{3}{2}x\right) + 27\sqrt{2}ax\sin\left(\frac{1}{2}x\right) + 2\sqrt{2}a\cos\left(\frac{3}{2}x\right) + 54\sqrt{2}a\cos\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

```
[In] integrate(x*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*(3*sqrt(2)*a*x*sin(3/2*x) + 27*sqrt(2)*a*x*sin(1/2*x) + 2*sqrt(2)*a*cos
(3/2*x) + 54*sqrt(2)*a*cos(1/2*x))*sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int x(a+a\cos(x))^{3/2}\,dx = \frac{1}{9}\sqrt{2}\left(3\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{3}{2}x\right) + 27\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right) + 2\,a\cos\left(\frac{3}{2}x\right)\right.$$

```
[In] integrate(x*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(2)*(3*a*x*sgn(cos(1/2*x))*sin(3/2*x) + 27*a*x*sgn(cos(1/2*x))*sin(
1/2*x) + 2*a*cos(3/2*x)*sgn(cos(1/2*x)) + 54*a*cos(1/2*x)*sgn(cos(1/2*x)))*
sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+a\cos(x))^{3/2}\,dx = \int x\,(a+a\,\cos(x))^{3/2}\,dx$$

```
[In] int(x*(a + a*cos(x))^(3/2),x)
```

```
[Out] int(x*(a + a*cos(x))^(3/2), x)
```

3.167 $\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx$

Optimal result . . . 987
Rubi [A] (verified) . . . 987
Mathematica [A] (verified) . . . 988
Maple [F] . . . 989
Fricas [F(-2)] . . . 989
Sympy [F] . . . 989
Maxima [C] (verification not implemented) . . . 989
Giac [A] (verification not implemented) . . . 990
Mupad [F(-1)] . . . 990

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \frac{3}{2}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{1}{2}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)$$

```
[Out] 3/2*a*Ci(1/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)+1/2*a*Ci(3/2*x)*sec(1/2*x)*(a
+a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3393, 3383}

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \frac{3}{2}a\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} + \frac{1}{2}a\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

```
[In] Int[(a + a*Cos[x])^(3/2)/x,x]
```

```
[Out] (3*a*Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2])/2 + (a*Sqrt[a + a*Cos[x]
]*CosIntegral[(3*x)/2]*Sec[x/2])/2
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x}\,dx \\
&= \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \left(\frac{3\cos\left(\frac{x}{2}\right)}{4x}+\frac{\cos\left(\frac{3x}{2}\right)}{4x}\right)\,dx \\
&= \frac{1}{2}\left(a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{3x}{2}\right)}{x}\,dx+\frac{1}{2}\left(3a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx \\
&= \frac{3}{2}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)+\frac{1}{2}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \frac{1}{2}a\sqrt{a(1+\cos(x))}\left(3\,\text{CosIntegral}\left(\frac{x}{2}\right)+\text{CosIntegral}\left(\frac{3x}{2}\right)\right)\sec\left(\frac{x}{2}\right)$$

```
[In] Integrate[(a + a*Cos[x])^(3/2)/x,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[x])]*(3*CosIntegral[x/2] + CosIntegral[(3*x)/2])*Sec[x/2
])/2
```

Maple [F]

$$\int \frac{(a+\cos(x)\,a)^{\frac{3}{2}}}{x}dx$$

```
[In] int((a+cos(x)*a)^(3/2)/x,x)
```

```
[Out] int((a+cos(x)*a)^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \int \frac{(a(\cos(x)+1))^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a+a*cos(x))**(3/2)/x,x)
```

```
[Out] Integral((a*(cos(x) + 1))**(3/2)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \frac{1}{4}\sqrt{2}a^{\frac{3}{2}}\left(\mathrm{Ei}\left(\frac{3}{2}i\,x\right)+3\,\mathrm{Ei}\left(\frac{1}{2}i\,x\right)+3\,\mathrm{Ei}\left(-\frac{1}{2}i\,x\right)+\mathrm{Ei}\left(-\frac{3}{2}i\,x\right)\right)$$

```
[In] integrate((a+a*cos(x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*a^(3/2)*(Ei(3/2*I*x) + 3*Ei(1/2*I*x) + 3*Ei(-1/2*I*x) + Ei(-3/2
*I*x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \frac{1}{2}\sqrt{2}\left(a\,\mathrm{Ci}\left(\frac{3}{2}x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)+3\,a\,\mathrm{Ci}\left(\frac{1}{2}x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\sqrt{a}$$

```
[In] integrate((a+a*cos(x))^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*cos_integral(3/2*x)*sgn(cos(1/2*x)) + 3*a*cos_integral(1/2*x
)*sgn(cos(1/2*x)))*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \int \frac{(a+a\,\cos(x))^{3/2}}{x}\,dx$$

```
[In] int((a + a*cos(x))^(3/2)/x,x)
```

```
[Out] int((a + a*cos(x))^(3/2)/x, x)
```

3.168 $\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx$

Optimal result . . . 991
Rubi [A] (verified) . . . 991
Mathematica [A] (verified) . . . 993
Maple [F] . . . 993
Fricas [F(-2)] . . . 993
Sympy [F] . . . 993
Maxima [C] (verification not implemented) . . . 994
Giac [A] (verification not implemented) . . . 994
Mupad [F(-1)] . . . 994

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x} - \frac{3}{4}a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right) - \frac{3}{4}a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{3x}{2}\right)$$

```
[Out] -2*a*cos(1/2*x)^2*(a+a*cos(x))^(1/2)/x-3/4*a*sec(1/2*x)*Si(1/2*x)*(a+a*cos(
x))^(1/2)-3/4*a*sec(1/2*x)*Si(3/2*x)*(a+a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3394, 3380}

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = -\frac{3}{4}a\operatorname{Si}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} - \frac{3}{4}a\operatorname{Si}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} - \frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{x}$$

```
[In] Int[(a + a*Cos[x])^(3/2)/x^2,x]
```

```
[Out] (-2*a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/x - (3*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*S
inIntegral[x/2])/4 - (3*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*SinIntegral[(3*x)/2])
/4
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x^2}\,dx \\
&= -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x} + \left(3a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\left(-\frac{\sin\left(\frac{x}{2}\right)}{4x} - \frac{\sin\left(\frac{3x}{2}\right)}{4x}\right)dx \\
&= -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x} - \frac{1}{4}\left(3a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\frac{\sin\left(\frac{x}{2}\right)}{x}\,dx \\
&\quad - \frac{1}{4}\left(3a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\frac{\sin\left(\frac{3x}{2}\right)}{x}\,dx \\
&= -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x} - \frac{3}{4}a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{x}{2}\right) \\
&\quad - \frac{3}{4}a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\operatorname{Si}\left(\frac{3x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.67

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a(1+\cos(x))}\sec\left(\frac{x}{2}\right)\left(8\cos^3\left(\frac{x}{2}\right)+3x\mathrm{Si}\left(\frac{x}{2}\right)+3x\mathrm{Si}\left(\frac{3x}{2}\right)\right)}{4x}$$

```
[In] Integrate[(a + a*Cos[x])^(3/2)/x^2,x]
[Out] -1/4*(a*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(8*Cos[x/2]^3 + 3*x*SinIntegral[x/2]
+ 3*x*SinIntegral[(3*x)/2]))/x
```

Maple [F]

$$\int \frac{(a+\cos(x)\,a)^{\frac{3}{2}}}{x^2}dx$$

```
[In] int((a+cos(x)*a)^(3/2)/x^2,x)
[Out] int((a+cos(x)*a)^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = \int \frac{(a(\cos(x)+1))^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+a*cos(x))**(3/2)/x**2,x)
[Out] Integral((a*(cos(x) + 1))**(3/2)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = \frac{3}{8}\sqrt{2}a^{\frac{3}{2}}\left(-i\,\Gamma\left(-1,\frac{3}{2}i\,x\right)-i\,\Gamma\left(-1,\frac{1}{2}i\,x\right)+i\,\Gamma\left(-1,-\frac{1}{2}i\,x\right)+i\,\Gamma\left(-1,-\frac{3}{2}i\,x\right)\right)$$

```
[In] integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*a^(3/2)*(-I*gamma(-1, 3/2*I*x) - I*gamma(-1, 1/2*I*x) + I*gamma
(-1, -1/2*I*x) + I*gamma(-1, -3/2*I*x))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx =$$
$$-\frac{\sqrt{2}\left(3\,a x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\mathrm{Si}\left(\frac{3}{2}\,x\right)+3\,a x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\mathrm{Si}\left(\frac{1}{2}\,x\right)+2\,a\cos\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+6\,a\cos\left(\frac{1}{2}\,x\right)\mathrm{s}\right.}{4\,x}$$

```
[In] integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(3*a*x*sgn(cos(1/2*x))*sin_integral(3/2*x) + 3*a*x*sgn(cos(1/2
*x))*sin_integral(1/2*x) + 2*a*cos(3/2*x)*sgn(cos(1/2*x)) + 6*a*cos(1/2*x)*
sgn(cos(1/2*x)))*sqrt(a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = \int \frac{(a+a\,\cos(x))^{3/2}}{x^2}\,dx$$

```
[In] int((a + a*cos(x))^(3/2)/x^2,x)
```

```
[Out] int((a + a*cos(x))^(3/2)/x^2, x)
```

3.169 $\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx$

Optimal result . . . 995
Rubi [A] (verified) . . . 995
Mathematica [A] (verified) . . . 997
Maple [F] . . . 998
Fricas [F(-2)] . . . 998
Sympy [F] . . . 998
Maxima [C] (verification not implemented) . . . 998
Giac [A] (verification not implemented) . . . 999
Mupad [F(-1)] . . . 999

Optimal result

Integrand size = 14, antiderivative size = 109

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} - \frac{3}{16}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) - \frac{9}{16}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)}{2x}$$

```
[Out] -a*cos(1/2*x)^2*(a+a*cos(x))^(1/2)/x^2-3/16*a*Ci(1/2*x)*sec(1/2*x)*(a+a*cos
(x))^(1/2)-9/16*a*Ci(3/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)+3/2*a*cos(1/2*x)*
sin(1/2*x)*(a+a*cos(x))^(1/2)/x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3400, 3395, 3383, 3393}

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = -\frac{3}{16}a\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} - \frac{9}{16}a\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a} - \frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{x^2} + \frac{3a\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}}{2x}$$

```
[In] Int[(a + a*Cos[x])^(3/2)/x^3,x]
```

```
[Out] -((a*Cos[x/2]^2*Sqrt[a + a*Cos[x]])/x^2) - (3*a*Sqrt[a + a*Cos[x]]*CosInteg
ral[x/2]*Sec[x/2])/16 - (9*a*Sqrt[a + a*Cos[x]]*CosIntegral[(3*x)/2]*Sec[x/
2])/16 + (3*a*Cos[x/2]*Sqrt[a + a*Cos[x]]*Sin[x/2])/(2*x)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \left(2a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x^3}\,dx$$

$$= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)}{2x}$$
$$+ \frac{1}{2}\left(3a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos\left(\frac{x}{2}\right)}{x}\,dx$$
$$- \frac{1}{4}\left(9a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int \frac{\cos^3\left(\frac{x}{2}\right)}{x}\,dx$$

$$
\begin{aligned}
&= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} + \frac{3}{2}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) \\
&\quad + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)}{2x} \\
&\quad - \frac{1}{4}\left(9a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\left(\frac{3\cos\left(\frac{x}{2}\right)}{4x} + \frac{\cos\left(\frac{3x}{2}\right)}{4x}\right)dx \\
&= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} + \frac{3}{2}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) \\
&\quad + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)}{2x} - \frac{1}{16}\left(9a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\frac{\cos\left(\frac{3x}{2}\right)}{x}\,dx \\
&\quad - \frac{1}{16}\left(27a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\right)\int\frac{\cos\left(\frac{x}{2}\right)}{x}\,dx \\
&= -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} - \frac{3}{16}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) \\
&\quad - \frac{9}{16}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right) \\
&\quad + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)}{2x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$
\int\frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \\
-\frac{(a(1+\cos(x)))^{3/2}\left(16+3x^2\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec^3\left(\frac{x}{2}\right)+9x^2\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec^3\left(\frac{x}{2}\right)-24x\tan\left(\frac{x}{2}\right)\right)}{32x^2}
$$

```
[In] Integrate[(a + a*Cos[x])^(3/2)/x^3,x]
```

```
[Out] -1/32*((a*(1 + Cos[x]))^(3/2)*(16 + 3*x^2*CosIntegral[x/2]*Sec[x/2]^3 + 9*x
^2*CosIntegral[(3*x)/2]*Sec[x/2]^3 - 24*x*Tan[x/2]))/x^2
```

Maple [F]

$$\int \frac{(a+\cos(x)\,a)^{\frac{3}{2}}}{x^3}dx$$

```
[In] int((a+cos(x)*a)^(3/2)/x^3,x)
[Out] int((a+cos(x)*a)^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \int \frac{(a(\cos(x)+1))^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((a+a*cos(x))**(3/2)/x**3,x)
[Out] Integral((a*(cos(x) + 1))**(3/2)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.30

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \frac{3}{16}\sqrt{2}a^{\frac{3}{2}}\left(3\,\Gamma\left(-2,\frac{3}{2}i\,x\right)+\Gamma\left(-2,\frac{1}{2}i\,x\right)+\Gamma\left(-2,-\frac{1}{2}i\,x\right)+3\,\Gamma\left(-2,-\frac{3}{2}i\,x\right)\right)$$

```
[In] integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="maxima")
[Out] 3/16*sqrt(2)*a^(3/2)*(3*gamma(-2, 3/2*I*x) + gamma(-2, 1/2*I*x) + gamma(-2,
 -1/2*I*x) + 3*gamma(-2, -3/2*I*x))
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx =$$
$$-\frac{\sqrt{2}\left(9\,ax^2\,\mathrm{Ci}\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+3\,ax^2\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-6\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{3}{2}\,x\right)-6\,ax\mathrm{sgn}\right.}{16\,x^2}$$

```
[In] integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(9*a*x^2*cos_integral(3/2*x)*sgn(cos(1/2*x)) + 3*a*x^2*cos_in
tegral(1/2*x)*sgn(cos(1/2*x)) - 6*a*x*sgn(cos(1/2*x))*sin(3/2*x) - 6*a*x*sg
n(cos(1/2*x))*sin(1/2*x) + 4*a*cos(3/2*x)*sgn(cos(1/2*x)) + 12*a*cos(1/2*x)
*sgn(cos(1/2*x)))*sqrt(a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \int \frac{(a+a\,\cos(x))^{3/2}}{x^3}\,dx$$

```
[In] int((a + a*cos(x))^(3/2)/x^3,x)
```

```
[Out] int((a + a*cos(x))^(3/2)/x^3, x)
```

3.170 $\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result . . . 1000
Rubi [A] (verified) . . . 1001
Mathematica [A] (verified) . . . 1005
Maple [F] . . . 1006
Fricas [F] . . . 1006
Sympy [F] . . . 1006
Maxima [F] . . . 1006
Giac [F] . . . 1007
Mupad [F(-1)] . . . 1007

Optimal result

Integrand size = 18, antiderivative size = 374

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = &-\frac{4ix^3\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}}\\ &+\frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}\\ &-\frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}\\ &-\frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}\\ &+\frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}\\ &-\frac{96i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(4,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^4\sqrt{a+a\cos(c+dx)}}\\ &+\frac{96i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(4,ie^{\frac{1}{2}i(c+dx)}\right)}{d^4\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

```
[Out] -4*I*x^3*arctan(exp(1/2*I*(d*x+c)))*cos(1/2*d*x+1/2*c)/d/(a+a*cos(d*x+c))^(
1/2)+12*I*x^2*cos(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*I*(d*x+c)))/d^2/(a+a*
cos(d*x+c))^(1/2)-12*I*x^2*cos(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*I*(d*x+c)
))/d^2/(a+a*cos(d*x+c))^(1/2)-48*x*cos(1/2*d*x+1/2*c)*polylog(3,-I*exp(1/2*
I*(d*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)+48*x*cos(1/2*d*x+1/2*c)*polylog(3,I*
```

```
exp(1/2*I*(d*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)-96*I*cos(1/2*d*x+1/2*c)*poly
log(4,-I*exp(1/2*I*(d*x+c)))/d^4/(a+a*cos(d*x+c))^(1/2)+96*I*cos(1/2*d*x+1/
2*c)*polylog(4,I*exp(1/2*I*(d*x+c)))/d^4/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3400, 4266, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = &-\frac{4ix^3\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cos(c+dx)+a}}\\ &-\frac{96i\,\text{PolyLog}\left(4,-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^4\sqrt{a\cos(c+dx)+a}}\\ &+\frac{96i\,\text{PolyLog}\left(4,ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^4\sqrt{a\cos(c+dx)+a}}\\ &-\frac{48x\,\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}\\ &+\frac{48x\,\text{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}\\ &+\frac{12ix^2\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}\\ &-\frac{12ix^2\,\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}\end{aligned}$$

```
[In] Int[x^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*x^3*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*C
os[c + d*x]]) + ((12*I)*x^2*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c
+ d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((12*I)*x^2*Cos[c/2 + (d*x)/2]*P
olyLog[2, I*E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - (48*x*Co
s[c/2 + (d*x)/2]*PolyLog[3, (-I)*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a + a*Cos[
c + d*x]]) + (48*x*Cos[c/2 + (d*x)/2]*PolyLog[3, I*E^((I/2)*(c + d*x))])/(d
^3*Sqrt[a + a*Cos[c + d*x]]) - ((96*I)*Cos[c/2 + (d*x)/2]*PolyLog[4, (-I)*E
^((I/2)*(c + d*x))])/(d^4*Sqrt[a + a*Cos[c + d*x]]) + ((96*I)*Cos[c/2 + (d*
x)/2]*PolyLog[4, I*E^((I/2)*(c + d*x))])/(d^4*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^3 \csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^3 \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(6\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2 \log\left(1-ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(6\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x^2 \log\left(1+ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^3 \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{\left(24i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\,\text{PolyLog}\left(2,-ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d^2\sqrt{a+a\cos(c+dx)}} + \frac{\left(24i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\,\text{PolyLog}\left(2,ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d^2\sqrt{a+a\cos(c+dx)}}$$

$$
\begin{aligned}
&= -\frac{4ix^3 \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(48\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \operatorname{PolyLog}\left(3,-ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)dx}{d^3\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(48\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \operatorname{PolyLog}\left(3,ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)dx}{d^3\sqrt{a+a\cos(c+dx)}} \\
&= -\frac{4ix^3 \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(96i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-ix)}{x}\,dx,x,e^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d^4\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(96i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,ix)}{x}\,dx,x,e^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d^4\sqrt{a+a\cos(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4ix^3 \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{96i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(4,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^4\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{96i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(4,ie^{\frac{1}{2}i(c+dx)}\right)}{d^4\sqrt{a+a\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time $= 0.14$ (sec) , antiderivative size $= 199$, normalized size of antiderivative $= 0.53$

$$
\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx =
$$

$$
-\frac{4i\cos\left(\frac{1}{2}(c+dx)\right)\left(d^3x^3\arctan\left(e^{\frac{1}{2}i(c+dx)}\right) - 3d^2x^2\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right) + 3d^2x^2\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(}\right.\right.
$$

```
[In] Integrate[x^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*Cos[(c + d*x)/2]*(d^3*x^3*ArcTan[E^((I/2)*(c + d*x))] - 3*d^2*x^2*P
olyLog[2, (-I)*E^((I/2)*(c + d*x))] + 3*d^2*x^2*PolyLog[2, I*E^((I/2)*(c +
d*x))] - (12*I)*d*x*PolyLog[3, (-I)*E^((I/2)*(c + d*x))] + (12*I)*d*x*PolyL
og[3, I*E^((I/2)*(c + d*x))] + 24*PolyLog[4, (-I)*E^((I/2)*(c + d*x))] - 24
*PolyLog[4, I*E^((I/2)*(c + d*x))]))/(d^4*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [F]

$$\int \frac{x^3}{\sqrt{a+\cos(dx+c)\,a}}dx$$

```
[In] int(x^3/(a+cos(d*x+c)*a)^(1/2),x)
[Out] int(x^3/(a+cos(d*x+c)*a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^3}{\sqrt{a\cos(dx+c)+a}}\,dx$$

```
[In] integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(x^3/sqrt(a*cos(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^3}{\sqrt{a\left(\cos(c+dx)+1\right)}}\,dx$$

```
[In] integrate(x**3/(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(x**3/sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^3}{\sqrt{a\cos(dx+c)+a}}\,dx$$

```
[In] integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*(6*sqrt(2)*d^2*x^2*cos(1/2*d*x + 1/2*c) + 24*(sqrt(2)*cos(d*x + c)^2 + sq
rt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan2(cos(1/2*d*
x + 1/2*c), sin(1/2*d*x + 1/2*c) + 1) + 24*(sqrt(2)*cos(d*x + c)^2 + sqrt(2
)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan2(cos(1/2*d*x +
1/2*c), -sin(1/2*d*x + 1/2*c) + 1) + (6*sqrt(2)*d^2*x^2*cos(1/2*d*x + 1/2*c
) - (sqrt(2)*d^3*x^3 - 24*sqrt(2)*d*x)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) +
 (sqrt(2)*a*d^7*cos(d*x + c)^2 + sqrt(2)*a*d^7*sin(d*x + c)^2 + 2*sqrt(2)*a
*d^7*cos(d*x + c) + sqrt(2)*a*d^7)*integrate((x^3*cos(2*d*x + 2*c)*cos(1/2*
```

```
d*x + 1/2*c) + 2*x^3*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + x^3*sin(2*d*x + 2*
c)*sin(1/2*d*x + 1/2*c) + 2*x^3*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + x^3*cos
(1/2*d*x + 1/2*c))/(a*d^3*cos(2*d*x + 2*c)^2 + 4*a*d^3*cos(d*x + c)^2 + a*d
^3*sin(2*d*x + 2*c)^2 + 4*a*d^3*sin(2*d*x + 2*c)*sin(d*x + c) + 4*a*d^3*sin
(d*x + c)^2 + 4*a*d^3*cos(d*x + c) + a*d^3 + 2*(2*a*d^3*cos(d*x + c) + a*d^
3)*cos(2*d*x + 2*c)), x) - 6*(sqrt(2)*a*d^6*cos(d*x + c)^2 + sqrt(2)*a*d^6*
sin(d*x + c)^2 + 2*sqrt(2)*a*d^6*cos(d*x + c) + sqrt(2)*a*d^6)*integrate((x
^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + 2*x^2*cos(1/2*d*x + 1/2*c)*sin(d
*x + c) - x^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 2*x^2*cos(d*x + c)*si
n(1/2*d*x + 1/2*c) - x^2*sin(1/2*d*x + 1/2*c))/(a*d^3*cos(2*d*x + 2*c)^2 +
4*a*d^3*cos(d*x + c)^2 + a*d^3*sin(2*d*x + 2*c)^2 + 4*a*d^3*sin(2*d*x + 2*c
)*sin(d*x + c) + 4*a*d^3*sin(d*x + c)^2 + 4*a*d^3*cos(d*x + c) + a*d^3 + 2*
(2*a*d^3*cos(d*x + c) + a*d^3)*cos(2*d*x + 2*c)), x) - 24*(sqrt(2)*a*d^5*co
s(d*x + c)^2 + sqrt(2)*a*d^5*sin(d*x + c)^2 + 2*sqrt(2)*a*d^5*cos(d*x + c)
+ sqrt(2)*a*d^5)*integrate((x*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + 2*x*c
os(d*x + c)*cos(1/2*d*x + 1/2*c) + x*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c)
+ 2*x*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + x*cos(1/2*d*x + 1/2*c))/(a*d^3*co
s(2*d*x + 2*c)^2 + 4*a*d^3*cos(d*x + c)^2 + a*d^3*sin(2*d*x + 2*c)^2 + 4*a*
d^3*sin(2*d*x + 2*c)*sin(d*x + c) + 4*a*d^3*sin(d*x + c)^2 + 4*a*d^3*cos(d*
x + c) + a*d^3 + 2*(2*a*d^3*cos(d*x + c) + a*d^3)*cos(2*d*x + 2*c)), x) + (
6*sqrt(2)*d^2*x^2*sin(1/2*d*x + 1/2*c) + (sqrt(2)*d^3*x^3 - 24*sqrt(2)*d*x)
*cos(1/2*d*x + 1/2*c))*sin(d*x + c) - (sqrt(2)*d^3*x^3 - 24*sqrt(2)*d*x)*si
n(1/2*d*x + 1/2*c))/((d^4*cos(d*x + c)^2 + d^4*sin(d*x + c)^2 + 2*d^4*cos(d
*x + c) + d^4)*sqrt(a))
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{x^3}{\sqrt{a\cos\left(dx + c\right) + a}}\,dx$$

```
[In] integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{x^3}{\sqrt{a + a\,\cos\left(c + d\,x\right)}}\,dx$$

```
[In] int(x^3/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(x^3/(a + a*cos(c + d*x))^(1/2), x)
```

3.171 $\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result . . . 1008
Rubi [A] (verified) . . . 1009
Mathematica [A] (verified) . . . 1012
Maple [F] . . . 1012
Fricas [F] . . . 1012
Sympy [F] . . . 1012
Maxima [F] . . . 1013
Giac [F] . . . 1013
Mupad [F(-1)] . . . 1014

Optimal result

Integrand size = 18, antiderivative size = 262

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = &-\frac{4ix^2\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}}\\ &+\frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}\\ &-\frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}\\ &-\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}\\ &+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

```
[Out] -4*I*x^2*arctan(exp(1/2*I*(d*x+c)))*cos(1/2*d*x+1/2*c)/d/(a+a*cos(d*x+c))^(
1/2)+8*I*x*cos(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*I*(d*x+c)))/d^2/(a+a*cos
(d*x+c))^(1/2)-8*I*x*cos(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*I*(d*x+c)))/d^2
/(a+a*cos(d*x+c))^(1/2)-16*cos(1/2*d*x+1/2*c)*polylog(3,-I*exp(1/2*I*(d*x+c
)))/d^3/(a+a*cos(d*x+c))^(1/2)+16*cos(1/2*d*x+1/2*c)*polylog(3,I*exp(1/2*I*
(d*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 4266, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = &-\frac{4ix^2\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cos(c+dx)+a}}\\ &-\frac{16\,\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}\\ &+\frac{16\,\text{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cos(c+dx)+a}}\\ &+\frac{8ix\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}\\ &-\frac{8ix\,\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cos(c+dx)+a}}\end{aligned}$$

```
[In] Int[x^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*x^2*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*C
os[c + d*x]]) + ((8*I)*x*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c + d
*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((8*I)*x*Cos[c/2 + (d*x)/2]*PolyLog
[2, I*E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - (16*Cos[c/2 +
(d*x)/2]*PolyLog[3, (-I)*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a + a*Cos[c + d*x]
]) + (16*Cos[c/2 + (d*x)/2]*PolyLog[3, I*E^((I/2)*(c + d*x))])/(d^3*Sqrt[a
+ a*Cos[c + d*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\int x^2 \csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^2 \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}}$$

$$- \frac{\left(4\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\log\left(1-ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d\sqrt{a+a\cos(c+dx)}}$$

$$+ \frac{\left(4\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int x\log\left(1+ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^2 \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}$$
$$- \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}$$
$$- \frac{\left(8i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \operatorname{PolyLog}\left(2,-ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d^2\sqrt{a+a\cos(c+dx)}}$$
$$+ \frac{\left(8i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int \operatorname{PolyLog}\left(2,ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d^2\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^2 \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}$$
$$- \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}$$
$$- \frac{\left(16\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx, x, e^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}$$
$$+ \frac{\left(16\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx, x, e^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{4ix^2 \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}$$
$$- \frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}$$
$$- \frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}$$
$$+ \frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(-id^2x^2\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)+2idx\,\mathrm{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)-2idx\,\mathrm{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)\right.}{d^3\sqrt{a(1+\cos(c+dx))}}$$

```
[In] Integrate[x^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]*((-I)*d^2*x^2*ArcTan[E^((I/2)*(c + d*x))] + (2*I)*d*x*P
olyLog[2, (-I)*E^((I/2)*(c + d*x))] - (2*I)*d*x*PolyLog[2, I*E^((I/2)*(c +
d*x))] - 4*PolyLog[3, (-I)*E^((I/2)*(c + d*x))] + 4*PolyLog[3, I*E^((I/2)*(
c + d*x))]))/(d^3*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a+\cos(dx+c)\,a}}\,dx$$

```
[In] int(x^2/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] int(x^2/(a+cos(d*x+c)*a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a\cos(dx+c)+a}}\,dx$$

```
[In] integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(a*cos(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a\left(\cos(c+dx)+1\right)}}\,dx$$

```
[In] integrate(x**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{a\cos\left(dx + c\right) + a}}\,dx$$

```
[In] integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(2)*d^2*x^2*sin(1/2*d*x + 1/2*c) - 4*sqrt(2)*d*x*cos(1/2*d*x + 1/2*
c) + (sqrt(2)*d^2*x^2*sin(1/2*d*x + 1/2*c) - 4*sqrt(2)*d*x*cos(1/2*d*x + 1/
2*c))*cos(d*x + c) - (sqrt(2)*a*d^5*cos(d*x + c)^2 + sqrt(2)*a*d^5*sin(d*x
+ c)^2 + 2*sqrt(2)*a*d^5*cos(d*x + c) + sqrt(2)*a*d^5)*integrate((x^2*cos(2
*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + 2*x^2*cos(d*x + c)*cos(1/2*d*x + 1/2*c)
+ x^2*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + 2*x^2*sin(d*x + c)*sin(1/2*d*
x + 1/2*c) + x^2*cos(1/2*d*x + 1/2*c))/(a*d^2*cos(2*d*x + 2*c)^2 + 4*a*d^2*
cos(d*x + c)^2 + a*d^2*sin(2*d*x + 2*c)^2 + 4*a*d^2*sin(2*d*x + 2*c)*sin(d*
x + c) + 4*a*d^2*sin(d*x + c)^2 + 4*a*d^2*cos(d*x + c) + a*d^2 + 2*(2*a*d^2
*cos(d*x + c) + a*d^2)*cos(2*d*x + 2*c)), x) + 4*(sqrt(2)*a*d^4*cos(d*x + c
)^2 + sqrt(2)*a*d^4*sin(d*x + c)^2 + 2*sqrt(2)*a*d^4*cos(d*x + c) + sqrt(2)
*a*d^4)*integrate((x*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + 2*x*cos(1/2*d*
x + 1/2*c)*sin(d*x + c) - x*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 2*x*cos
(d*x + c)*sin(1/2*d*x + 1/2*c) - x*sin(1/2*d*x + 1/2*c))/(a*d^2*cos(2*d*x +
 2*c)^2 + 4*a*d^2*cos(d*x + c)^2 + a*d^2*sin(2*d*x + 2*c)^2 + 4*a*d^2*sin(2
*d*x + 2*c)*sin(d*x + c) + 4*a*d^2*sin(d*x + c)^2 + 4*a*d^2*cos(d*x + c) +
a*d^2 + 2*(2*a*d^2*cos(d*x + c) + a*d^2)*cos(2*d*x + 2*c)), x) + 2*(sqrt(2)
*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2)
)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 2*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*c
os(d*x + c) + sqrt(2))*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - (sqrt(2)*d^2*x^2*cos(1/2*d*x + 1/2*c) + 4*s
qrt(2)*d*x*sin(1/2*d*x + 1/2*c))*sin(d*x + c))/((d^3*cos(d*x + c)^2 + d^3*s
in(d*x + c)^2 + 2*d^3*cos(d*x + c) + d^3)*sqrt(a))
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{a\cos\left(dx + c\right) + a}}\,dx$$

```
[In] integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(x^2/(a + a*cos(c + d*x))^(1/2),x)
[Out] int(x^2/(a + a*cos(c + d*x))^(1/2), x)
```

3.172 $\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result . . . 1015
Rubi [A] (verified) . . . 1015
Mathematica [A] (verified) . . . 1017
Maple [F] . . . 1018
Fricas [F] . . . 1018
Sympy [F] . . . 1018
Maxima [F] . . . 1018
Giac [F] . . . 1019
Mupad [F(-1)] . . . 1019

Optimal result

Integrand size = 16, antiderivative size = 156

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{4ix\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}$$

```
[Out] -4*I*x*arctan(exp(1/2*I*(d*x+c)))*cos(1/2*d*x+1/2*c)/d/(a+a*cos(d*x+c))^(1/
2)+4*I*cos(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*I*(d*x+c)))/d^2/(a+a*cos(d*x
+c))^(1/2)-4*I*cos(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*I*(d*x+c)))/d^2/(a+a*
cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{3400, 4266, 2317, 2438\}$

$$\int \frac{x}{\sqrt{a + a\cos(c + dx)}}\,dx = -\frac{4ix \arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a\cos(c + dx) + a}} + \frac{4i\,\mathrm{PolyLog}\left(2, -ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a\cos(c + dx) + a}} - \frac{4i\,\mathrm{PolyLog}\left(2, ie^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a\cos(c + dx) + a}}$$

```
[In] Int[x/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*x*ArcTan[E^((I/2)*(c + d*x))]*Cos[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cos
[c + d*x]]) + ((4*I)*Cos[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^((I/2)*(c + d*x))
])/(d^2*Sqrt[a + a*Cos[c + d*x]]) - ((4*I)*Cos[c/2 + (d*x)/2]*PolyLog[2, I*
E^((I/2)*(c + d*x))])/(d^2*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\int x\csc\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{\sqrt{a+a\cos(c+dx)}} \\
&= -\frac{4ix\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad -\frac{\left(2\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\log\left(1-ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad +\frac{\left(2\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\int\log\left(1+ie^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,dx}{d\sqrt{a+a\cos(c+dx)}} \\
&= -\frac{4ix\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad +\frac{\left(4i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\
&\quad -\frac{\left(4i\sin\left(\frac{1}{2}\left(c+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{dx}{2}\right)\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{i\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\
&= -\frac{4ix\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}}+\frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\
&\quad -\frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int\frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{4i\cos\left(\frac{1}{2}(c+dx)\right)\left(dx\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)-\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)+\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)\right)}{d^2\sqrt{a(1+\cos(c+dx))}}$$

```
[In] Integrate[x/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-4*I)*Cos[(c + d*x)/2]*(d*x*ArcTan[E^((I/2)*(c + d*x))] - PolyLog[2, (-I)
*E^((I/2)*(c + d*x))] + PolyLog[2, I*E^((I/2)*(c + d*x))]))/(d^2*Sqrt[a*(1
+ Cos[c + d*x])])
```

Maple [F]

$$\int \frac{x}{\sqrt{a+\cos\left(dx+c\right)a}}dx$$

```
[In] int(x/(a+cos(d*x+c)*a)^(1/2),x)
[Out] int(x/(a+cos(d*x+c)*a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(a*cos(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

```
[In] integrate(x/(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(x/sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*(sqrt(2)*d*x*cos(1/2*d*x + 1/2*c)*sin(d*x + c) - sqrt(2)*d*x*cos(d*x + c)
*sin(1/2*d*x + 1/2*c) - sqrt(2)*d*x*sin(1/2*d*x + 1/2*c) - (sqrt(2)*cos(d*x
 + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan
2(cos(1/2*d*x + 1/2*c), sin(1/2*d*x + 1/2*c) + 1) - (sqrt(2)*cos(d*x + c)^2
 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan2(cos(1
/2*d*x + 1/2*c), -sin(1/2*d*x + 1/2*c) + 1) + (sqrt(2)*a*d^3*cos(d*x + c)^2
 + sqrt(2)*a*d^3*sin(d*x + c)^2 + 2*sqrt(2)*a*d^3*cos(d*x + c) + sqrt(2)*a*
d^3)*integrate((x*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + 2*x*cos(d*x + c)*
```

```
cos(1/2*d*x + 1/2*c) + x*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + 2*x*sin(d*
x + c)*sin(1/2*d*x + 1/2*c) + x*cos(1/2*d*x + 1/2*c))/(a*d*cos(2*d*x + 2*c)
^2 + 4*a*d*cos(d*x + c)^2 + a*d*sin(2*d*x + 2*c)^2 + 4*a*d*sin(2*d*x + 2*c)
*sin(d*x + c) + 4*a*d*sin(d*x + c)^2 + 4*a*d*cos(d*x + c) + a*d + 2*(2*a*d*
cos(d*x + c) + a*d)*cos(2*d*x + 2*c)), x))/((d^2*cos(d*x + c)^2 + d^2*sin(d
*x + c)^2 + 2*d^2*cos(d*x + c) + d^2)*sqrt(a))
```

Giac [F]

$$\int \frac{x}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{x}{\sqrt{a\cos\left(dx + c\right) + a}}\,dx$$

```
[In] integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(a*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a + a\cos(c + dx)}}\,dx = \int \frac{x}{\sqrt{a + a\,\cos\left(c + d\,x\right)}}\,dx$$

```
[In] int(x/(a + a*cos(c + d*x))^(1/2),x)
[Out] int(x/(a + a*cos(c + d*x))^(1/2), x)
```

3.173 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result 1020
Rubi [A] (verified) 1020
Mathematica [A] (verified) 1021
Maple [C] (warning: unable to verify) 1021
Fricas [A] (verification not implemented) 1022
Sympy [F] 1022
Maxima [B] (verification not implemented) 1022
Giac [B] (verification not implemented) 1023
Mupad [B] (verification not implemented) 1023

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2728, 212}

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$

$$= \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

```
[In] Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])
])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\sqrt{2}\ \text{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert 1\right)}{d\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \text{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	56

```
[In] int(1/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/sec(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/csgn(cos(1/2*
d*x+1/2*c))*InverseJacobiAM(1/2*d*x+1/2*c,1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d},\right.$$
$$\left.-\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}}{\sin(dx+c)}\right)}{d}\right]$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)
*sqrt(-1/a)/sin(d*x + c))/d]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos(c+dx)+a}}\,dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cos(c + d*x) + a), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\right.\right.}{2\sqrt{a}d}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(37) = 74$.

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{a + a\cos(c + dx)}}\,dx$$
$$= \frac{\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{4\,d}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2))/(s
qrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c)
+ sin(1/2*d*x + 1/2*c) - 2))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [B] (verification not implemented)

Time = 14.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a + a\cos(c + dx)}}\,dx = \frac{\mathrm{F}\left(\frac{c}{2} + \frac{dx}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\cos(c+dx))}{a}}}{d\,\sqrt{a + a\,\cos\left(c + d\,x\right)}}$$

```
[In] int(1/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a*
cos(c + d*x))^(1/2))
```

3.174 $\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal result . . . 1024
Rubi [N/A] . . . 1024
Mathematica [N/A] . . . 1025
Maple [N/A] (verified) . . . 1025
Fricas [N/A] . . . 1025
Sympy [N/A] . . . 1025
Maxima [N/A] . . . 1026
Giac [N/A] . . . 1026
Mupad [N/A] . . . 1026

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{a+a\cos(c+dx)}}, x\right)$$

```
[Out] Unintegrable(1/x/(a+a*cos(d*x+c))^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

```
[In] Int[1/(x*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[a + a*Cos[c + d*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

```
[In] Integrate[1/(x*Sqrt[a + a*Cos[c + d*x]]),x]
[Out] Integrate[1/(x*Sqrt[a + a*Cos[c + d*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a+\cos(dx+c)\,a}}dx$$

```
[In] int(1/x/(a+cos(d*x+c)*a)^(1/2),x)
[Out] int(1/x/(a+cos(d*x+c)*a)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos(dx+c)+ax}}\,dx$$

```
[In] integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cos(d*x + c) + a)/(a*x*cos(d*x + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a\left(\cos(c+dx)+1\right)}}\,dx$$

```
[In] integrate(1/x/(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(x*sqrt(a*(cos(c + d*x) + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos\left(dx+c\right)+ax}}\,dx$$

```
[In] integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos\left(dx+c\right)+ax}}\,dx$$

```
[In] integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 14.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{x\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(1/(x*(a + a*cos(c + d*x))^(1/2)),x)
[Out] int(1/(x*(a + a*cos(c + d*x))^(1/2)), x)
```

3.175 $\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx$

Optimal result . . . 1027
Rubi [A] (verified) . . . 1028
Mathematica [A] (verified) . . . 1031
Maple [F] . . . 1031
Fricas [F] . . . 1031
Sympy [F] . . . 1032
Maxima [F] . . . 1032
Giac [F] . . . 1032
Mupad [F(-1)] . . . 1032

Optimal result

Integrand size = 15, antiderivative size = 235

$$\begin{aligned}\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx = &-\frac{4x^3\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{12ix^2\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}\\&-\frac{12ix^2\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}\\&-\frac{48x\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{48x\operatorname{PolyLog}\left(3,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}\\&-\frac{96i\operatorname{PolyLog}\left(4,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{96i\operatorname{PolyLog}\left(4,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}\end{aligned}$$

```
[Out] -4*x^3*arctanh(exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+12*I*x^2*polylog
(2,-exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-12*I*x^2*polylog(2,exp(1/2*
I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-48*x*polylog(3,-exp(1/2*I*x))*sin(1/2*x
)/(a-a*cos(x))^(1/2)+48*x*polylog(3,exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(
1/2)-96*I*polylog(4,-exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+96*I*polyl
og(4,exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3400, 4268, 2611, 6744, 2320, 6724}

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx = -\frac{4x^3 \operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} + \frac{12ix^2\,\operatorname{PolyLog}\left(2, -e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{12ix^2\,\operatorname{PolyLog}\left(2, e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{48x\,\operatorname{PolyLog}\left(3, -e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} + \frac{48x\,\operatorname{PolyLog}\left(3, e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{96i\,\operatorname{PolyLog}\left(4, -e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} + \frac{96i\,\operatorname{PolyLog}\left(4, e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}}$$

```
[In] Int[x^3/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-4*x^3*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((12*I)*x^2*Pol
yLog[2, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((12*I)*x^2*PolyLog[2,
 E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - (48*x*PolyLog[3, -E^((I/2)*x)]
*Sin[x/2])/Sqrt[a - a*Cos[x]] + (48*x*PolyLog[3, E^((I/2)*x)]*Sin[x/2])/Sqr
t[a - a*Cos[x]] - ((96*I)*PolyLog[4, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos
[x]] + ((96*I)*PolyLog[4, E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
```

```
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin\left(\frac{x}{2}\right)\int x^3\csc\left(\frac{x}{2}\right)\,dx}{\sqrt{a-a\cos(x)}} \\
&= -\frac{4x^3\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(6\sin\left(\frac{x}{2}\right)\right)\int x^2\log\left(1-e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}} \\
&\quad + \frac{\left(6\sin\left(\frac{x}{2}\right)\right)\int x^2\log\left(1+e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}} \\
&= -\frac{4x^3\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} \\
&\quad - \frac{12ix^2\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(24i\sin\left(\frac{x}{2}\right)\right)\int x\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}} \\
&\quad + \frac{\left(24i\sin\left(\frac{x}{2}\right)\right)\int x\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4x^3 \operatorname{arctanh}\left(e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} + \frac{12ix^2 \operatorname{PolyLog}\left(2, -e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} \\
&\quad - \frac{12ix^2 \operatorname{PolyLog}\left(2, e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{48x \operatorname{PolyLog}\left(3, -e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} \\
&\quad + \frac{48x \operatorname{PolyLog}\left(3, e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} + \frac{\left(48 \sin\left(\frac{x}{2}\right)\right) \int \operatorname{PolyLog}\left(3, -e^{\frac{ix}{2}}\right) dx}{\sqrt{a - a\cos(x)}} \\
&\quad - \frac{\left(48 \sin\left(\frac{x}{2}\right)\right) \int \operatorname{PolyLog}\left(3, e^{\frac{ix}{2}}\right) dx}{\sqrt{a - a\cos(x)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4x^3 \operatorname{arctanh}\left(e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} + \frac{12ix^2 \operatorname{PolyLog}\left(2, -e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} \\
&\quad - \frac{12ix^2 \operatorname{PolyLog}\left(2, e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{48x \operatorname{PolyLog}\left(3, -e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} \\
&\quad + \frac{48x \operatorname{PolyLog}\left(3, e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{\left(96i \sin\left(\frac{x}{2}\right)\right) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, -x)}{x} dx, x, e^{\frac{ix}{2}}\right)}{\sqrt{a - a\cos(x)}} \\
&\quad + \frac{\left(96i \sin\left(\frac{x}{2}\right)\right) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, x)}{x} dx, x, e^{\frac{ix}{2}}\right)}{\sqrt{a - a\cos(x)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4x^3 \operatorname{arctanh}\left(e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} + \frac{12ix^2 \operatorname{PolyLog}\left(2, -e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} \\
&\quad - \frac{12ix^2 \operatorname{PolyLog}\left(2, e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{48x \operatorname{PolyLog}\left(3, -e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} \\
&\quad + \frac{48x \operatorname{PolyLog}\left(3, e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{96i \operatorname{PolyLog}\left(4, -e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} \\
&\quad + \frac{96i \operatorname{PolyLog}\left(4, e^{\frac{ix}{2}}\right) \sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx =$$
$$-\frac{i\left(8\pi^4 - x^4 + 8ix^3\log\left(1 - e^{-\frac{ix}{2}}\right) - 8ix^3\log\left(1 + e^{\frac{ix}{2}}\right) - 48x^2\,\mathrm{PolyLog}\left(2, e^{-\frac{ix}{2}}\right) - 48x^2\,\mathrm{PolyLog}\left(2, -\right.\right.}{}$$

```
[In] Integrate[x^3/Sqrt[a - a*Cos[x]],x]
```

```
[Out] ((-1/4*I)*(8*Pi^4 - x^4 + (8*I)*x^3*Log[1 - E^((-1/2*I)*x)] - (8*I)*x^3*Log
[1 + E^((I/2)*x)] - 48*x^2*PolyLog[2, E^((-1/2*I)*x)] - 48*x^2*PolyLog[2, -
E^((I/2)*x)] + (192*I)*x*PolyLog[3, E^((-1/2*I)*x)] - (192*I)*x*PolyLog[3,
-E^((I/2)*x)] + 384*PolyLog[4, E^((-1/2*I)*x)] + 384*PolyLog[4, -E^((I/2)*x
)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{a - \cos(x)\,a}}\,dx$$

```
[In] int(x^3/(a-cos(x)*a)^(1/2),x)
```

```
[Out] int(x^3/(a-cos(x)*a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{-a\cos(x) + a}}\,dx$$

```
[In] integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*cos(x) + a)*x^3/(a*cos(x) - a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{-a\left(\cos\left(x\right) - 1\right)}}\,dx$$

```
[In] integrate(x**3/(a-a*cos(x))**(1/2),x)
[Out] Integral(x**3/sqrt(-a*(cos(x) - 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{-a\cos\left(x\right) + a}}\,dx$$

```
[In] integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(-a*cos(x) + a), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{-a\cos\left(x\right) + a}}\,dx$$

```
[In] integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(-a*cos(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{a - a\,\cos\left(x\right)}}\,dx$$

```
[In] int(x^3/(a - a*cos(x))^(1/2),x)
[Out] int(x^3/(a - a*cos(x))^(1/2), x)
```

3.176 $\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx$

Optimal result . . . 1033
Rubi [A] (verified) . . . 1033
Mathematica [A] (verified) . . . 1036
Maple [F] . . . 1036
Fricas [F] . . . 1036
Sympy [F] . . . 1036
Maxima [F] . . . 1037
Giac [F] . . . 1037
Mupad [F(-1)] . . . 1037

Optimal result

Integrand size = 15, antiderivative size = 163

$$\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx = -\frac{4x^2\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{8ix\,\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{8ix\,\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{16\,\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{16\,\operatorname{PolyLog}\left(3,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

```
[Out] -4*x^2*arctanh(exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+8*I*x*polylog(2,
-exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-8*I*x*polylog(2,exp(1/2*I*x))*
sin(1/2*x)/(a-a*cos(x))^(1/2)-16*polylog(3,-exp(1/2*I*x))*sin(1/2*x)/(a-a*c
os(x))^(1/2)+16*polylog(3,exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{3400, 4268, 2611, 2320, 6724\}$

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = -\frac{4x^2 \operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} + \frac{8ix\,\operatorname{PolyLog}\left(2, -e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{8ix\,\operatorname{PolyLog}\left(2, e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} - \frac{16\,\operatorname{PolyLog}\left(3, -e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}} + \frac{16\,\operatorname{PolyLog}\left(3, e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}}$$

```
[In] Int[x^2/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-4*x^2*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((8*I)*x*PolyLo
g[2, -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((8*I)*x*PolyLog[2, E^((I
/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - (16*PolyLog[3, -E^((I/2)*x)]*Sin[x/2
])/Sqrt[a - a*Cos[x]] + (16*PolyLog[3, E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Co
s[x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\sin\left(\frac{x}{2}\right)\int x^2 \csc\left(\frac{x}{2}\right)\,dx}{\sqrt{a-a\cos(x)}}$$

$$= -\frac{4x^2 \operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(4\sin\left(\frac{x}{2}\right)\right)\int x\log\left(1-e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}} + \frac{\left(4\sin\left(\frac{x}{2}\right)\right)\int x\log\left(1+e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}}$$

$$= -\frac{4x^2 \operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{8ix\,\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{8ix\,\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(8i\sin\left(\frac{x}{2}\right)\right)\int \operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}} + \frac{\left(8i\sin\left(\frac{x}{2}\right)\right)\int \operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}}$$

$$= -\frac{4x^2 \operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{8ix\,\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{8ix\,\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(16\sin\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}} + \frac{\left(16\sin\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}$$

$$= -\frac{4x^2 \operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{8ix\,\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{8ix\,\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{16\,\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{16\,\operatorname{PolyLog}\left(3,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx$$
$$= \frac{2\left(x^2 \log\left(1 - e^{\frac{ix}{2}}\right) - x^2 \log\left(1 + e^{\frac{ix}{2}}\right) + 4ix\,\mathrm{PolyLog}\left(2, -e^{\frac{ix}{2}}\right) - 4ix\,\mathrm{PolyLog}\left(2, e^{\frac{ix}{2}}\right) - 8\,\mathrm{PolyLog}\left(3, -\right.\right.}{\sqrt{a - a\cos(x)}}$$

```
[In] Integrate[x^2/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (2*(x^2*Log[1 - E^((I/2)*x)] - x^2*Log[1 + E^((I/2)*x)] + (4*I)*x*PolyLog[2
, -E^((I/2)*x)] - (4*I)*x*PolyLog[2, E^((I/2)*x)] - 8*PolyLog[3, -E^((I/2)*
x)] + 8*PolyLog[3, E^((I/2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a - \cos(x)\,a}}\,dx$$

```
[In] int(x^2/(a-cos(x)*a)^(1/2),x)
```

```
[Out] int(x^2/(a-cos(x)*a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{-a\cos(x) + a}}\,dx$$

```
[In] integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*cos(x) + a)*x^2/(a*cos(x) - a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{-a\left(\cos(x) - 1\right)}}\,dx$$

```
[In] integrate(x**2/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-a*(cos(x) - 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{-a\cos(x) + a}}\,dx$$

```
[In] integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(-a*cos(x) + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{-a\cos(x) + a}}\,dx$$

```
[In] integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(-a*cos(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx$$

```
[In] int(x^2/(a - a*cos(x))^(1/2),x)
[Out] int(x^2/(a - a*cos(x))^(1/2), x)
```

3.177 $\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx$

Optimal result . . . 1038
Rubi [A] (verified) . . . 1038
Mathematica [A] (verified) . . . 1040
Maple [F] . . . 1040
Fricas [F] . . . 1040
Sympy [F] . . . 1040
Maxima [F] . . . 1041
Giac [F] . . . 1041
Mupad [F(-1)] . . . 1041

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx = -\frac{4x\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{4i\,\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{4i\,\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

```
[Out] -4*x*arctanh(exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+4*I*polylog(2,-exp
(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-4*I*polylog(2,exp(1/2*I*x))*sin(1/
2*x)/(a-a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3400, 4268, 2317, 2438}

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx = -\frac{4x\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{4i\,\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{4i\,\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

```
[In] Int[x/Sqrt[a - a*Cos[x]],x]
```

```
[Out] (-4*x*ArcTanh[E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] + ((4*I)*PolyLog[2,
 -E^((I/2)*x)]*Sin[x/2])/Sqrt[a - a*Cos[x]] - ((4*I)*PolyLog[2, E^((I/2)*x)
]*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\sin\left(\frac{x}{2}\right)\int x\csc\left(\frac{x}{2}\right)\,dx}{\sqrt{a-a\cos(x)}}$$

$$= -\frac{4x\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(2\sin\left(\frac{x}{2}\right)\right)\int\log\left(1-e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}} + \frac{\left(2\sin\left(\frac{x}{2}\right)\right)\int\log\left(1+e^{\frac{ix}{2}}\right)\,dx}{\sqrt{a-a\cos(x)}}$$

$$= -\frac{4x\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{\left(4i\sin\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}} - \frac{\left(4i\sin\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{\frac{ix}{2}}\right)}{\sqrt{a-a\cos(x)}}$$

$$= -\frac{4x\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{4i\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{4i\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{x}{\sqrt{a - a\cos(x)}}\,dx$$
$$= \frac{2\left(x\left(\log\left(1 - e^{\frac{ix}{2}}\right) - \log\left(1 + e^{\frac{ix}{2}}\right)\right) + 2i\,\mathrm{PolyLog}\left(2, -e^{\frac{ix}{2}}\right) - 2i\,\mathrm{PolyLog}\left(2, e^{\frac{ix}{2}}\right)\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}}$$

```
[In] Integrate[x/Sqrt[a - a*Cos[x]],x]
[Out] (2*(x*(Log[1 - E^((I/2)*x)] - Log[1 + E^((I/2)*x)]) + (2*I)*PolyLog[2, -E^(
(I/2)*x)] - (2*I)*PolyLog[2, E^((I/2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [F]

$$\int \frac{x}{\sqrt{a - \cos(x)\,a}}dx$$

```
[In] int(x/(a-cos(x)*a)^(1/2),x)
[Out] int(x/(a-cos(x)*a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x}{\sqrt{-a\cos(x) + a}}\,dx$$

```
[In] integrate(x/(a-a*cos(x))^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a*cos(x) + a)*x/(a*cos(x) - a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x}{\sqrt{-a\left(\cos(x) - 1\right)}}\,dx$$

```
[In] integrate(x/(a-a*cos(x))**(1/2),x)
[Out] Integral(x/sqrt(-a*(cos(x) - 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x}{\sqrt{-a\cos(x) + a}}\,dx$$

```
[In] integrate(x/(a-a*cos(x))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(-a*cos(x) + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x}{\sqrt{-a\cos(x) + a}}\,dx$$

```
[In] integrate(x/(a-a*cos(x))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(-a*cos(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x}{\sqrt{a - a\cos(x)}}\,dx$$

```
[In] int(x/(a - a*cos(x))^(1/2),x)
[Out] int(x/(a - a*cos(x))^(1/2), x)
```

3.178 $\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx$

Optimal result . . . 1042
Rubi [A] (verified) . . . 1042
Mathematica [A] (verified) . . . 1043
Maple [A] (verified) . . . 1043
Fricas [A] (verification not implemented) . . . 1044
Sympy [F] . . . 1044
Maxima [B] (verification not implemented) . . . 1044
Giac [A] (verification not implemented) . . . 1045
Mupad [F(-1)] . . . 1045

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*sin(x)*a^(1/2)*2^(1/2)/(a-a*cos(x))^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2728, 212}

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}$$

```
[In] Int[1/Sqrt[a - a*Cos[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a - a*Cos[x]])])/Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{2a - x^2}\, dx, x, \frac{a\sin(x)}{\sqrt{a - a\cos(x)}}\right)\right)$$

$$= -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{a - a\cos(x)}}\, dx = \frac{2\left(-\log\left(\cos\left(\frac{x}{4}\right)\right) + \log\left(\sin\left(\frac{x}{4}\right)\right)\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a - a\cos(x)}}$$

```
[In] Integrate[1/Sqrt[a - a*Cos[x]],x]
[Out] (2*(-Log[Cos[x/4]] + Log[Sin[x/4]])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{\sin\left(\frac{x}{2}\right)\text{arctanh}\left(\cos\left(\frac{x}{2}\right)\right)\sqrt{2}}{\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}}$	25

```
[In] int(1/(a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -sin(1/2*x)*arctanh(cos(1/2*x))*2^(1/2)/(a*sin(1/2*x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx$$
$$= \left[\frac{\sqrt{2}\log\left(-\frac{(\cos(x)+3)\sin(x)-\frac{2\sqrt{2}\sqrt{-a\cos(x)+a}(\cos(x)+1)}{\sqrt{a}}}{(\cos(x)-1)\sin(x)}\right)}{2\sqrt{a}}, \sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{-a\cos(x)+a}\sqrt{-\frac{1}{a}}}{\sin(x)}\right)\right]$$

```
[In] integrate(1/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-((cos(x) + 3)*sin(x) - 2*sqrt(2)*sqrt(-a*cos(x) + a)*(cos
(x) + 1)/sqrt(a))/((cos(x) - 1)*sin(x)))/sqrt(a), sqrt(2)*sqrt(-1/a)*arctan
(sqrt(2)*sqrt(-a*cos(x) + a)*sqrt(-1/a)/sin(x))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{\sqrt{-a\cos(x)+a}}\,dx$$

```
[In] integrate(1/(a-a*cos(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*cos(x) + a), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(28) = 56.

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx =$$
$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)\right)\right)^2+\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)\right)\right)^2+2\cos\left(\frac{1}{2}\arctan\left(\sin(x),c\right.\right.\right.}{}$$

```
[In] integrate(1/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(x), cos(x)))^2 + sin(1/2*arctan2(sin(
x), cos(x)))^2 + 2*cos(1/2*arctan2(sin(x), cos(x))) + 1) - sqrt(2)*log(cos(
1/2*arctan2(sin(x), cos(x)))^2 + sin(1/2*arctan2(sin(x), cos(x)))^2 - 2*cos
(1/2*arctan2(sin(x), cos(x))) + 1))/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a - a\cos(x)}}\,dx = \frac{\sqrt{2}\log\left(\left|\tan\left(\frac{1}{4}\,x\right)\right|\right)}{\sqrt{a}\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)}$$

```
[In] integrate(1/(a-a*cos(x))^(1/2),x, algorithm="giac")
[Out] sqrt(2)*log(abs(tan(1/4*x)))/(sqrt(a)*sgn(sin(1/2*x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{1}{\sqrt{a - a\,\cos\left(x\right)}}\,dx$$

```
[In] int(1/(a - a*cos(x))^(1/2),x)
[Out] int(1/(a - a*cos(x))^(1/2), x)
```

3.179 $\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$

Optimal result . . . 1046
Rubi [N/A] . . . 1046
Mathematica [N/A] . . . 1047
Maple [N/A] (verified) . . . 1047
Fricas [N/A] . . . 1047
Sympy [N/A] . . . 1047
Maxima [N/A] . . . 1048
Giac [N/A] . . . 1048
Mupad [N/A] . . . 1048

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{a-a\cos(x)}}, x\right)$$

```
[Out] Unintegrable(1/x/(a-a*cos(x))^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

```
[In] Int[1/(x*Sqrt[a - a*Cos[x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[a - a*Cos[x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

```
[In] Integrate[1/(x*Sqrt[a - a*Cos[x]]),x]
[Out] Integrate[1/(x*Sqrt[a - a*Cos[x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{a-\cos(x)\,a}}dx$$

```
[In] int(1/x/(a-cos(x)*a)^(1/2),x)
[Out] int(1/x/(a-cos(x)*a)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{\sqrt{-a\cos(x)+ax}}\,dx$$

```
[In] integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a*cos(x) + a)/(a*x*cos(x) - a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{x\sqrt{-a\left(\cos(x)-1\right)}}\,dx$$

```
[In] integrate(1/x/(a-a*cos(x))**(1/2),x)
[Out] Integral(1/(x*sqrt(-a*(cos(x) - 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{\sqrt{-a\cos(x)+ax}}\,dx$$

```
[In] integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a*cos(x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{\sqrt{-a\cos(x)+ax}}\,dx$$

```
[In] integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-a*cos(x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 13.98 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

```
[In] int(1/(x*(a - a*cos(x))^(1/2)),x)
[Out] int(1/(x*(a - a*cos(x))^(1/2)), x)
```

3.180 $\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx$

Optimal result . . . 1049
Rubi [A] (verified) . . . 1050
Mathematica [A] (verified) . . . 1055
Maple [F] . . . 1055
Fricas [F] . . . 1055
Sympy [F] . . . 1056
Maxima [F] . . . 1056
Giac [F] . . . 1078
Mupad [F(-1)] . . . 1078

Optimal result

Integrand size = 14, antiderivative size = 423

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(4,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(4,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}$$

```
[Out] -3*x^2/a/(a+a*cos(x))^(1/2)-24*I*x*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*c
os(x))^(1/2)-I*x^3*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*cos(x))^(1/2)+24*
I*cos(1/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+3*I*x^2*cos(1/
2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-24*I*cos(1/2*x)*polylo
g(2,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-3*I*x^2*cos(1/2*x)*polylog(2,I*exp
(1/2*I*x))/a/(a+a*cos(x))^(1/2)-12*x*cos(1/2*x)*polylog(3,-I*exp(1/2*I*x))/
a/(a+a*cos(x))^(1/2)+12*x*cos(1/2*x)*polylog(3,I*exp(1/2*I*x))/a/(a+a*cos(x
))^(1/2)-24*I*cos(1/2*x)*polylog(4,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+24
```

```
*I*cos(1/2*x)*polylog(4,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+1/2*x^3*tan(1/
2*x)/a/(a+a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3400, 4271, 4266, 2317, 2438, 2611, 6744, 2320, 6724}

$$\begin{aligned}
\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx &= -\frac{ix^3 \arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} \\
&- \frac{24ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} \\
&- \frac{3ix^2\,\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} - \frac{12x\,\text{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} \\
&+ \frac{12x\,\text{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} + \frac{24i\,\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} \\
&- \frac{24i\,\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} - \frac{24i\,\text{PolyLog}\left(4,-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} \\
&+ \frac{24i\,\text{PolyLog}\left(4,ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a\cos(x)+a}} - \frac{3x^2}{a\sqrt{a\cos(x)+a}}
\end{aligned}$$

```
[In] Int[x^3/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (-3*x^2)/(a*Sqrt[a + a*Cos[x]]) - ((24*I)*x*ArcTan[E^((I/2)*x)]*Cos[x/2])/(
a*Sqrt[a + a*Cos[x]]) - (I*x^3*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqrt[a + a*
Cos[x]]) + ((24*I)*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos
[x]]) + ((3*I)*x^2*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos
[x]]) - ((24*I)*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]])
- ((3*I)*x^2*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - (
12*x*Cos[x/2]*PolyLog[3, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (12*x*
Cos[x/2]*PolyLog[3, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - ((24*I)*Cos[x/
2]*PolyLog[4, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + ((24*I)*Cos[x/2]*
PolyLog[4, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x^3*Tan[x/2])/(2*a*Sqr
t[a + a*Cos[x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
 - 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
```

```
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\cos\left(\frac{x}{2}\right)\int x^3\sec^3\left(\frac{x}{2}\right)\,dx}{2a\sqrt{a+a\cos(x)}}$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int x^3\sec\left(\frac{x}{2}\right)\,dx}{4a\sqrt{a+a\cos(x)}} + \frac{\left(6\cos\left(\frac{x}{2}\right)\right)\int x\sec\left(\frac{x}{2}\right)\,dx}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}$$
$$- \frac{ix^3\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}$$
$$- \frac{\left(3\cos\left(\frac{x}{2}\right)\right)\int x^2\log\left(1-ie^{\frac{ix}{2}}\right)\,dx}{2a\sqrt{a+a\cos(x)}} + \frac{\left(3\cos\left(\frac{x}{2}\right)\right)\int x^2\log\left(1+ie^{\frac{ix}{2}}\right)\,dx}{2a\sqrt{a+a\cos(x)}}$$
$$- \frac{\left(12\cos\left(\frac{x}{2}\right)\right)\int \log\left(1-ie^{\frac{ix}{2}}\right)\,dx}{a\sqrt{a+a\cos(x)}} + \frac{\left(12\cos\left(\frac{x}{2}\right)\right)\int \log\left(1+ie^{\frac{ix}{2}}\right)\,dx}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} - \frac{\left(6i\cos\left(\frac{x}{2}\right)\right)\int x\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)dx}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{\left(6i\cos\left(\frac{x}{2}\right)\right)\int x\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)dx}{a\sqrt{a+a\cos(x)}} + \frac{\left(24i\cos\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}dx,x,e^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$- \frac{\left(24i\cos\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}dx,x,e^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}$$
$$- \frac{ix^3\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$- \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}$$
$$+ \frac{\left(12\cos\left(\frac{x}{2}\right)\right)\int\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)dx}{a\sqrt{a+a\cos(x)}} - \frac{\left(12\cos\left(\frac{x}{2}\right)\right)\int\operatorname{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)dx}{a\sqrt{a+a\cos(x)}}$$

$$
\begin{aligned}
&= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} - \frac{ix^3\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad + \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad - \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad - \frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} - \frac{\left(24i\cos\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-ix)}{x}\,dx,x,e^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad + \frac{\left(24i\cos\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,ix)}{x}\,dx,x,e^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3x^2}{a\sqrt{a+a\cos(x)}} - \frac{24ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad - \frac{ix^3\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad + \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad - \frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad + \frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(4,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} \\
&\quad + \frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(4,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx =$$

$$-\frac{i\cos\left(\frac{x}{2}\right)\left(-6ix^2\cos\left(\frac{x}{2}\right)+48x\arctan\left(e^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)+2x^3\arctan\left(e^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)-6(8+x^2)\cos^2\left(\frac{x}{2}\right)\mathrm{Pol}\right.}{}$$

```
[In] Integrate[x^3/(a + a*Cos[x])^(3/2),x]
```

```
[Out] ((-I)*Cos[x/2]*((-6*I)*x^2*Cos[x/2] + 48*x*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 +
 2*x^3*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 - 6*(8 + x^2)*Cos[x/2]^2*PolyLog[2, (
-I)*E^((I/2)*x)] + 6*(8 + x^2)*Cos[x/2]^2*PolyLog[2, I*E^((I/2)*x)] - (24*I
)*x*Cos[x/2]^2*PolyLog[3, (-I)*E^((I/2)*x)] + (24*I)*x*Cos[x/2]^2*PolyLog[3
, I*E^((I/2)*x)] + 48*Cos[x/2]^2*PolyLog[4, (-I)*E^((I/2)*x)] - 48*Cos[x/2]
^2*PolyLog[4, I*E^((I/2)*x)] + I*x^3*Sin[x/2]))/(a*(1 + Cos[x]))^(3/2)
```

Maple [F]

$$\int \frac{x^3}{(a+\cos(x)\,a)^{\frac{3}{2}}}\,dx$$

```
[In] int(x^3/(a+cos(x)*a)^(3/2),x)
```

```
[Out] int(x^3/(a+cos(x)*a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x) + a)*x^3/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^3}{(a + a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{\left(a\left(\cos\left(x\right) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(a+a*cos(x))**(3/2),x)
[Out] Integral(x**3/(a*(cos(x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(a + a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{\left(a\cos\left(x\right) + a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="maxima")
[Out] 4/9*((6*sqrt(2)*x^2*cos(3/2*x) - (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*
cos(3*x)^3 + (6*sqrt(2)*x^2*sin(3/2*x) + (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(
3/2*x))*sin(3*x)^3 + 48*sqrt(2)*cos(2*x)^2*cos(3/2*x) + 48*sqrt(2)*cos(3/2*
x)*sin(2*x)^2 + ((2*(9*sqrt(2)*x^2 + 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x
^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)*x^2 + 2*(9*sqrt(2)*x^2
 + 8*sqrt(2))*cos(x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x))*cos(3/2*x) +
 (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2)
)*sin(3/2*x))*sin(2*x) - (9*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*c
os(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 24*sqrt(2)*x)*sin(3/2*x))*co
s(3*x)^2 + 3*((2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3
 - 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) + 3*(2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(
3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)
*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(x) + 9*(3*sqrt(2)*x^3 - 8*sqrt(2)*
x)*sin(x) - 16*sqrt(2))*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2
)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt
(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^
2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt
(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*
a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*int
egrate(1/9*(x^3*cos(4*x)*cos(3/2*x) + 4*x^3*cos(3*x)*cos(3/2*x) + 6*x^3*cos
(2*x)*cos(3/2*x) + x^3*sin(4*x)*sin(3/2*x) + 4*x^3*sin(3*x)*sin(3/2*x) + 6*
x^3*sin(2*x)*sin(3/2*x) + 4*x^3*sin(3/2*x)*sin(x) + (4*x^3*cos(x) + x^3)*co
s(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*
cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*
sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x)
+ 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2
```

```
*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3
*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*
sin(x))*sin(3*x)), x) - 486*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x
)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2
*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)
*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x
) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x)
 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x
^2*cos(3/2*x)*sin(4*x) + 4*x^2*cos(3/2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2
*x) - x^2*cos(4*x)*sin(3/2*x) - 4*x^2*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*
sin(3/2*x) + 4*x^2*cos(3/2*x)*sin(x) - (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^
2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^
2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(
x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2
*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2
)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*s
in(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3
*x)), x) - 648*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(
2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sq
rt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) +
 sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a
^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)
*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x*cos(4*x)*cos
(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*si
n(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*
x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^
2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^
2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*co
s(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4
*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) +
 a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x
) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + (3*(3*sqrt(2)*x^3 -
8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(3*x
) + 3*(3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sq
rt(2))*sin(3/2*x))*sin(2*x) - 3*(3*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(
2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 8*sqrt(2)*x)*sin(3/2*
x))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 3*((2*(9*sqrt(2)*x^2 - 8*s
qrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) +
 3*(2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)
*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*
cos(x) + 9*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x) - 16*sqrt(2))*cos(3/2*x) +
243*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(
x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*s
in(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^
2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x
```

```
) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*
x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x^3*cos(4*x)*cos(3/2*x) +
 4*x^3*cos(3*x)*cos(3/2*x) + 6*x^3*cos(2*x)*cos(3/2*x) + x^3*sin(4*x)*sin(3
/2*x) + 4*x^3*sin(3*x)*sin(3/2*x) + 6*x^3*sin(2*x)*sin(3/2*x) + 4*x^3*sin(3
/2*x)*sin(x) + (4*x^3*cos(x) + x^3)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*co
s(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*si
n(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8
*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2
)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*c
os(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*
sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 486*(sqrt(2)*
a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2
)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x
) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt
(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt
(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*
a^2*sin(x))*sin(3*x))*integrate(1/9*(x^2*cos(3/2*x)*sin(4*x) + 4*x^2*cos(3/
2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2*x) - x^2*cos(4*x)*sin(3/2*x) - 4*x^2
*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*sin(3/2*x) + 4*x^2*cos(3/2*x)*sin(x)
 - (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36
*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36
*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) +
 a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) +
8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*
cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16
*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 648*(sqrt(2)*a^2*cos(3*x)^
2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x
)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)
*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2
*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x
) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*s
in(3*x))*integrate(1/9*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6
*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) +
6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*
x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)
^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*
x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^
2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x
) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) +
3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x)
)*sin(3*x)), x) + (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2
)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(3*x) + 3*(3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x
)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(2*x) - 3*(3*sq
rt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*s
qrt(2))*sin(x) - 8*sqrt(2)*x)*sin(3/2*x))*cos(2/3*arctan2(sin(3/2*x), cos(3
```

```
/2*x)))^2 + ((6*sqrt(2)*x^2*cos(3/2*x) - (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(
3/2*x))*cos(3*x) + (2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)
*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (6*sqrt(2)*x^2 + 2*(9*sqrt(2)*x^
2 - 8*sqrt(2))*cos(x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x))*cos(3/2*x)
+ (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 + 8*sqrt(2
))*sin(3/2*x))*sin(2*x) - (3*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*
cos(x) - 2*(9*sqrt(2)*x^2 + 8*sqrt(2))*sin(x) - 8*sqrt(2)*x)*sin(3/2*x))*si
n(3*x)^2 + 3*((2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3
- 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) + 3*(2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(
3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)
*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(x) + 9*(3*sqrt(2)*x^3 - 8*sqrt(2)*
x)*sin(x) - 16*sqrt(2))*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2
)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt
(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^
2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt
(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*
a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*int
egrate(1/9*(x^3*cos(4*x)*cos(3/2*x) + 4*x^3*cos(3*x)*cos(3/2*x) + 6*x^3*cos
(2*x)*cos(3/2*x) + x^3*sin(4*x)*sin(3/2*x) + 4*x^3*sin(3*x)*sin(3/2*x) + 6*
x^3*sin(2*x)*sin(3/2*x) + 4*x^3*sin(3/2*x)*sin(x) + (4*x^3*cos(x) + x^3)*co
s(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*
cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*
sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x)
+ 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2
*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3
*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*
sin(x))*sin(3*x)), x) - 486*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x
)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2
*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)
*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x
) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x)
 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x
^2*cos(3/2*x)*sin(4*x) + 4*x^2*cos(3/2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2
*x) - x^2*cos(4*x)*sin(3/2*x) - 4*x^2*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*
sin(3/2*x) + 4*x^2*cos(3/2*x)*sin(x) - (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^
2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^
2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(
x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2
*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2
)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*s
in(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3
*x)), x) - 648*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(
2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sq
rt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) +
 sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a
```

```
^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)
*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x*cos(4*x)*cos
(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*si
n(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*
x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^
2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^
2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*co
s(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4
*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) +
 a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x
) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + (3*(3*sqrt(2)*x^3 -
8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(3*x
) + 3*(3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sq
rt(2))*sin(3/2*x))*sin(2*x) - 3*(3*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(
2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 8*sqrt(2)*x)*sin(3/2*
x))*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 3*((2*(9*sqrt(2)*x^2 - 8*s
qrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) +
 3*(2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)
*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*
cos(x) + 9*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x) - 16*sqrt(2))*cos(3/2*x) +
243*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(
x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*s
in(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^
2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x
) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*
x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x^3*cos(4*x)*cos(3/2*x) +
 4*x^3*cos(3*x)*cos(3/2*x) + 6*x^3*cos(2*x)*cos(3/2*x) + x^3*sin(4*x)*sin(3
/2*x) + 4*x^3*sin(3*x)*sin(3/2*x) + 6*x^3*sin(2*x)*sin(3/2*x) + 4*x^3*sin(3
/2*x)*sin(x) + (4*x^3*cos(x) + x^3)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*co
s(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*si
n(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8
*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2
)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*c
os(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*
sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 486*(sqrt(2)*
a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2
)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x
) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt
(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt
(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*
a^2*sin(x))*sin(3*x))*integrate(1/9*(x^2*cos(3/2*x)*sin(4*x) + 4*x^2*cos(3/
2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2*x) - x^2*cos(4*x)*sin(3/2*x) - 4*x^2
*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*sin(3/2*x) + 4*x^2*cos(3/2*x)*sin(x)
- (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36
*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36
```

```
*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) +
 a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) +
8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*
cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16
*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 648*(sqrt(2)*a^2*cos(3*x)^
2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x
)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)
*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2
*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x
) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*s
in(3*x))*integrate(1/9*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6
*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) +
6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*
x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)
^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*
x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^
2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x
) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) +
3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x)
)*sin(3*x)), x) + (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2
)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(3*x) + 3*(3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x
)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(2*x) - 3*(3*sq
rt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*s
qrt(2))*sin(x) - 8*sqrt(2)*x)*sin(3/2*x))*sin(2/3*arctan2(sin(3/2*x), cos(3
/2*x)))^2 + (sqrt(2)*cos(3*x)^4 + sqrt(2)*sin(3*x)^4 + 2*(3*sqrt(2)*cos(2*x
) + 3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x)^3 + 6*(sqrt(2)*sin(2*x) + sqrt(2
)*sin(x))*sin(3*x)^3 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqr
t(2)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2 + 6*(sqrt(
2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x)^2 +
9*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)
*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)
*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) +
6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin
(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(4/3*arctan2(sin(3/2*x), cos
(3/2*x)))^2 + 9*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(
x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(
x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2
))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x)
 + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(2/3*arctan2(s
in(3/2*x), cos(3/2*x)))^2 + (2*sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 +
9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*
sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + 2*sqrt(2))*co
s(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + 2*sqr
t(2))*sin(3*x)^2 + 9*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)
*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)
```

```
*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + s
qrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin
(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(4/3*arct
an2(sin(3/2*x), cos(3/2*x)))^2 + 9*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)
^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sq
rt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt
(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) +
6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2)
)*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqr
t(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(
2)*sin(x)^2 + 2*(9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(
2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 9*(2*sqrt(2)*cos
(x) + sqrt(2))*cos(2*x) + 9*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x) + 6*(3*sqr
t(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(
2*x) + 2*sqrt(2)*cos(x) + sqrt(2))*cos(3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2)
)*sin(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*
x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2
*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(
x) + 3*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqr
t(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) +
 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2
)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2
)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) +
 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*si
n(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), co
s(3/2*x))) + 6*((sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(
2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(4/3*arcta
n2(sin(3/2*x), cos(3/2*x))) + 6*(sqrt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x)
 + 2*sqrt(2)*cos(x) + sqrt(2))*cos(3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*si
n(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2
 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^
2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) +
 3*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)
*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(
(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*s
in(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), c
os(3/2*x))) + 6*((sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(3*x)^2 + 2*(sqrt(2
)*sin(2*x) + sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*
sin(3*x) + 6*(sqrt(2)*sin(3*x)^3 + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*si
n(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2
+ 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 +
2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)
*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(3*x) + 3*(sqr
t(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3
*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x
```

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)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*s
qrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*s
in(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x
))))*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(sqrt(2)*sin(3*x)^3 + 6*(
sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqr
t(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*si
n(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(
x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2
)*cos(x) + sqrt(2))*sin(3*x))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*
sqrt(2)*cos(x) + sqrt(2))*arctan2(cos(1/3*arctan2(sin(3/2*x), cos(3/2*x))),
 sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))) + 1) + (sqrt(2)*cos(3*x)^4 + sqrt
(2)*sin(3*x)^4 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(
3*x)^3 + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x)^3 + 3*(3*sqrt(2)*co
s(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*s
in(x) + 3*sqrt(2)*sin(x)^2 + 6*(sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt
(2)*cos(x) + 2*sqrt(2))*cos(3*x)^2 + 9*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(
2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 1
8*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*
sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x
) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqr
t(2))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*(sqrt(2)*cos(3*x)^2 +
9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*
sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)
*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sq
rt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2
)*cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + (2*sqrt(2)
*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x
)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*
x) + 3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2)
)*cos(2*x) + 6*sqrt(2)*cos(x) + 2*sqrt(2))*sin(3*x)^2 + 9*(sqrt(2)*cos(3*x)
^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqr
t(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sq
rt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x)
 + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*s
qrt(2)*cos(x) + sqrt(2))*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*(sq
rt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(
3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(
x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*
sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*
sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*
x)))^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 +
 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(9*sqrt(2)*cos(2*x)^2
+ 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) +
9*sqrt(2)*sin(x)^2 + 9*(2*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 9*sqrt(2)*co
s(x) + 2*sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(s
```

```
qrt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqrt(2)*cos(x) + sqrt(2))*cos
(3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 +
9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*
sqrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2)
*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)
*cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3
*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos
(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 +
18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3
*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*
x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sq
rt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*((sqrt(2)*sin(2*x) + sq
rt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sq
rt(2)*cos(x) + sqrt(2))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(sqrt(
2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqrt(2)*cos(x) + sqrt(2))*cos(3*x
)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sq
rt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt
(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2)*sin
(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos
(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqr
t(2)*cos(x) + sqrt(2))*cos(2*x) + 6*((sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*co
s(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + s
qrt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*((sqrt(2)*sin(2*x) + s
qrt(2)*sin(x))*cos(3*x)^2 + 2*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(3*x)
+ sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*(sqrt(2)*sin(3*x)^3 + 6*(
sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqr
t(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*si
n(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(
x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2
)*cos(x) + sqrt(2))*sin(3*x) + 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2
 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt
(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2
)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*
(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*
sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))))*sin(4/3*arctan2(sin(3/2*x), cos(3
/2*x))) + 6*(sqrt(2)*sin(3*x)^3 + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin
(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 +
 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2
*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*
cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(3*x))*sin(2/3*
arctan2(sin(3/2*x), cos(3/2*x))) + 6*sqrt(2)*cos(x) + sqrt(2))*arctan2(cos(
1/3*arctan2(sin(3/2*x), cos(3/2*x))), -sin(1/3*arctan2(sin(3/2*x), cos(3/2*
x))) + 1) + (48*sqrt(2)*cos(2*x)^2*cos(3/2*x) + 48*sqrt(2)*cos(3/2*x)*sin(2
*x)^2 + 2*(2*(9*sqrt(2)*x^2 + 24*sqrt(2)*cos(x) + 8*sqrt(2))*cos(3/2*x) - 3
*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + 2*(9*sqrt(2)*x^2 + 24
```

```
*sqrt(2)*cos(x)^2 + 24*sqrt(2)*sin(x)^2 + 2*(9*sqrt(2)*x^2 + 8*sqrt(2))*cos
(x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x))*cos(3/2*x) + 2*(3*(3*sqrt(2)*
x^3 - 8*sqrt(2)*x + 16*sqrt(2)*sin(x))*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sq
rt(2))*sin(3/2*x))*sin(2*x) - (9*sqrt(2)*x^3 + 6*(3*sqrt(2)*x^3 - 8*sqrt(2)
*x)*cos(x) - 4*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 24*sqrt(2)*x)*sin(3/2*x
))*cos(3*x) + (2*(9*sqrt(2)*x^2 + 48*sqrt(2)*cos(x) + 8*sqrt(2))*cos(3/2*x)
 - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (6*sqrt(2)*x^2 +
48*sqrt(2)*cos(x)^2 + 48*sqrt(2)*sin(x)^2 + 2*(9*sqrt(2)*x^2 + 8*sqrt(2))*c
os(x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x))*cos(3/2*x) + 2*(sqrt(2)*cos
(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqrt(2)*cos(x) + sqrt(2))*cos(3*x)^2 +
(sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*
cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*si
n(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2)*sin(2*x)^
2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(x) +
2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*c
os(x) + sqrt(2))*cos(2*x) + 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 +
9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)
*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*c
os(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sq
rt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos
(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*c
os(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2
+ 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) +
 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(
2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) +
sqrt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*((sqrt(2)*sin(2*x) +
sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*
sqrt(2)*cos(x) + sqrt(2))*cos(5/3*arctan2(sin(3/2*x), cos(3/2*x))) + 2*(72*
sqrt(2)*cos(2*x)^2*cos(3/2*x) + 8*sqrt(2)*cos(3/2*x)*sin(3*x)^2 + 72*sqrt(2
)*cos(3/2*x)*sin(2*x)^2 + (2*(9*sqrt(2)*x^2 - 4*sqrt(2))*cos(3/2*x) - 3*(3*
sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x)^2 + (9*(6*sqrt(2)*x^2*cos(3
/2*x) - (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (54*sqrt(2)*x^
2*cos(x) + 36*sqrt(2)*x^2 + 9*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x) - 16*sqr
t(2))*cos(3/2*x) + 3*(3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqr
t(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(2*x) - 3*(6*sqrt(2)*x^3 + 3*(3*sqrt(2
)*x^3 - 8*sqrt(2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 16*sqr
t(2)*x)*sin(3/2*x))*cos(3*x) + 9*(2*(3*sqrt(2)*x^2 + 8*sqrt(2)*cos(x))*cos(
3/2*x) - (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (54*sqrt(2)*x
^2*cos(x) + 18*sqrt(2)*x^2 + 72*sqrt(2)*cos(x)^2 + 72*sqrt(2)*sin(x)^2 + 9*
(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x) - 8*sqrt(2))*cos(3/2*x) + 3*((2*(9*sqr
t(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*
x))*cos(3*x) + 3*(2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x
^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)*x^2 + 6*(9*sqrt(2)*x^2
 - 8*sqrt(2))*cos(x) + 9*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x) - 16*sqrt(2))
*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sq
```

```
rt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18
*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x
) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2
)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt
(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x^3*cos(4*x
)*cos(3/2*x) + 4*x^3*cos(3*x)*cos(3/2*x) + 6*x^3*cos(2*x)*cos(3/2*x) + x^3*
sin(4*x)*sin(3/2*x) + 4*x^3*sin(3*x)*sin(3/2*x) + 6*x^3*sin(2*x)*sin(3/2*x)
 + 4*x^3*sin(3/2*x)*sin(x) + (4*x^3*cos(x) + x^3)*cos(3/2*x))/(a^2*cos(4*x)
^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)
^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^
2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^
2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x)
 + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) +
2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) -
 486*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos
(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*
sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a
^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*
x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2
*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x^2*cos(3/2*x)*sin(4*x)
+ 4*x^2*cos(3/2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2*x) - x^2*cos(4*x)*sin(
3/2*x) - 4*x^2*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*sin(3/2*x) + 4*x^2*cos(
3/2*x)*sin(x) - (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*c
os(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*s
in(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 +
8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^
2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*
cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))
*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 648*(sqrt(2)
*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(
2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(
x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqr
t(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqr
t(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)
*a^2*sin(x))*sin(3*x))*integrate(1/9*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*
cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)
*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x)
 + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 +
 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 +
 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*c
os(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x)
 + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a
^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x)
+ 2*a^2*sin(x))*sin(3*x)), x) + (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x)
 + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(3*x) + 3*(3*(3*sqrt(2)*x^3
```

```
 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(
2*x) - 3*(3*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(x) - 2*(9*sqr
t(2)*x^2 - 8*sqrt(2))*sin(x) - 8*sqrt(2)*x)*sin(3/2*x))*cos(2/3*arctan2(sin
(3/2*x), cos(3/2*x))) - 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sq
rt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin
(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x
) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2
)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(1/3
*arctan2(sin(3/2*x), cos(3/2*x))) + 243*(sqrt(2)*a^2*cos(3*x)^3 + 9*sqrt(2)
*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sq
rt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) +
 3*(2*sqrt(2)*a^2*cos(2*x) + 2*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x)^2
 + (sqrt(2)*a^2*cos(3*x) + sqrt(2)*a^2)*sin(3*x)^2 + sqrt(2)*a^2 + 3*(3*sqr
t(2)*a^2*cos(2*x)^2 + 3*sqrt(2)*a^2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6
*sqrt(2)*a^2*sin(2*x)*sin(x) + 3*sqrt(2)*a^2*sin(x)^2 + 4*sqrt(2)*a^2*cos(x
) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2)*cos(2*x))*cos(3*
x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2
*x) + sqrt(2)*a^2*sin(x) + (sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(
3*x))*sin(3*x))*integrate(1/9*(x^3*cos(4*x)*cos(3/2*x) + 4*x^3*cos(3*x)*cos
(3/2*x) + 6*x^3*cos(2*x)*cos(3/2*x) + x^3*sin(4*x)*sin(3/2*x) + 4*x^3*sin(3
*x)*sin(3/2*x) + 6*x^3*sin(2*x)*sin(3/2*x) + 4*x^3*sin(3/2*x)*sin(x) + (4*x
^3*cos(x) + x^3)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*c
os(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*s
in(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 +
 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a
^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*
x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^
2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 486*(sqrt(2)*a^2*cos(3*x)^3 + 9*
sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2
+ 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*c
os(x) + 3*(2*sqrt(2)*a^2*cos(2*x) + 2*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos
(3*x)^2 + (sqrt(2)*a^2*cos(3*x) + sqrt(2)*a^2)*sin(3*x)^2 + sqrt(2)*a^2 + 3
*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqrt(2)*a^2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x
)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x) + 3*sqrt(2)*a^2*sin(x)^2 + 4*sqrt(2)*a^
2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2)*cos(2*x))
*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^
2*sin(2*x) + sqrt(2)*a^2*sin(x) + (sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x
))*cos(3*x))*sin(3*x))*integrate(1/9*(x^2*cos(3/2*x)*sin(4*x) + 4*x^2*cos(3
/2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2*x) - x^2*cos(4*x)*sin(3/2*x) - 4*x^
2*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*sin(3/2*x) + 4*x^2*cos(3/2*x)*sin(x)
 - (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 3
6*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 3
6*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x)
+ a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) +
 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)
```

```
*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 1
6*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 648*(sqrt(2)*a^2*cos(3*x)
^3 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(
2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2
)*a^2*cos(x) + 3*(2*sqrt(2)*a^2*cos(2*x) + 2*sqrt(2)*a^2*cos(x) + sqrt(2)*a
^2)*cos(3*x)^2 + (sqrt(2)*a^2*cos(3*x) + sqrt(2)*a^2)*sin(3*x)^2 + sqrt(2)*
a^2 + 3*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqrt(2)*a^2*cos(x)^2 + 3*sqrt(2)*a^2*
sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x) + 3*sqrt(2)*a^2*sin(x)^2 + 4*sqr
t(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2)*co
s(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqr
t(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) + (sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^
2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x*cos(4*x)*cos(3/2*x) + 4*x*co
s(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*s
in(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x
*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2
*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2
*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(
4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*c
os(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) +
 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*si
n(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + (48*sqrt(2)*cos(3/2*x)*sin(2*x) + (3
*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*s
in(3/2*x))*cos(3*x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x + 16*sqrt(2)*sin(x))*c
os(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(3*x) + 3*(3*(3*sq
rt(2)*x^3 - 8*sqrt(2)*x + 16*sqrt(2)*sin(x))*cos(3/2*x) + 2*(9*sqrt(2)*x^2
 - 8*sqrt(2))*sin(3/2*x))*sin(2*x) - 3*(3*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8
*sqrt(2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 8*sqrt(2)*x)*si
n(3/2*x))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 2*(72*sqrt(2)*cos(2*x)
^2*cos(3/2*x) + 8*sqrt(2)*cos(3/2*x)*sin(3*x)^2 + 72*sqrt(2)*cos(3/2*x)*sin
(2*x)^2 + (2*(9*sqrt(2)*x^2 - 4*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*
sqrt(2)*x)*sin(3/2*x))*cos(3*x)^2 + (9*(6*sqrt(2)*x^2*cos(3/2*x) - (3*sqrt(
2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (54*sqrt(2)*x^2*cos(x) + 36*sq
rt(2)*x^2 + 9*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x) - 16*sqrt(2))*cos(3/2*x)
 + 3*(3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqr
t(2))*sin(3/2*x))*sin(2*x) - 3*(6*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(2
)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 16*sqrt(2)*x)*sin(3/2*
x))*cos(3*x) + 9*(2*(3*sqrt(2)*x^2 + 8*sqrt(2)*cos(x))*cos(3/2*x) - (3*sqrt
(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (54*sqrt(2)*x^2*cos(x) + 18*s
qrt(2)*x^2 + 72*sqrt(2)*cos(x)^2 + 72*sqrt(2)*sin(x)^2 + 9*(3*sqrt(2)*x^3 -
 8*sqrt(2)*x)*sin(x) - 8*sqrt(2))*cos(3/2*x) - 3*(sqrt(2)*cos(3*x)^2 + 9*sq
rt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(
2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos
(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2
))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*co
s(x) + sqrt(2))*cos(1/3*arctan2(sin(3/2*x), cos(3/2*x))) + 243*(sqrt(2)*a^2
```

```
*cos(3*x)^3 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)
*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 +
 6*sqrt(2)*a^2*cos(x) + 3*(2*sqrt(2)*a^2*cos(2*x) + 2*sqrt(2)*a^2*cos(x) +
sqrt(2)*a^2)*cos(3*x)^2 + (sqrt(2)*a^2*cos(3*x) + sqrt(2)*a^2)*sin(3*x)^2 +
 sqrt(2)*a^2 + 3*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqrt(2)*a^2*cos(x)^2 + 3*sqr
t(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x) + 3*sqrt(2)*a^2*sin(x)^
2 + 4*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(x) + 2*sqrt(2
)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x)
 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) + (sqrt(2)*a^2*sin(2*x) + s
qrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x^3*cos(4*x)*cos(3/2*
x) + 4*x^3*cos(3*x)*cos(3/2*x) + 6*x^3*cos(2*x)*cos(3/2*x) + x^3*sin(4*x)*s
in(3/2*x) + 4*x^3*sin(3*x)*sin(3/2*x) + 6*x^3*sin(2*x)*sin(3/2*x) + 4*x^3*s
in(3/2*x)*sin(x) + (4*x^3*cos(x) + x^3)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^
2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^
2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2
 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) +
 a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a
^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(
x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 486*(sqrt
(2)*a^2*cos(3*x)^3 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*
sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin
(x)^2 + 6*sqrt(2)*a^2*cos(x) + 3*(2*sqrt(2)*a^2*cos(2*x) + 2*sqrt(2)*a^2*co
s(x) + sqrt(2)*a^2)*cos(3*x)^2 + (sqrt(2)*a^2*cos(3*x) + sqrt(2)*a^2)*sin(3
*x)^2 + sqrt(2)*a^2 + 3*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqrt(2)*a^2*cos(x)^2
+ 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x) + 3*sqrt(2)*a^2*
sin(x)^2 + 4*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(x) + 2
*sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*c
os(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) + (sqrt(2)*a^2*sin(2
*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x^2*cos(3/2*x)
*sin(4*x) + 4*x^2*cos(3/2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2*x) - x^2*cos
(4*x)*sin(3/2*x) - 4*x^2*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*sin(3/2*x) +
4*x^2*cos(3/2*x)*sin(x) - (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^2*cos(4*x)^2
+ 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2
+ 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*s
in(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*c
os(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) +
12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a
^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 64
8*(sqrt(2)*a^2*cos(3*x)^3 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)
^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*
a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 3*(2*sqrt(2)*a^2*cos(2*x) + 2*sqrt(2)
*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x)^2 + (sqrt(2)*a^2*cos(3*x) + sqrt(2)*a^2
)*sin(3*x)^2 + sqrt(2)*a^2 + 3*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqrt(2)*a^2*co
s(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x) + 3*sqrt(
2)*a^2*sin(x)^2 + 4*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos
```

```
(x) + 2*sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)
*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) + (sqrt(2)*a^
2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x*cos(4
*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(
4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*s
in(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*co
s(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*si
n(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8
*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2
)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*c
os(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*
sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + (48*sqrt(2)*c
os(3/2*x)*sin(2*x) + (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqr
t(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(3*x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x
 + 16*sqrt(2)*sin(x))*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x)
)*sin(3*x) + 3*(3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x + 16*sqrt(2)*sin(x))*cos(3/2
*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(2*x) - 3*(3*sqrt(2)*x^3
 + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*s
in(x) - 8*sqrt(2)*x)*sin(3/2*x))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) -
 2*(sqrt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqrt(2)*cos(x) + sqrt(2)
)*cos(3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2 + 9*sqrt(2)*cos(2*x)
^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x)
 + 9*sqrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sq
rt(2)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2 + 2*(3*sq
rt(2)*cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) +
 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*((sqrt(2)*sin(2*x) + sqrt(2)*s
in(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*c
os(x) + sqrt(2))*cos(1/3*arctan2(sin(3/2*x), cos(3/2*x))) + 81*(sqrt(2)*a^2
*cos(3*x)^4 + sqrt(2)*a^2*sin(3*x)^4 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)
*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) +
 9*sqrt(2)*a^2*sin(x)^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x)
+ 2*sqrt(2)*a^2)*cos(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))
*sin(3*x)^3 + 6*sqrt(2)*a^2*cos(x) + 3*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqrt(2
)*a^2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x) +
 3*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 6*(sqrt(2)
*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x)^2 + (2*sqrt(2)*a^2*cos(3*x)^2
 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*
x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*
a^2*cos(x) + 2*sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(
x) + 2*sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2
*x))*sin(3*x)^2 + sqrt(2)*a^2 + 2*(9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2
*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*s
qrt(2)*a^2*sin(x)^2 + 9*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 9*(2*sqrt(2)*a
^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqr
t(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) + (sqrt(2
```

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)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x)^2 + 2*(sqrt(2)*a^2*sin(2*x) +
 sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x^3*cos(4*x)*cos(3/
2*x) + 4*x^3*cos(3*x)*cos(3/2*x) + 6*x^3*cos(2*x)*cos(3/2*x) + x^3*sin(4*x)
*sin(3/2*x) + 4*x^3*sin(3*x)*sin(3/2*x) + 6*x^3*sin(2*x)*sin(3/2*x) + 4*x^3
*sin(3/2*x)*sin(x) + (4*x^3*cos(x) + x^3)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*
a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*
a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)
^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x)
 + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4
*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*si
n(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 162*(sq
rt(2)*a^2*cos(3*x)^4 + sqrt(2)*a^2*sin(3*x)^4 + 9*sqrt(2)*a^2*cos(2*x)^2 +
9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)
*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^
2*cos(x) + 2*sqrt(2)*a^2)*cos(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^
2*sin(x))*sin(3*x)^3 + 6*sqrt(2)*a^2*cos(x) + 3*(3*sqrt(2)*a^2*cos(2*x)^2 +
 3*sqrt(2)*a^2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)
*sin(x) + 3*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 6
*(sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x)^2 + (2*sqrt(2)*a^2*c
os(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a
^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6
*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)
*a^2*cos(x) + 2*sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a
^2)*cos(2*x))*sin(3*x)^2 + sqrt(2)*a^2 + 2*(9*sqrt(2)*a^2*cos(2*x)^2 + 9*sq
rt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin
(x) + 9*sqrt(2)*a^2*sin(x)^2 + 9*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 9*(2*
sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos
(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x)
+ (sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x)^2 + 2*(sqrt(2)*a^2*s
in(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x^2*cos(3/
2*x)*sin(4*x) + 4*x^2*cos(3/2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2*x) - x^2
*cos(4*x)*sin(3/2*x) - 4*x^2*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*sin(3/2*x
) + 4*x^2*cos(3/2*x)*sin(x) - (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^2*cos(4*x
)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x
)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a
^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a
^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x
) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) +
 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x)
- 216*(sqrt(2)*a^2*cos(3*x)^4 + sqrt(2)*a^2*sin(3*x)^4 + 9*sqrt(2)*a^2*cos(
2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2
*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*s
qrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2)*cos(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + s
qrt(2)*a^2*sin(x))*sin(3*x)^3 + 6*sqrt(2)*a^2*cos(x) + 3*(3*sqrt(2)*a^2*cos
(2*x)^2 + 3*sqrt(2)*a^2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2
```

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*sin(2*x)*sin(x) + 3*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 2*sqrt(2
)*a^2 + 6*(sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x)^2 + (2*sqrt
(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*
sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin
(x)^2 + 6*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) +
3*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) +
sqrt(2)*a^2)*cos(2*x))*sin(3*x)^2 + sqrt(2)*a^2 + 2*(9*sqrt(2)*a^2*cos(2*x)
^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin
(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 9*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^
2 + 9*(2*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2
)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^
2*sin(x) + (sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x)^2 + 2*(sqrt
(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x
*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) +
x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) +
 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*
a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*
a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)
^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x)
 + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4
*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*si
n(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + (48*sqr
t(2)*cos(2*x)^2*sin(3/2*x) + 48*sqrt(2)*sin(2*x)^2*sin(3/2*x) + (6*sqrt(2)*
x^2*sin(3/2*x) + (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x))*cos(3*x)^2 + 32*
(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x)*sin(3/2*x) + 2*(16*sqrt(2)*cos(3/2*x)
*sin(2*x) + 16*sqrt(2)*cos(2*x)*sin(3/2*x) + (3*sqrt(2)*x^3 - 8*sqrt(2)*x +
 16*sqrt(2)*sin(x))*cos(3/2*x) + 2*(3*sqrt(2)*x^2 + 8*sqrt(2)*cos(x))*sin(3
/2*x))*cos(3*x) + (3*sqrt(2)*x^3 - 8*sqrt(2)*x + 32*sqrt(2)*sin(x))*cos(3/2
*x) + 32*(3*sqrt(2)*sin(3/2*x)*sin(x) + sqrt(2)*cos(3/2*x))*sin(2*x) + 2*(3
*sqrt(2)*x^2 + 24*sqrt(2)*cos(x)^2 + 24*sqrt(2)*sin(x)^2 + 16*sqrt(2)*cos(x
))*sin(3/2*x))*sin(3*x) + (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x + 32*sqrt(2)*sin(
x))*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(2*x) - (3*sq
rt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*s
qrt(2))*sin(x) - 8*sqrt(2)*x)*sin(3/2*x) + 2*(sqrt(2)*sin(3*x)^3 + 6*(sqrt(
2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*
cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x
)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) +
sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos
(x) + sqrt(2))*sin(3*x) + 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*
sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*s
in(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos
(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt
(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(4
/3*arctan2(sin(3/2*x), cos(3/2*x))) + 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos
(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 +
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18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3
*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*
x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sq
rt(2))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))))*sin(5/3*arctan2(sin(3/2*x)
, cos(3/2*x))) + 2*(8*sqrt(2)*cos(3*x)^2*sin(3/2*x) + 72*sqrt(2)*cos(2*x)^2
*sin(3/2*x) + 72*sqrt(2)*sin(2*x)^2*sin(3/2*x) + 144*sqrt(2)*sin(2*x)*sin(3
/2*x)*sin(x) + (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x
^2 - 4*sqrt(2))*sin(3/2*x))*sin(3*x)^2 + 48*(3*sqrt(2)*cos(x) + sqrt(2))*co
s(2*x)*sin(3/2*x) + 16*(3*sqrt(2)*cos(2*x)*sin(3/2*x) + (3*sqrt(2)*cos(x) +
 sqrt(2))*sin(3/2*x))*cos(3*x) + 243*(sqrt(2)*a^2*sin(3*x)^3 + 6*(sqrt(2)*a
^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x)^2 + (sqrt(2)*a^2*cos(3*x)^2 + 9*
sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2
+ 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*c
os(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sq
rt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*sin(
3*x))*integrate(1/9*(x^3*cos(4*x)*cos(3/2*x) + 4*x^3*cos(3*x)*cos(3/2*x) +
6*x^3*cos(2*x)*cos(3/2*x) + x^3*sin(4*x)*sin(3/2*x) + 4*x^3*sin(3*x)*sin(3/
2*x) + 6*x^3*sin(2*x)*sin(3/2*x) + 4*x^3*sin(3/2*x)*sin(x) + (4*x^3*cos(x)
+ x^3)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2
+ 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2
+ 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*
cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x
) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*
a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x)
 + 2*a^2*sin(x))*sin(3*x)), x) - 486*(sqrt(2)*a^2*sin(3*x)^3 + 6*(sqrt(2)*a
^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x)^2 + (sqrt(2)*a^2*cos(3*x)^2 + 9*
sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2
+ 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*c
os(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sq
rt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*sin(
3*x))*integrate(1/9*(x^2*cos(3/2*x)*sin(4*x) + 4*x^2*cos(3/2*x)*sin(3*x) +
6*x^2*cos(3/2*x)*sin(2*x) - x^2*cos(4*x)*sin(3/2*x) - 4*x^2*cos(3*x)*sin(3/
2*x) - 6*x^2*cos(2*x)*sin(3/2*x) + 4*x^2*cos(3/2*x)*sin(x) - (4*x^2*cos(x)
+ x^2)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2
+ 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2
+ 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*
cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x
) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*
a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x)
 + 2*a^2*sin(x))*sin(3*x)), x) - 648*(sqrt(2)*a^2*sin(3*x)^3 + 6*(sqrt(2)*a
^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x)^2 + (sqrt(2)*a^2*cos(3*x)^2 + 9*
sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2
+ 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*c
os(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sq
rt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*sin(
```

```
3*x))*integrate(1/9*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*
cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x
*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))
/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2
+ a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*
sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*c
os(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) +
 a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a
^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*s
in(3*x)), x) + ((2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^
3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) + 3*(2*(9*sqrt(2)*x^2 - 8*sqrt(2))*co
s(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(
2)*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(x) + 9*(3*sqrt(2)*x^3 - 8*sqrt(2
)*x)*sin(x) - 16*sqrt(2))*cos(3/2*x) + 9*(6*sqrt(2)*x^2*sin(3/2*x) + (3*sqr
t(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x))*sin(2*x) - 3*(3*sqrt(2)*x^3 - 18*sqrt(2
)*x^2*sin(x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(x) - 8*sqrt(2)*x)*sin(3/
2*x))*sin(3*x) + 8*(9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(x)^2 + 6*sqrt(2)*cos
(x) + sqrt(2))*sin(3/2*x) + 3*((2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) -
3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) + 3*(2*(9*sqrt(2)*x^2
- 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2
*x) + (18*sqrt(2)*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(x) + 9*(3*sqrt(2)
*x^3 - 8*sqrt(2)*x)*sin(x) - 16*sqrt(2))*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(
3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*si
n(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sq
rt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*
cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*
cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(
x))*sin(3*x))*integrate(1/9*(x^3*cos(4*x)*cos(3/2*x) + 4*x^3*cos(3*x)*cos(3
/2*x) + 6*x^3*cos(2*x)*cos(3/2*x) + x^3*sin(4*x)*sin(3/2*x) + 4*x^3*sin(3*x
)*sin(3/2*x) + 6*x^3*sin(2*x)*sin(3/2*x) + 4*x^3*sin(3/2*x)*sin(x) + (4*x^3
*cos(x) + x^3)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos
(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin
(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2
*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2
*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x)
 + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*
sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 486*(sqrt(2)*a^2*cos(3*x)^2 + 9*sq
rt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*
sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin
(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*
sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt
(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))
*integrate(1/9*(x^2*cos(3/2*x)*sin(4*x) + 4*x^2*cos(3/2*x)*sin(3*x) + 6*x^2
*cos(3/2*x)*sin(2*x) - x^2*cos(4*x)*sin(3/2*x) - 4*x^2*cos(3*x)*sin(3/2*x)
- 6*x^2*cos(2*x)*sin(3/2*x) + 4*x^2*cos(3/2*x)*sin(x) - (4*x^2*cos(x) + x^2
```

```
)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*
a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*
a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3
*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4
*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*s
in(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*
a^2*sin(x))*sin(3*x)), x) - 648*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos
(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*s
in(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqr
t(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*c
os(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(
2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/
9*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x
) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*
x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 +
 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 +
 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*si
n(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*co
s(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 1
2*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^
2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + (3*
(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*si
n(3/2*x))*sin(3*x) + 3*(3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*s
qrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*sin(2*x) - 3*(3*sqrt(2)*x^3 + 3*(3*sqrt
(2)*x^3 - 8*sqrt(2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 8*sq
rt(2)*x)*sin(3/2*x))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))) - 3*(sqrt(2)*
cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2
 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 +
 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2
)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*
x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))))*
sin(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 2*(8*sqrt(2)*cos(3*x)^2*sin(3/2*
x) + 72*sqrt(2)*cos(2*x)^2*sin(3/2*x) + 72*sqrt(2)*sin(2*x)^2*sin(3/2*x) +
144*sqrt(2)*sin(2*x)*sin(3/2*x)*sin(x) + (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*c
os(3/2*x) + 2*(9*sqrt(2)*x^2 - 4*sqrt(2))*sin(3/2*x))*sin(3*x)^2 + 48*(3*sq
rt(2)*cos(x) + sqrt(2))*cos(2*x)*sin(3/2*x) + 16*(3*sqrt(2)*cos(2*x)*sin(3/
2*x) + (3*sqrt(2)*cos(x) + sqrt(2))*sin(3/2*x))*cos(3*x) + 243*(sqrt(2)*a^2
*sin(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x)^2 + (s
qrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 +
 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*
sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) +
 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + s
qrt(2)*a^2)*cos(2*x))*sin(3*x))*integrate(1/9*(x^3*cos(4*x)*cos(3/2*x) + 4*
x^3*cos(3*x)*cos(3/2*x) + 6*x^3*cos(2*x)*cos(3/2*x) + x^3*sin(4*x)*sin(3/2*
x) + 4*x^3*sin(3*x)*sin(3/2*x) + 6*x^3*sin(2*x)*sin(3/2*x) + 4*x^3*sin(3/2*
```

```
x)*sin(x) + (4*x^3*cos(x) + x^3)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3
*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3
*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^
2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*c
os(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(
x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin
(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 486*(sqrt(2)*a^2
*sin(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x)^2 + (s
qrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 +
 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*
sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) +
 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + s
qrt(2)*a^2)*cos(2*x))*sin(3*x))*integrate(1/9*(x^2*cos(3/2*x)*sin(4*x) + 4*
x^2*cos(3/2*x)*sin(3*x) + 6*x^2*cos(3/2*x)*sin(2*x) - x^2*cos(4*x)*sin(3/2*
x) - 4*x^2*cos(3*x)*sin(3/2*x) - 6*x^2*cos(2*x)*sin(3/2*x) + 4*x^2*cos(3/2*
x)*sin(x) - (4*x^2*cos(x) + x^2)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3
*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3
*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^
2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*c
os(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(
x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin
(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 648*(sqrt(2)*a^2
*sin(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x)^2 + (s
qrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 +
 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*
sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) +
 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + s
qrt(2)*a^2)*cos(2*x))*sin(3*x))*integrate(1/9*(x*cos(4*x)*cos(3/2*x) + 4*x*
cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x
*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4
*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos
(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin
(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2
*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2
*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x)
 + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*
sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + ((2*(9*sqrt(2)*x^2 - 8*sqrt(2))*co
s(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) + 3*(2*(9*s
qrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/
2*x))*cos(2*x) + (18*sqrt(2)*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(x) + 9
*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x) - 16*sqrt(2))*cos(3/2*x) + 9*(6*sqrt(
2)*x^2*sin(3/2*x) + (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x))*sin(2*x) - 3*
(3*sqrt(2)*x^3 - 18*sqrt(2)*x^2*sin(x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*co
s(x) - 8*sqrt(2)*x)*sin(3/2*x))*sin(3*x) + 8*(9*sqrt(2)*cos(x)^2 + 9*sqrt(2
)*sin(x)^2 + 6*sqrt(2)*cos(x) + sqrt(2))*sin(3/2*x) - 3*(sqrt(2)*cos(3*x)^2
```

```
 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(
2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt
(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) +
 sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqr
t(2)*cos(x) + sqrt(2))*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))))*sin(2/3*ar
ctan2(sin(3/2*x), cos(3/2*x))) - 2*(sqrt(2)*sin(3*x)^3 + 6*(sqrt(2)*sin(2*x
) + sqrt(2)*sin(x))*sin(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2
 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) +
 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*c
os(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + sqrt
(2))*sin(3*x))*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))))/((a*cos(3*x)^4 + a
*sin(3*x)^4 + 2*(3*a*cos(2*x) + 3*a*cos(x) + 2*a)*cos(3*x)^3 + 6*(a*sin(2*x
) + a*sin(x))*sin(3*x)^3 + 3*(3*a*cos(2*x)^2 + 3*a*cos(x)^2 + 3*a*sin(2*x)^
2 + 6*a*sin(2*x)*sin(x) + 3*a*sin(x)^2 + 6*(a*cos(x) + a)*cos(2*x) + 6*a*co
s(x) + 2*a)*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*cos(x)^2 + 9*(a*cos(3*x)^2 +
9*a*cos(2*x)^2 + 9*a*cos(x)^2 + a*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*
x)*sin(x) + 9*a*sin(x)^2 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(
3*a*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*sin(2*x) + a*sin(x))*sin(3*x)
+ a)*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*(a*cos(3*x)^2 + 9*a*cos
(2*x)^2 + 9*a*cos(x)^2 + a*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(
x) + 9*a*sin(x)^2 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos
(x) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*sin(2*x) + a*sin(x))*sin(3*x) + a)*co
s(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + (2*a*cos(3*x)^2 + 9*a*cos(2*x)^2
 + 9*a*cos(x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2 + 2*
(3*a*cos(2*x) + 3*a*cos(x) + 2*a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x) +
6*a*cos(x) + 2*a)*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*
sin(x)^2 + 9*(a*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*cos(x)^2 + a*sin(3*x)^2 +
 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2 + 2*(3*a*cos(2*x) + 3
*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*s
in(2*x) + a*sin(x))*sin(3*x) + a)*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^
2 + 9*(a*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*cos(x)^2 + a*sin(3*x)^2 + 9*a*si
n(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2 + 2*(3*a*cos(2*x) + 3*a*cos(
x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*sin(2*x)
 + a*sin(x))*sin(3*x) + a)*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 2*(
9*a*cos(2*x)^2 + 9*a*cos(x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a
*sin(x)^2 + 9*(2*a*cos(x) + a)*cos(2*x) + 9*a*cos(x) + 2*a)*cos(3*x) + 6*(3
*a*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*cos(3*x)^3 + 3*(2*a*cos(2*x) +
2*a*cos(x) + a)*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*cos(x)^2 + (a*cos(3*x) +
a)*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2 + 3*(3
*a*cos(2*x)^2 + 3*a*cos(x)^2 + 3*a*sin(2*x)^2 + 6*a*sin(2*x)*sin(x) + 3*a*s
in(x)^2 + 2*(3*a*cos(x) + 2*a)*cos(2*x) + 4*a*cos(x) + a)*cos(3*x) + 6*(3*a
*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 3*(a*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a
*cos(x)^2 + a*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(
x)^2 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(
2*x) + 6*a*cos(x) + 6*(a*sin(2*x) + a*sin(x))*sin(3*x) + a)*cos(2/3*arctan2
```

```
(sin(3/2*x), cos(3/2*x))) + 6*((a*sin(2*x) + a*sin(x))*cos(3*x) + a*sin(2*x
) + a*sin(x))*sin(3*x) + a)*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(a
*cos(3*x)^3 + 3*(2*a*cos(2*x) + 2*a*cos(x) + a)*cos(3*x)^2 + 9*a*cos(2*x)^2
 + 9*a*cos(x)^2 + (a*cos(3*x) + a)*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2
*x)*sin(x) + 9*a*sin(x)^2 + 3*(3*a*cos(2*x)^2 + 3*a*cos(x)^2 + 3*a*sin(2*x)
^2 + 6*a*sin(2*x)*sin(x) + 3*a*sin(x)^2 + 2*(3*a*cos(x) + 2*a)*cos(2*x) + 4
*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*((a*
sin(2*x) + a*sin(x))*cos(3*x) + a*sin(2*x) + a*sin(x))*sin(3*x) + a)*cos(2/
3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*((a*sin(2*x) + a*sin(x))*cos(3*x)^2
 + 2*(a*sin(2*x) + a*sin(x))*cos(3*x) + a*sin(2*x) + a*sin(x))*sin(3*x) + 6*
(a*sin(3*x)^3 + 6*(a*sin(2*x) + a*sin(x))*sin(3*x)^2 + (a*cos(3*x)^2 + 9*a*
cos(2*x)^2 + 9*a*cos(x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin
(x)^2 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos
(2*x) + 6*a*cos(x) + a)*sin(3*x) + 3*(a*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*c
os(x)^2 + a*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)
^2 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*
x) + 6*a*cos(x) + 6*(a*sin(2*x) + a*sin(x))*sin(3*x) + a)*sin(2/3*arctan2(s
in(3/2*x), cos(3/2*x))))*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(a*si
n(3*x)^3 + 6*(a*sin(2*x) + a*sin(x))*sin(3*x)^2 + (a*cos(3*x)^2 + 9*a*cos(2
*x)^2 + 9*a*cos(x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2
 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x)
 + 6*a*cos(x) + a)*sin(3*x))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + a)*
sqrt(a))
```

Giac [F]

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="giac")
[Out] integrate(x^3/(a*cos(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{(a+a\,\cos(x))^{3/2}}\,dx$$

```
[In] int(x^3/(a + a*cos(x))^(3/2),x)
[Out] int(x^3/(a + a*cos(x))^(3/2), x)
```

3.181 $\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx$

Optimal result . . . 1079
Rubi [A] (verified) . . . 1080
Mathematica [A] (verified) . . . 1083
Maple [F] . . . 1083
Fricas [F] . . . 1083
Sympy [F] . . . 1083
Maxima [F] . . . 1084
Giac [F] . . . 1101
Mupad [F(-1)] . . . 1102

Optimal result

Integrand size = 14, antiderivative size = 257

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\operatorname{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{2ix\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{2ix\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{4\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}$$

```
[Out] -2*x/a/(a+a*cos(x))^(1/2)-I*x^2*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*cos(
x))^(1/2)+4*arctanh(sin(1/2*x))*cos(1/2*x)/a/(a+a*cos(x))^(1/2)+2*I*x*cos(1
/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-2*I*x*cos(1/2*x)*poly
log(2,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-4*cos(1/2*x)*polylog(3,-I*exp(1/
2*I*x))/a/(a+a*cos(x))^(1/2)+4*cos(1/2*x)*polylog(3,I*exp(1/2*I*x))/a/(a+a*
cos(x))^(1/2)+1/2*x^2*tan(1/2*x)/a/(a+a*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3400, 4271, 3855, 4266, 2611, 2320, 6724}

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = -\frac{ix^2\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} + \frac{4\cos\left(\frac{x}{2}\right)\operatorname{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)}{a\sqrt{a\cos(x)+a}} + \frac{2ix\,\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} - \frac{2ix\,\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} - \frac{4\,\text{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} + \frac{4\,\text{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} + \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a\cos(x)+a}} - \frac{2x}{a\sqrt{a\cos(x)+a}}$$

```
[In] Int[x^2/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (-2*x)/(a*Sqrt[a + a*Cos[x]]) - (I*x^2*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqr
t[a + a*Cos[x]]) + (4*ArcTanh[Sin[x/2]]*Cos[x/2])/(a*Sqrt[a + a*Cos[x]]) +
((2*I)*x*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - ((
2*I)*x*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) - (4*Cos[
x/2]*PolyLog[3, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (4*Cos[x/2]*Pol
yLog[3, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x^2*Tan[x/2])/(2*a*Sqrt[a
 + a*Cos[x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\cos\left(\frac{x}{2}\right)\int x^2\sec^3\left(\frac{x}{2}\right)\,dx}{2a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} + \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int x^2\sec\left(\frac{x}{2}\right)\,dx}{4a\sqrt{a+a\cos(x)}} + \frac{\left(2\cos\left(\frac{x}{2}\right)\right)\int \sec\left(\frac{x}{2}\right)\,dx}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2 \arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\operatorname{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} - \frac{\cos\left(\frac{x}{2}\right)\int x\log\left(1-ie^{\frac{ix}{2}}\right)dx}{a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int x\log\left(1+ie^{\frac{ix}{2}}\right)dx}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2 \arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\operatorname{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{2ix\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{2ix\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} - \frac{\left(2i\cos\left(\frac{x}{2}\right)\right)\int \operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)dx}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{\left(2i\cos\left(\frac{x}{2}\right)\right)\int \operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)dx}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2 \arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\operatorname{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{2ix\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{2ix\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} - \frac{\left(4\cos\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx, x, e^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{\left(4\cos\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx, x, e^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{2x}{a\sqrt{a+a\cos(x)}} - \frac{ix^2 \arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\operatorname{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{2ix\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{2ix\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$- \frac{4\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{4\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \frac{\cos\left(\frac{x}{2}\right)\left(-4x\cos\left(\frac{x}{2}\right) - 2ix^2\arctan\left(e^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right) + 8\text{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos^2\left(\frac{x}{2}\right) +$$

```
[In] Integrate[x^2/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (Cos[x/2]*(-4*x*Cos[x/2] - (2*I)*x^2*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 + 8*Arc
Tanh[Sin[x/2]]*Cos[x/2]^2 + (4*I)*x*Cos[x/2]^2*PolyLog[2, (-I)*E^((I/2)*x)]
 - (4*I)*x*Cos[x/2]^2*PolyLog[2, I*E^((I/2)*x)] - 8*Cos[x/2]^2*PolyLog[3, (
-I)*E^((I/2)*x)] + 8*Cos[x/2]^2*PolyLog[3, I*E^((I/2)*x)] + x^2*Sin[x/2]))/
(a*(1 + Cos[x]))^(3/2)
```

Maple [F]

$$\int \frac{x^2}{(a+\cos(x)\,a)^{\frac{3}{2}}}\,dx$$

```
[In] int(x^2/(a+cos(x)*a)^(3/2),x)
```

```
[Out] int(x^2/(a+cos(x)*a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x) + a)*x^2/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a\,(\cos(x)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral(x**2/(a*(cos(x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*(4*(3*sqrt(2)*x^2*sin(3/2*x) - 4*sqrt(2)*x*cos(3/2*x))*cos(3*x)^3 - 4*
(3*sqrt(2)*x^2*cos(3/2*x) + 4*sqrt(2)*x*sin(3/2*x))*sin(3*x)^3 + 96*sqrt(2)
*cos(2*x)^2*sin(3/2*x) + 96*sqrt(2)*sin(2*x)^2*sin(3/2*x) - 4*((12*sqrt(2)*
x*cos(3/2*x) - (9*sqrt(2)*x^2 + 8*sqrt(2))*sin(3/2*x))*cos(2*x) + (12*sqrt(
2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) + 12*sqrt(2)*x)*cos(3/2*x)
 + (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))*sin(2
*x) - (9*sqrt(2)*x^2 - 12*sqrt(2)*x*sin(x) + (9*sqrt(2)*x^2 + 8*sqrt(2))*co
s(x))*sin(3/2*x))*cos(3*x)^2 - 12*((12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^
2 - 8*sqrt(2))*sin(3/2*x))*cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(
2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(2*x) + 3*(12*sqrt(2)*x*cos(x) + (9*sqrt
(2)*x^2 - 8*sqrt(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) + 243*(sqrt(2)*a^2*co
s(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*
sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*
sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^
2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^
2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*si
n(x))*sin(3*x))*integrate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos
(3/2*x) + 6*x^2*cos(2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3
*x)*sin(3/2*x) + 6*x^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x
^2*cos(x) + x^2)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*c
os(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*s
in(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 +
 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a
^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*
x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^
2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 324*(sqrt(2)*a^2*cos(3*x)^2 + 9*
sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 +
9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*s
in(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) +
3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sq
rt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x
))*integrate(1/9*(x*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos
(3/2*x)*sin(2*x) - x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*co
s(2*x)*sin(3/2*x) + 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a
^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a
^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin
(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(
```

```
2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^
2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*
sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(
3*x)), x) + (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*
x))*sin(3*x) + 3*(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos
(3/2*x))*sin(2*x) - (9*sqrt(2)*x^2 - 36*sqrt(2)*x*sin(x) + 3*(9*sqrt(2)*x^2
 - 8*sqrt(2))*cos(x) - 8*sqrt(2))*sin(3/2*x))*cos(4/3*arctan2(sin(3/2*x), c
os(3/2*x)))^2 - 12*((12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt(2))*
sin(3/2*x))*cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt
(2))*sin(3/2*x))*cos(2*x) + 3*(12*sqrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sqr
t(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(3*x)^2 + 9*sq
rt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*
sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin
(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*
sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt
(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))
*integrate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos(3/2*x) + 6*x^2
*cos(2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3*x)*sin(3/2*x)
+ 6*x^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x^2*cos(x) + x^2
)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*
a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*
a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3
*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4
*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*s
in(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*
a^2*sin(x))*sin(3*x)), x) - 324*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos
(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*s
in(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqr
t(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*c
os(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(
2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/
9*(x*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos(3/2*x)*sin(2*x
) - x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*cos(2*x)*sin(3/2*
x) + 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a^2*cos(4*x)^2 +
 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 +
 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*si
n(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*co
s(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 1
2*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^
2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + (12
*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))*sin(3*x) +
3*(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))*sin(2*
x) - (9*sqrt(2)*x^2 - 36*sqrt(2)*x*sin(x) + 3*(9*sqrt(2)*x^2 - 8*sqrt(2))*c
os(x) - 8*sqrt(2))*sin(3/2*x))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 +
 4*((3*sqrt(2)*x^2*sin(3/2*x) - 4*sqrt(2)*x*cos(3/2*x))*cos(3*x) - (12*sqrt
```

```
(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(2*x) - (12*s
qrt(2)*x*cos(x) + (9*sqrt(2)*x^2 + 8*sqrt(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2
*x) - (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 + 8*sqrt(2))*cos(3/2*x))*si
n(2*x) + (3*sqrt(2)*x^2 - 12*sqrt(2)*x*sin(x) + (9*sqrt(2)*x^2 - 8*sqrt(2))
*cos(x))*sin(3/2*x))*sin(3*x)^2 - 12*((12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)
*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x) - (9*sq
rt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(2*x) + 3*(12*sqrt(2)*x*cos(x) + (9*s
qrt(2)*x^2 - 8*sqrt(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) + 243*(sqrt(2)*a^2
*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a
^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) +
 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)
*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)
*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2
*sin(x))*sin(3*x))*integrate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*
cos(3/2*x) + 6*x^2*cos(2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*si
n(3*x)*sin(3/2*x) + 6*x^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (
4*x^2*cos(x) + x^2)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^
2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^
2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^
2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(
6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos
(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3
*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 324*(sqrt(2)*a^2*cos(3*x)^2 +
 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2
 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^
2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x)
 + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) +
 sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(
3*x))*integrate(1/9*(x*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*
cos(3/2*x)*sin(2*x) - x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x
*cos(2*x)*sin(3/2*x) + 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))
/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2
+ a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*
sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*c
os(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) +
 a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a
^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*s
in(3*x)), x) + (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3
/2*x))*sin(3*x) + 3*(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*
cos(3/2*x))*sin(2*x) - (9*sqrt(2)*x^2 - 36*sqrt(2)*x*sin(x) + 3*(9*sqrt(2)*
x^2 - 8*sqrt(2))*cos(x) - 8*sqrt(2))*sin(3/2*x))*sin(4/3*arctan2(sin(3/2*x)
, cos(3/2*x)))^2 - 12*((12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt(2
))*sin(3/2*x))*cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*s
qrt(2))*sin(3/2*x))*cos(2*x) + 3*(12*sqrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*
sqrt(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(3*x)^2 + 9
```

```
*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 +
 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*
sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) +
 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + s
qrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*
x))*integrate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos(3/2*x) + 6*
x^2*cos(2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3*x)*sin(3/2*
x) + 6*x^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x^2*cos(x) +
x^2)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 +
16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 +
48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*co
s(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x)
+ 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^
2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) +
 2*a^2*sin(x))*sin(3*x)), x) - 324*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*
cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^
2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*
sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^
2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*c
os(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate
(1/9*(x*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos(3/2*x)*sin(
2*x) - x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*cos(2*x)*sin(3
/2*x) + 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a^2*cos(4*x)^
2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^
2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2
*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2
*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x)
+ 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2
*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) +
(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))*sin(3*x)
 + 3*(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))*sin
(2*x) - (9*sqrt(2)*x^2 - 36*sqrt(2)*x*sin(x) + 3*(9*sqrt(2)*x^2 - 8*sqrt(2)
)*cos(x) - 8*sqrt(2))*sin(3/2*x))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^
2 + 4*(24*sqrt(2)*cos(2*x)^2*sin(3/2*x) + 24*sqrt(2)*sin(2*x)^2*sin(3/2*x)
- 2*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 + 24*sqrt(2)*cos(x) + 8*sqrt(
2))*sin(3/2*x))*cos(2*x) - 2*(12*sqrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sqrt
(2))*sin(x) + 6*sqrt(2)*x)*cos(3/2*x) - 2*((9*sqrt(2)*x^2 - 8*sqrt(2))*cos(
3/2*x) + 12*(sqrt(2)*x - 2*sqrt(2)*sin(x))*sin(3/2*x))*sin(2*x) + (9*sqrt(2
)*x^2 + 24*sqrt(2)*cos(x)^2 - 24*sqrt(2)*x*sin(x) + 24*sqrt(2)*sin(x)^2 + 2
*(9*sqrt(2)*x^2 + 8*sqrt(2))*cos(x))*sin(3/2*x))*cos(3*x) - 4*(12*sqrt(2)*x
*cos(3/2*x) - (9*sqrt(2)*x^2 + 48*sqrt(2)*cos(x) + 8*sqrt(2))*sin(3/2*x))*c
os(2*x) - 4*(12*sqrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) + 4*s
qrt(2)*x)*cos(3/2*x) - 4*(sqrt(2)*sin(3*x)^3 + 6*(sqrt(2)*sin(2*x) + sqrt(2
)*sin(x))*sin(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(
2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)
```

```
*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) +
6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(3
*x) + 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + s
qrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*s
qrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3
*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(
2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(4/3*arctan2(sin(3/2*x
), cos(3/2*x))) + 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*
cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*
sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sq
rt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(
2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(2/3*arcta
n2(sin(3/2*x), cos(3/2*x))))*cos(5/3*arctan2(sin(3/2*x), cos(3/2*x))) + 4*(
72*sqrt(2)*cos(2*x)^2*sin(3/2*x) + 8*sqrt(2)*sin(3*x)^2*sin(3/2*x) + 72*sqr
t(2)*sin(2*x)^2*sin(3/2*x) - 2*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 -
4*sqrt(2))*sin(3/2*x))*cos(3*x)^2 + 2*(9*(3*sqrt(2)*x^2*sin(3/2*x) - 4*sqrt
(2)*x*cos(3/2*x))*cos(2*x) - 3*(12*sqrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sq
rt(2))*sin(x) + 8*sqrt(2)*x)*cos(3/2*x) - 3*(12*sqrt(2)*x*sin(3/2*x) + (9*s
qrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))*sin(2*x) + (27*sqrt(2)*x^2*cos(x) + 18*
sqrt(2)*x^2 - 36*sqrt(2)*x*sin(x) - 8*sqrt(2))*sin(3/2*x))*cos(3*x) - 18*(4
*sqrt(2)*x*cos(3/2*x) - (3*sqrt(2)*x^2 + 8*sqrt(2)*cos(x))*sin(3/2*x))*cos(
2*x) - 6*(12*sqrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) + 4*sqrt
(2)*x)*cos(3/2*x) - 6*((12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt(2
))*sin(3/2*x))*cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*s
qrt(2))*sin(3/2*x))*cos(2*x) + 3*(12*sqrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*
sqrt(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(3*x)^2 + 9
*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 +
 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*
sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) +
 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + s
qrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*
x))*integrate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos(3/2*x) + 6*
x^2*cos(2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3*x)*sin(3/2*
x) + 6*x^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x^2*cos(x) +
x^2)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 +
16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 +
48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*co
s(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x)
+ 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^
2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) +
 2*a^2*sin(x))*sin(3*x)), x) - 324*(sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*
cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^
2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*
sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^
2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*c
```

```
os(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*sin(3*x))*integrate
(1/9*(x*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos(3/2*x)*sin(
2*x) - x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*cos(2*x)*sin(3
/2*x) + 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a^2*cos(4*x)^
2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^
2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2
*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2
*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x)
+ 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2
*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) +
(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))*sin(3*x)
 + 3*(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))*sin
(2*x) - (9*sqrt(2)*x^2 - 36*sqrt(2)*x*sin(x) + 3*(9*sqrt(2)*x^2 - 8*sqrt(2)
)*cos(x) - 8*sqrt(2))*sin(3/2*x))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x)))
- 486*(sqrt(2)*a^2*cos(3*x)^3 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*co
s(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt
(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 3*(2*sqrt(2)*a^2*cos(2*x) + 2*sqr
t(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x)^2 + (sqrt(2)*a^2*cos(3*x) + sqrt(2)
*a^2)*sin(3*x)^2 + sqrt(2)*a^2 + 3*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqrt(2)*a^
2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x) + 3*s
qrt(2)*a^2*sin(x)^2 + 4*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2
*cos(x) + 2*sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqr
t(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) + (sqrt(2
)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x^2
*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos(3/2*x) + 6*x^2*cos(2*x)*cos(3/2*x
) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3*x)*sin(3/2*x) + 6*x^2*sin(2*x)*si
n(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x^2*cos(x) + x^2)*cos(3/2*x))/(a^2*
cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*
sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x)
 + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x
) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*
cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin
(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x
)), x) + 648*(sqrt(2)*a^2*cos(3*x)^3 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)
*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) +
 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 3*(2*sqrt(2)*a^2*cos(2*x)
+ 2*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x)^2 + (sqrt(2)*a^2*cos(3*x) +
sqrt(2)*a^2)*sin(3*x)^2 + sqrt(2)*a^2 + 3*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqr
t(2)*a^2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x
) + 3*sqrt(2)*a^2*sin(x)^2 + 4*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt
(2)*a^2*cos(x) + 2*sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x
) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) +
(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1
/9*(x*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos(3/2*x)*sin(2*
x) - x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*cos(2*x)*sin(3/2
```

```
*x) + 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a^2*cos(4*x)^2
+ 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2
+ 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*s
in(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*c
os(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) +
12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a
^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + 2*
(24*sqrt(2)*sin(2*x)*sin(3/2*x) - (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2
 - 8*sqrt(2))*cos(3/2*x))*cos(3*x) - (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x)
 - 12*(sqrt(2)*x - 2*sqrt(2)*sin(x))*sin(3/2*x))*sin(3*x) - 6*((9*sqrt(2)*x
^2 - 8*sqrt(2))*cos(3/2*x) + 12*(sqrt(2)*x - 2*sqrt(2)*sin(x))*sin(3/2*x))*
sin(2*x) + 2*(27*sqrt(2)*x^2*cos(x) + 9*sqrt(2)*x^2 + 36*sqrt(2)*cos(x)^2 -
 36*sqrt(2)*x*sin(x) + 36*sqrt(2)*sin(x)^2 - 4*sqrt(2))*sin(3/2*x) - 3*(sqr
t(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3
*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x
)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*s
qrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*s
in(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x
))))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 4*(72*sqrt(2)*cos(2*x)^2*si
n(3/2*x) + 8*sqrt(2)*sin(3*x)^2*sin(3/2*x) + 72*sqrt(2)*sin(2*x)^2*sin(3/2*
x) - 2*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 4*sqrt(2))*sin(3/2*x))*c
os(3*x)^2 + 2*(9*(3*sqrt(2)*x^2*sin(3/2*x) - 4*sqrt(2)*x*cos(3/2*x))*cos(2*
x) - 3*(12*sqrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) + 8*sqrt(2
)*x)*cos(3/2*x) - 3*(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*
cos(3/2*x))*sin(2*x) + (27*sqrt(2)*x^2*cos(x) + 18*sqrt(2)*x^2 - 36*sqrt(2)
*x*sin(x) - 8*sqrt(2))*sin(3/2*x))*cos(3*x) - 18*(4*sqrt(2)*x*cos(3/2*x) -
(3*sqrt(2)*x^2 + 8*sqrt(2)*cos(x))*sin(3/2*x))*cos(2*x) - 6*(12*sqrt(2)*x*c
os(x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) - 486*
(sqrt(2)*a^2*cos(3*x)^3 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2
 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^
2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 3*(2*sqrt(2)*a^2*cos(2*x) + 2*sqrt(2)*a
^2*cos(x) + sqrt(2)*a^2)*cos(3*x)^2 + (sqrt(2)*a^2*cos(3*x) + sqrt(2)*a^2)*
sin(3*x)^2 + sqrt(2)*a^2 + 3*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqrt(2)*a^2*cos(
x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x) + 3*sqrt(2)
*a^2*sin(x)^2 + 4*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(x
) + 2*sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a
^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) + (sqrt(2)*a^2*
sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x^2*cos(4
*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos(3/2*x) + 6*x^2*cos(2*x)*cos(3/2*x) + x^
2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3*x)*sin(3/2*x) + 6*x^2*sin(2*x)*sin(3/2*
x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x^2*cos(x) + x^2)*cos(3/2*x))/(a^2*cos(4*
x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*
x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*
a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*
a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*
```

```
x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x)
+ 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x)
 + 648*(sqrt(2)*a^2*cos(3*x)^3 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*c
os(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqr
t(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 3*(2*sqrt(2)*a^2*cos(2*x) + 2*sq
rt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x)^2 + (sqrt(2)*a^2*cos(3*x) + sqrt(2
)*a^2)*sin(3*x)^2 + sqrt(2)*a^2 + 3*(3*sqrt(2)*a^2*cos(2*x)^2 + 3*sqrt(2)*a
^2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*sin(x) + 3*
sqrt(2)*a^2*sin(x)^2 + 4*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^
2*cos(x) + 2*sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sq
rt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) + (sqrt(
2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x*
cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos(3/2*x)*sin(2*x) - x
*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*cos(2*x)*sin(3/2*x) +
4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*a
^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a
^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^
2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x)
+ a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*
a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin
(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + 2*(24*sq
rt(2)*sin(2*x)*sin(3/2*x) - (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*s
qrt(2))*cos(3/2*x))*cos(3*x) - (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 12*
(sqrt(2)*x - 2*sqrt(2)*sin(x))*sin(3/2*x))*sin(3*x) - 6*((9*sqrt(2)*x^2 - 8
*sqrt(2))*cos(3/2*x) + 12*(sqrt(2)*x - 2*sqrt(2)*sin(x))*sin(3/2*x))*sin(2*
x) + 2*(27*sqrt(2)*x^2*cos(x) + 9*sqrt(2)*x^2 + 36*sqrt(2)*cos(x)^2 - 36*sq
rt(2)*x*sin(x) + 36*sqrt(2)*sin(x)^2 - 4*sqrt(2))*sin(3/2*x) - 3*(sqrt(2)*c
os(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2
+ 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 +
2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)
*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x
) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))))*c
os(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 4*(sqrt(2)*sin(3*x)^3 + 6*(sqrt(2
)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*c
os(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)
*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + s
qrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(
x) + sqrt(2))*sin(3*x))*cos(1/3*arctan2(sin(3/2*x), cos(3/2*x))) - 324*(sqr
t(2)*a^2*cos(3*x)^4 + sqrt(2)*a^2*sin(3*x)^4 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9
*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*
sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2
*cos(x) + 2*sqrt(2)*a^2)*cos(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2
*sin(x))*sin(3*x)^3 + 6*sqrt(2)*a^2*cos(x) + 3*(3*sqrt(2)*a^2*cos(2*x)^2 +
3*sqrt(2)*a^2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*sin(2*x)*
sin(x) + 3*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 6*
```

```
(sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x)^2 + (2*sqrt(2)*a^2*co
s(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^
2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*
sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*
a^2*cos(x) + 2*sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^
2)*cos(2*x))*sin(3*x)^2 + sqrt(2)*a^2 + 2*(9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqr
t(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(
x) + 9*sqrt(2)*a^2*sin(x)^2 + 9*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 9*(2*s
qrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(
x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x) +
 (sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x)^2 + 2*(sqrt(2)*a^2*si
n(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x^2*cos(4*x
)*cos(3/2*x) + 4*x^2*cos(3*x)*cos(3/2*x) + 6*x^2*cos(2*x)*cos(3/2*x) + x^2*
sin(4*x)*sin(3/2*x) + 4*x^2*sin(3*x)*sin(3/2*x) + 6*x^2*sin(2*x)*sin(3/2*x)
 + 4*x^2*sin(3/2*x)*sin(x) + (4*x^2*cos(x) + x^2)*cos(3/2*x))/(a^2*cos(4*x)
^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)
^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^
2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^
2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x)
 + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) +
2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) +
 432*(sqrt(2)*a^2*cos(3*x)^4 + sqrt(2)*a^2*sin(3*x)^4 + 9*sqrt(2)*a^2*cos(2
*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*
sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sq
rt(2)*a^2*cos(x) + 2*sqrt(2)*a^2)*cos(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + sq
rt(2)*a^2*sin(x))*sin(3*x)^3 + 6*sqrt(2)*a^2*cos(x) + 3*(3*sqrt(2)*a^2*cos(
2*x)^2 + 3*sqrt(2)*a^2*cos(x)^2 + 3*sqrt(2)*a^2*sin(2*x)^2 + 6*sqrt(2)*a^2*
sin(2*x)*sin(x) + 3*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + 2*sqrt(2)
*a^2 + 6*(sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x)^2 + (2*sqrt(
2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*s
qrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(
x)^2 + 6*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3
*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + s
qrt(2)*a^2)*cos(2*x))*sin(3*x)^2 + sqrt(2)*a^2 + 2*(9*sqrt(2)*a^2*cos(2*x)^
2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(
2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 9*sqrt(2)*a^2*cos(x) + 2*sqrt(2)*a^2
 + 9*(2*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*cos(3*x) + 6*(3*sqrt(2)
*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2
*sin(x) + (sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x)^2 + 2*(sqrt(
2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/9*(x*
cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos(3/2*x)*sin(2*x) - x
*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*cos(2*x)*sin(3/2*x) +
4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*a
^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a
^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^
```

```
2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x)
+ a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*
a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin
(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + (sqrt(2)
*cos(3*x)^4 + sqrt(2)*sin(3*x)^4 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x)
 + 2*sqrt(2))*cos(3*x)^3 + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x)^3
 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2)*sin(2*x)^2 + 6*
sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2 + 6*(sqrt(2)*cos(x) + sqrt(2))
*cos(2*x) + 6*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x)^2 + 9*(sqrt(2)*cos(3*x)^
2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt
(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqr
t(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x)
+ sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sq
rt(2)*cos(x) + sqrt(2))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*(sqr
t(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3
*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x
)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*s
qrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*s
in(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x
)))^2 + (2*sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 +
 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2
*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x) + 6*(3*sqrt(2
)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + 2*sqrt(2))*sin(3*x)^2 + 9
*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*
sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*
sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6
*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(
x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(4/3*arctan2(sin(3/2*x), cos(
3/2*x)))^2 + 9*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x
)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x
) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2)
)*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x)
+ sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(2/3*arctan2(si
n(3/2*x), cos(3/2*x)))^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sq
rt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(9*s
qrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*
sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 9*(2*sqrt(2)*cos(x) + sqrt(2))*cos(2
*x) + 9*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2
))*cos(2*x) + 6*(sqrt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqrt(2)*cos
(x) + sqrt(2))*cos(3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2 + 9*sqr
t(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*si
n(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*co
s(x)^2 + 3*sqrt(2)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x
)^2 + 2*(3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x) + sqrt(2
))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 3*(sqrt(2)*cos(3*x)
```

```
^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqr
t(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sq
rt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x)
 + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*s
qrt(2)*cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*((sqr
t(2)*sin(2*x) + sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x
))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(4/3*arctan2(sin(3/2*x), cos(3
/2*x))) + 6*(sqrt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqrt(2)*cos(x)
+ sqrt(2))*cos(3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2 + 9*sqrt(2)
*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*
x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)
^2 + 3*sqrt(2)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2
+ 2*(3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x) + sqrt(2))*c
os(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*((sqrt(2)*sin(2*x) +
sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*
sqrt(2)*cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*((sq
rt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(3*x)^2 + 2*(sqrt(2)*sin(2*x) + sqrt(2)
*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*(sqrt(2
)*sin(3*x)^3 + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x)^2 + (sqrt(2)*
cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)
^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x
) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*c
os(2*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(3*x) + 3*(sqrt(2)*cos(3*x)^2 + 9*
sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*si
n(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*c
os(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt
(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*
cos(x) + sqrt(2))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))))*sin(4/3*arctan2
(sin(3/2*x), cos(3/2*x))) + 6*(sqrt(2)*sin(3*x)^3 + 6*(sqrt(2)*sin(2*x) + s
qrt(2)*sin(x))*sin(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*
sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sq
rt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*
x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + sqrt(2))*
sin(3*x))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*sqrt(2)*cos(x) + sqr
t(2))*log(cos(1/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + sin(1/3*arctan2(sin(
3/2*x), cos(3/2*x)))^2 + 2*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))) + 1) -
(sqrt(2)*cos(3*x)^4 + sqrt(2)*sin(3*x)^4 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2
)*cos(x) + 2*sqrt(2))*cos(3*x)^3 + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*si
n(3*x)^3 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2)*sin(2*x
)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2 + 6*(sqrt(2)*cos(x) +
sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x)^2 + 9*(sqrt(2)*c
os(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2
+ 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 +
2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)
*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x
```

```
) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2
+ 9*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(
2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(
2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x)
+ 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*s
in(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), c
os(3/2*x)))^2 + (2*sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*co
s(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(
x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x) + 6*(
3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + 2*sqrt(2))*sin(3*
x)^2 + 9*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 +
sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*
sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(
3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt
(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(4/3*arctan2(sin(3/2*
x), cos(3/2*x)))^2 + 9*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(
2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*
x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) +
 sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*s
in(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(2/3*ar
ctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^
2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2
+ 2*(9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*
sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 9*(2*sqrt(2)*cos(x) + sqrt(2
))*cos(2*x) + 9*sqrt(2)*cos(x) + 2*sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x)
+ sqrt(2))*cos(2*x) + 6*(sqrt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqr
t(2)*cos(x) + sqrt(2))*cos(3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2
 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sq
rt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sq
rt(2)*cos(x)^2 + 3*sqrt(2)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(
2)*sin(x)^2 + 2*(3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x)
+ sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 3*(sqrt(2)*
cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2
 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 +
 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2
)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*
x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) +
 6*((sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(
2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(4/3*arctan2(sin(3/2*x
), cos(3/2*x))) + 6*(sqrt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqrt(2)
*cos(x) + sqrt(2))*cos(3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2 + 9
*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2
)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2
)*cos(x)^2 + 3*sqrt(2)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*s
in(x)^2 + 2*(3*sqrt(2)*cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x) + sq
```

```
rt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*((sqrt(2)*sin
(2*x) + sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3
*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x)))
+ 6*((sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(3*x)^2 + 2*(sqrt(2)*sin(2*x) +
 sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6
*(sqrt(2)*sin(3*x)^3 + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x)^2 + (
sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*
sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)
*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sq
rt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(3*x) + 3*(sqrt(2)*cos(3*x
)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sq
rt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*s
qrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x
) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*
sqrt(2)*cos(x) + sqrt(2))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))))*sin(4/3
*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(sqrt(2)*sin(3*x)^3 + 6*(sqrt(2)*sin(
2*x) + sqrt(2)*sin(x))*sin(3*x)^2 + (sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x
)^2 + 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x
) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2)
)*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*sqrt(2)*cos(x) + s
qrt(2))*sin(3*x))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*sqrt(2)*cos(
x) + sqrt(2))*log(cos(1/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + sin(1/3*arct
an2(sin(3/2*x), cos(3/2*x)))^2 - 2*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x)))
 + 1) - 4*(24*sqrt(2)*cos(2*x)^2*cos(3/2*x) + 24*sqrt(2)*cos(3/2*x)*sin(2*x
)^2 + (3*sqrt(2)*x^2*cos(3/2*x) + 4*sqrt(2)*x*sin(3/2*x))*cos(3*x)^2 + 16*(
3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x)*cos(3/2*x) + 2*(8*sqrt(2)*cos(2*x)*cos
(3/2*x) - 8*sqrt(2)*sin(2*x)*sin(3/2*x) + (3*sqrt(2)*x^2 + 8*sqrt(2)*cos(x)
)*cos(3/2*x) + 4*(sqrt(2)*x - 2*sqrt(2)*sin(x))*sin(3/2*x))*cos(3*x) + (3*s
qrt(2)*x^2 + 24*sqrt(2)*cos(x)^2 + 24*sqrt(2)*sin(x)^2 + 16*sqrt(2)*cos(x))
*cos(3/2*x) + 16*(3*sqrt(2)*cos(3/2*x)*sin(x) - sqrt(2)*sin(3/2*x))*sin(2*x
) + 4*(sqrt(2)*x - 4*sqrt(2)*sin(x))*sin(3/2*x))*sin(3*x) - 4*((9*sqrt(2)*x
^2 - 8*sqrt(2))*cos(3/2*x) + 12*(sqrt(2)*x - 4*sqrt(2)*sin(x))*sin(3/2*x))*
sin(2*x) + 4*(3*sqrt(2)*x^2 + 24*sqrt(2)*cos(x)^2 - 12*sqrt(2)*x*sin(x) + 2
4*sqrt(2)*sin(x)^2 + (9*sqrt(2)*x^2 + 8*sqrt(2))*cos(x))*sin(3/2*x) + 4*(sq
rt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqrt(2)*cos(x) + sqrt(2))*cos(
3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9
*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*s
qrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2)*
sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*
cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*
sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(
2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 1
8*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*
sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x
) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqr
```

```
t(2))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 3*(sqrt(2)*cos(3*x)^2 + 9*
sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*si
n(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*c
os(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt
(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*
cos(x) + sqrt(2))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*((sqrt(2)*si
n(2*x) + sqrt(2)*sin(x))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(
3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*sin(5/3*arctan2(sin(3/2*x), cos(3/2*x)))
 - 4*(8*sqrt(2)*cos(3*x)^2*cos(3/2*x) + 72*sqrt(2)*cos(2*x)^2*cos(3/2*x) +
72*sqrt(2)*cos(3/2*x)*sin(2*x)^2 + 144*sqrt(2)*cos(3/2*x)*sin(2*x)*sin(x) +
 48*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x)*cos(3/2*x) + 2*(12*sqrt(2)*x*sin(
3/2*x) + (9*sqrt(2)*x^2 - 4*sqrt(2))*cos(3/2*x))*sin(3*x)^2 + 16*(3*sqrt(2)
*cos(2*x)*cos(3/2*x) + (3*sqrt(2)*cos(x) + sqrt(2))*cos(3/2*x))*cos(3*x) +
8*(9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(x)^2 + 6*sqrt(2)*cos(x) + sqrt(2))*co
s(3/2*x) - 3*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^
2 + sqrt(2)*sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x)
+ 9*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*
cos(3*x) + 6*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) +
sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(1/3*arctan2(sin(
3/2*x), cos(3/2*x))) + 486*(sqrt(2)*a^2*sin(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x
) + sqrt(2)*a^2*sin(x))*sin(3*x)^2 + (sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^
2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(
2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sq
rt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)
*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*sin(3*x))*inte
grate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos(3/2*x) + 6*x^2*cos(
2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3*x)*sin(3/2*x) + 6*x
^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x^2*cos(x) + x^2)*cos
(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*c
os(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*s
in(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) +
 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*
cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*
x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*s
in(x))*sin(3*x)), x) - 648*(sqrt(2)*a^2*sin(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x
) + sqrt(2)*a^2*sin(x))*sin(3*x)^2 + (sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^
2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(
2)*a^2*sin(2*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sq
rt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)
*cos(3*x) + 6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*sin(3*x))*inte
grate(1/9*(x*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos(3/2*x)
*sin(2*x) - x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*cos(2*x)*
sin(3/2*x) + 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a^2*cos(
4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(
4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 1
```

```
6*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) +
4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(
3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x
) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)),
x) + 2*((12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*
cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2
*x))*cos(2*x) + 3*(9*sqrt(2)*x^2*sin(x) + 12*sqrt(2)*x*cos(x) + 4*sqrt(2)*x
)*cos(3/2*x) + 9*(3*sqrt(2)*x^2*cos(3/2*x) + 4*sqrt(2)*x*sin(3/2*x))*sin(2*
x) - (9*sqrt(2)*x^2 - 36*sqrt(2)*x*sin(x) + 3*(9*sqrt(2)*x^2 - 8*sqrt(2))*c
os(x) - 8*sqrt(2))*sin(3/2*x))*sin(3*x) + 6*((12*sqrt(2)*x*cos(3/2*x) - (9*
sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x)
- (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(2*x) + 3*(12*sqrt(2)*x*cos(x)
 + (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) + 243*(sqrt
(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sq
rt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*s
in(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*
sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*
sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt
(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4*x^2*co
s(3*x)*cos(3/2*x) + 6*x^2*cos(2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/2*x) + 4
*x^2*sin(3*x)*sin(3/2*x) + 6*x^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3/2*x)*sin
(x) + (4*x^2*cos(x) + x^2)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2
+ 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2
+ 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(
x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x
) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a
^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x)
+ 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 324*(sqrt(2)*a^2*cos(3
*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin
(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqr
t(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*c
os(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2*c
os(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*sin(x
))*sin(3*x))*integrate(1/9*(x*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x)
 + 6*x*cos(3/2*x)*sin(2*x) - x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x
) - 6*x*cos(2*x)*sin(3/2*x) + 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(
3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*co
s(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*si
n(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) +
6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*c
os(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x
) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*si
n(x))*sin(3*x)), x) + (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2)
)*cos(3/2*x))*sin(3*x) + 3*(12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sq
rt(2))*cos(3/2*x))*sin(2*x) - (9*sqrt(2)*x^2 - 36*sqrt(2)*x*sin(x) + 3*(9*s
```

```
qrt(2)*x^2 - 8*sqrt(2))*cos(x) - 8*sqrt(2))*sin(3/2*x))*sin(2/3*arctan2(sin
(3/2*x), cos(3/2*x))))*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x))) - 4*(8*sqrt
(2)*cos(3*x)^2*cos(3/2*x) + 72*sqrt(2)*cos(2*x)^2*cos(3/2*x) + 72*sqrt(2)*c
os(3/2*x)*sin(2*x)^2 + 144*sqrt(2)*cos(3/2*x)*sin(2*x)*sin(x) + 48*(3*sqrt(
2)*cos(x) + sqrt(2))*cos(2*x)*cos(3/2*x) + 2*(12*sqrt(2)*x*sin(3/2*x) + (9*
sqrt(2)*x^2 - 4*sqrt(2))*cos(3/2*x))*sin(3*x)^2 + 16*(3*sqrt(2)*cos(2*x)*co
s(3/2*x) + (3*sqrt(2)*cos(x) + sqrt(2))*cos(3/2*x))*cos(3*x) + 8*(9*sqrt(2)
*cos(x)^2 + 9*sqrt(2)*sin(x)^2 + 6*sqrt(2)*cos(x) + sqrt(2))*cos(3/2*x) - 3
*(sqrt(2)*cos(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 + 9*sqrt(2)*cos(x)^2 + sqrt(2)*
sin(3*x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9*sqrt(2)*
sin(x)^2 + 2*(3*sqrt(2)*cos(2*x) + 3*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6
*(3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*(sqrt(2)*sin(2*x) + sqrt(2)*sin(
x))*sin(3*x) + 6*sqrt(2)*cos(x) + sqrt(2))*cos(1/3*arctan2(sin(3/2*x), cos(
3/2*x))) + 486*(sqrt(2)*a^2*sin(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*
a^2*sin(x))*sin(3*x)^2 + (sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2
 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2
*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 +
2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) +
6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*sin(3*x))*integrate(1/9*(x
^2*cos(4*x)*cos(3/2*x) + 4*x^2*cos(3*x)*cos(3/2*x) + 6*x^2*cos(2*x)*cos(3/2
*x) + x^2*sin(4*x)*sin(3/2*x) + 4*x^2*sin(3*x)*sin(3/2*x) + 6*x^2*sin(2*x)*
sin(3/2*x) + 4*x^2*sin(3/2*x)*sin(x) + (4*x^2*cos(x) + x^2)*cos(3/2*x))/(a^
2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^
2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(
x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2
*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2
)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*s
in(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3
*x)), x) - 648*(sqrt(2)*a^2*sin(3*x)^3 + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*
a^2*sin(x))*sin(3*x)^2 + (sqrt(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2
 + 9*sqrt(2)*a^2*cos(x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2
*x)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 +
2*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) +
6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x))*sin(3*x))*integrate(1/9*(x
*cos(3/2*x)*sin(4*x) + 4*x*cos(3/2*x)*sin(3*x) + 6*x*cos(3/2*x)*sin(2*x) -
x*cos(4*x)*sin(3/2*x) - 4*x*cos(3*x)*sin(3/2*x) - 6*x*cos(2*x)*sin(3/2*x) +
 4*x*cos(3/2*x)*sin(x) - (4*x*cos(x) + x)*sin(3/2*x))/(a^2*cos(4*x)^2 + 16*
a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*
a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)
^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x)
 + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4
*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*si
n(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + 2*((12*
sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(3*x) + 3
*(12*sqrt(2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(2*x
```

```
) + 3*(9*sqrt(2)*x^2*sin(x) + 12*sqrt(2)*x*cos(x) + 4*sqrt(2)*x)*cos(3/2*x)
 + 9*(3*sqrt(2)*x^2*cos(3/2*x) + 4*sqrt(2)*x*sin(3/2*x))*sin(2*x) - (9*sqrt
(2)*x^2 - 36*sqrt(2)*x*sin(x) + 3*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(x) - 8*sq
rt(2))*sin(3/2*x))*sin(3*x))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))) - 4*(
sqrt(2)*cos(3*x)^3 + 3*(2*sqrt(2)*cos(2*x) + 2*sqrt(2)*cos(x) + sqrt(2))*co
s(3*x)^2 + (sqrt(2)*cos(3*x) + sqrt(2))*sin(3*x)^2 + 9*sqrt(2)*cos(2*x)^2 +
 9*sqrt(2)*cos(x)^2 + 9*sqrt(2)*sin(2*x)^2 + 18*sqrt(2)*sin(2*x)*sin(x) + 9
*sqrt(2)*sin(x)^2 + 3*(3*sqrt(2)*cos(2*x)^2 + 3*sqrt(2)*cos(x)^2 + 3*sqrt(2
)*sin(2*x)^2 + 6*sqrt(2)*sin(2*x)*sin(x) + 3*sqrt(2)*sin(x)^2 + 2*(3*sqrt(2
)*cos(x) + 2*sqrt(2))*cos(2*x) + 4*sqrt(2)*cos(x) + sqrt(2))*cos(3*x) + 6*(
3*sqrt(2)*cos(x) + sqrt(2))*cos(2*x) + 6*((sqrt(2)*sin(2*x) + sqrt(2)*sin(x
))*cos(3*x) + sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(3*x) + 6*sqrt(2)*cos(x
) + sqrt(2))*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))))/((a*cos(3*x)^4 + a*s
in(3*x)^4 + 2*(3*a*cos(2*x) + 3*a*cos(x) + 2*a)*cos(3*x)^3 + 6*(a*sin(2*x)
+ a*sin(x))*sin(3*x)^3 + 3*(3*a*cos(2*x)^2 + 3*a*cos(x)^2 + 3*a*sin(2*x)^2
+ 6*a*sin(2*x)*sin(x) + 3*a*sin(x)^2 + 6*(a*cos(x) + a)*cos(2*x) + 6*a*cos(
x) + 2*a)*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*cos(x)^2 + 9*(a*cos(3*x)^2 + 9*
a*cos(2*x)^2 + 9*a*cos(x)^2 + a*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)
*sin(x) + 9*a*sin(x)^2 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*
a*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*sin(2*x) + a*sin(x))*sin(3*x) +
a)*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*(a*cos(3*x)^2 + 9*a*cos(2
*x)^2 + 9*a*cos(x)^2 + a*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x)
 + 9*a*sin(x)^2 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x
) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*sin(2*x) + a*sin(x))*sin(3*x) + a)*cos(
2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + (2*a*cos(3*x)^2 + 9*a*cos(2*x)^2 +
 9*a*cos(x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2 + 2*(3
*a*cos(2*x) + 3*a*cos(x) + 2*a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x) + 6*
a*cos(x) + 2*a)*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*si
n(x)^2 + 9*(a*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*cos(x)^2 + a*sin(3*x)^2 + 9
*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2 + 2*(3*a*cos(2*x) + 3*a
*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*sin
(2*x) + a*sin(x))*sin(3*x) + a)*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2
+ 9*(a*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*cos(x)^2 + a*sin(3*x)^2 + 9*a*sin(
2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2 + 2*(3*a*cos(2*x) + 3*a*cos(x)
 + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*sin(2*x) +
 a*sin(x))*sin(3*x) + a)*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 2*(9*
a*cos(2*x)^2 + 9*a*cos(x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*s
in(x)^2 + 9*(2*a*cos(x) + a)*cos(2*x) + 9*a*cos(x) + 2*a)*cos(3*x) + 6*(3*a
*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*(a*cos(3*x)^3 + 3*(2*a*cos(2*x) + 2*
a*cos(x) + a)*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*cos(x)^2 + (a*cos(3*x) + a)
*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2 + 3*(3*a
*cos(2*x)^2 + 3*a*cos(x)^2 + 3*a*sin(2*x)^2 + 6*a*sin(2*x)*sin(x) + 3*a*sin
(x)^2 + 2*(3*a*cos(x) + 2*a)*cos(2*x) + 4*a*cos(x) + a)*cos(3*x) + 6*(3*a*c
os(x) + a)*cos(2*x) + 6*a*cos(x) + 3*(a*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*c
os(x)^2 + a*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)
```

```
^2 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*
x) + 6*a*cos(x) + 6*(a*sin(2*x) + a*sin(x))*sin(3*x) + a)*cos(2/3*arctan2(s
in(3/2*x), cos(3/2*x))) + 6*((a*sin(2*x) + a*sin(x))*cos(3*x) + a*sin(2*x)
 + a*sin(x))*sin(3*x) + a)*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(a*c
os(3*x)^3 + 3*(2*a*cos(2*x) + 2*a*cos(x) + a)*cos(3*x)^2 + 9*a*cos(2*x)^2 +
 9*a*cos(x)^2 + (a*cos(3*x) + a)*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x
)*sin(x) + 9*a*sin(x)^2 + 3*(3*a*cos(2*x)^2 + 3*a*cos(x)^2 + 3*a*sin(2*x)^2
 + 6*a*sin(2*x)*sin(x) + 3*a*sin(x)^2 + 2*(3*a*cos(x) + 2*a)*cos(2*x) + 4*a
*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x) + 6*a*cos(x) + 6*((a*si
n(2*x) + a*sin(x))*cos(3*x) + a*sin(2*x) + a*sin(x))*sin(3*x) + a)*cos(2/3*
arctan2(sin(3/2*x), cos(3/2*x))) + 6*((a*sin(2*x) + a*sin(x))*cos(3*x)^2 +
2*(a*sin(2*x) + a*sin(x))*cos(3*x) + a*sin(2*x) + a*sin(x))*sin(3*x) + 6*(a
*sin(3*x)^3 + 6*(a*sin(2*x) + a*sin(x))*sin(3*x)^2 + (a*cos(3*x)^2 + 9*a*co
s(2*x)^2 + 9*a*cos(x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x
)^2 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2
*x) + 6*a*cos(x) + a)*sin(3*x) + 3*(a*cos(3*x)^2 + 9*a*cos(2*x)^2 + 9*a*cos
(x)^2 + a*sin(3*x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2
 + 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x)
 + 6*a*cos(x) + 6*(a*sin(2*x) + a*sin(x))*sin(3*x) + a)*sin(2/3*arctan2(sin
(3/2*x), cos(3/2*x))))*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(a*sin(
3*x)^3 + 6*(a*sin(2*x) + a*sin(x))*sin(3*x)^2 + (a*cos(3*x)^2 + 9*a*cos(2*x
)^2 + 9*a*cos(x)^2 + 9*a*sin(2*x)^2 + 18*a*sin(2*x)*sin(x) + 9*a*sin(x)^2 +
 2*(3*a*cos(2*x) + 3*a*cos(x) + a)*cos(3*x) + 6*(3*a*cos(x) + a)*cos(2*x) +
 6*a*cos(x) + a)*sin(3*x))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + a)*sq
rt(a))
```

Giac [F]

$$\int \frac{x^2}{(a + a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a\cos(x) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(a*cos(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a+a\,\cos(x))^{3/2}}\,dx$$

```
[In] int(x^2/(a + a*cos(x))^(3/2),x)
[Out] int(x^2/(a + a*cos(x))^(3/2), x)
```

3.182 $\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx$

Optimal result . . . 1103
Rubi [A] (verified) . . . 1103
Mathematica [A] (verified) . . . 1105
Maple [F] . . . 1106
Fricas [F] . . . 1106
Sympy [F] . . . 1106
Maxima [F] . . . 1106
Giac [F] . . . 1117
Mupad [F(-1)] . . . 1117

Optimal result

Integrand size = 12, antiderivative size = 150

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = -\frac{1}{a\sqrt{a+a\cos(x)}} - \frac{ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}$$

```
[Out] -1/a/(a+a*cos(x))^(1/2)-I*x*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*cos(x))^
(1/2)+I*cos(1/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-I*cos(1/
2*x)*polylog(2,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+1/2*x*tan(1/2*x)/a/(a+a
*cos(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3400, 4270, 4266, 2317, 2438}

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = -\frac{ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} + \frac{i\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} - \frac{i\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a\cos(x)+a}} - \frac{1}{a\sqrt{a\cos(x)+a}} + \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a\cos(x)+a}}$$

```
[In] Int[x/(a + a*Cos[x])^(3/2),x]
```

```
[Out] -(1/(a*Sqrt[a + a*Cos[x]])) - (I*x*ArcTan[E^((I/2)*x)]*Cos[x/2])/(a*Sqrt[a
+ a*Cos[x]]) + (I*Cos[x/2]*PolyLog[2, (-I)*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[
x]]) - (I*Cos[x/2]*PolyLog[2, I*E^((I/2)*x)])/(a*Sqrt[a + a*Cos[x]]) + (x*T
an[x/2])/(2*a*Sqrt[a + a*Cos[x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\text{integral} = \frac{\cos\left(\frac{x}{2}\right)\int x\sec^3\left(\frac{x}{2}\right)\,dx}{2a\sqrt{a+a\cos(x)}}$$

$$= -\frac{1}{a\sqrt{a+a\cos(x)}} + \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int x\sec\left(\frac{x}{2}\right)\,dx}{4a\sqrt{a+a\cos(x)}}$$

$$= -\frac{1}{a\sqrt{a+a\cos(x)}} - \frac{ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}$$
$$- \frac{\cos\left(\frac{x}{2}\right)\int \log\left(1-ie^{\frac{ix}{2}}\right)\,dx}{2a\sqrt{a+a\cos(x)}} + \frac{\cos\left(\frac{x}{2}\right)\int \log\left(1+ie^{\frac{ix}{2}}\right)\,dx}{2a\sqrt{a+a\cos(x)}}$$

$$= -\frac{1}{a\sqrt{a+a\cos(x)}} - \frac{ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}$$
$$+ \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}} + \frac{\left(i\cos\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$- \frac{\left(i\cos\left(\frac{x}{2}\right)\right)\operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$

$$= -\frac{1}{a\sqrt{a+a\cos(x)}} - \frac{ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}$$
$$- \frac{i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \frac{\sec\left(\frac{x}{2}\right)\left(-4\cos\left(\frac{x}{2}\right)+x\log\left(1-ie^{\frac{ix}{2}}\right)+x\cos(x)\log\left(1-ie^{\frac{ix}{2}}\right)-x\log\left(1+ie^{\frac{ix}{2}}\right.\right.}{}$$

```
[In] Integrate[x/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (Sec[x/2]*(-4*Cos[x/2] + x*Log[1 - I*E^((I/2)*x)] + x*Cos[x]*Log[1 - I*E^((
I/2)*x)] - x*Log[1 + I*E^((I/2)*x)] - x*Cos[x]*Log[1 + I*E^((I/2)*x)] + (2*
I)*(1 + Cos[x])*PolyLog[2, (-I)*E^((I/2)*x)] - (2*I)*(1 + Cos[x])*PolyLog[2
, I*E^((I/2)*x)] + 2*x*Sin[x/2]))/(4*a*Sqrt[a*(1 + Cos[x])])
```

Maple [F]

$$\int \frac{x}{(a+\cos(x)\,a)^{\frac{3}{2}}}dx$$

```
[In] int(x/(a+cos(x)*a)^(3/2),x)
[Out] int(x/(a+cos(x)*a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+a*cos(x))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cos(x) + a)*x/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

Sympy [F]

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x}{(a(\cos(x)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+a*cos(x))**(3/2),x)
[Out] Integral(x/(a*(cos(x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+a*cos(x))^(3/2),x, algorithm="maxima")
[Out] 1/3*(8*x*cos(3/2*x)*sin(3*x)^3 - 8*x*cos(3*x)^3*sin(3/2*x) - 8*((3*x*sin(3/
2*x) + 2*cos(3/2*x))*cos(2*x) - (3*x*sin(x) - 2*cos(x))*cos(3/2*x) - (3*x*c
os(3/2*x) + 2*sin(3/2*x))*sin(2*x) + (3*x*cos(x) + 3*x - 2*sin(x))*sin(3/2*
x))*cos(3*x)^2 - 48*cos(2*x)^2*cos(3/2*x) - 24*((3*x*sin(3/2*x) - 2*cos(3/2
*x))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(x) +
6*cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*a^2*co
s(x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2
*sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + a^2)*co
```

```
s(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x))*si
n(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*
x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6
*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x
))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^
2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x
)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2
*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x)
 + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3
*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))
*sin(3*x)), x) - (3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(3*x) - 3*(3*x*cos(3/2*
x) + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + x - 2*sin(x))*sin(3/2*x))*cos
(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 - 24*((3*x*sin(3/2*x) - 2*cos(3/2*x
))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(x) + 6*
cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*a^2*cos(
x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2*s
in(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + a^2)*cos(
3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x))*sin(
3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*
cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x
*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))
/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2
+ a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*
sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*c
os(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) +
 a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a
^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*s
in(3*x)), x) - (3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(3*x) - 3*(3*x*cos(3/2*x)
 + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + x - 2*sin(x))*sin(3/2*x))*cos(2
/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 - 8*(x*cos(3*x)*sin(3/2*x) + (3*x*sin
(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (3*x*sin(x) + 2*cos(x))*cos(3/2*x) - (3*
x*cos(3/2*x) - 2*sin(3/2*x))*sin(2*x) + (3*x*cos(x) + x + 2*sin(x))*sin(3/2
*x))*sin(3*x)^2 - 48*cos(3/2*x)*sin(2*x)^2 - 24*((3*x*sin(3/2*x) - 2*cos(3/
2*x))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(x) +
 6*cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*a^2*c
os(x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^
2*sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + a^2)*c
os(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x))*s
in(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6
*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) +
6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*
x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)
^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*
x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^
2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x
```

```
) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) +
3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x)
)*sin(3*x)), x) - (3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(3*x) - 3*(3*x*cos(3/2
*x) + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + x - 2*sin(x))*sin(3/2*x))*si
n(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 - 24*((3*x*sin(3/2*x) - 2*cos(3/2*
x))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(x) + 6
*cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*a^2*cos
(x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2*
sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + a^2)*cos
(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x))*sin
(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x
*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*
x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x)
)/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2
 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)
*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*
cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x)
+ a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*
a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*
sin(3*x)), x) - (3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(3*x) - 3*(3*x*cos(3/2*x
) + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + x - 2*sin(x))*sin(3/2*x))*sin(
2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 - (2*(3*cos(2*x) + 3*cos(x) + 2)*cos
(3*x)^3 + cos(3*x)^4 + 6*(sin(2*x) + sin(x))*sin(3*x)^3 + sin(3*x)^4 + 3*(6
*(cos(x) + 1)*cos(2*x) + 3*cos(2*x)^2 + 3*cos(x)^2 + 3*sin(2*x)^2 + 6*sin(2
*x)*sin(x) + 3*sin(x)^2 + 6*cos(x) + 2)*cos(3*x)^2 + 9*(2*(3*cos(2*x) + 3*c
os(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2
 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2
+ 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(4/3*arctan2(sin(3/2*x
), cos(3/2*x)))^2 + 9*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2
+ 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin
(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2
 + 6*cos(x) + 1)*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + (2*(3*cos(2*x
) + 3*cos(x) + 2)*cos(3*x) + 2*cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*c
os(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6
*cos(x) + 2)*sin(3*x)^2 + 9*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3
*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x)
 + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*si
n(x)^2 + 6*cos(x) + 1)*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*(2*(3
*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x)
 + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2
+ 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(2/3*ar
ctan2(sin(3/2*x), cos(3/2*x)))^2 + 2*(9*(2*cos(x) + 1)*cos(2*x) + 9*cos(2*x
)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 9*cos(x
) + 2)*cos(3*x) + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6
*(3*(2*cos(2*x) + 2*cos(x) + 1)*cos(3*x)^2 + cos(3*x)^3 + (cos(3*x) + 1)*si
```

```
n(3*x)^2 + 3*(2*(3*cos(x) + 2)*cos(2*x) + 3*cos(2*x)^2 + 3*cos(x)^2 + 3*sin
(2*x)^2 + 6*sin(2*x)*sin(x) + 3*sin(x)^2 + 4*cos(x) + 1)*cos(3*x) + 6*(3*co
s(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 3*(2*(3*cos(2*x) + 3*cos(x
) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9
*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18
*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(2/3*arctan2(sin(3/2*x), c
os(3/2*x))) + 6*((sin(2*x) + sin(x))*cos(3*x) + sin(2*x) + sin(x))*sin(3*x)
 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(4/3*a
rctan2(sin(3/2*x), cos(3/2*x))) + 6*(3*(2*cos(2*x) + 2*cos(x) + 1)*cos(3*x)
^2 + cos(3*x)^3 + (cos(3*x) + 1)*sin(3*x)^2 + 3*(2*(3*cos(x) + 2)*cos(2*x)
 + 3*cos(2*x)^2 + 3*cos(x)^2 + 3*sin(2*x)^2 + 6*sin(2*x)*sin(x) + 3*sin(x)^2
 + 4*cos(x) + 1)*cos(3*x) + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*co
s(x)^2 + 6*((sin(2*x) + sin(x))*cos(3*x) + sin(2*x) + sin(x))*sin(3*x) + 9*
sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(2/3*arctan
2(sin(3/2*x), cos(3/2*x))) + 6*((sin(2*x) + sin(x))*cos(3*x)^2 + 2*(sin(2*x
) + sin(x))*cos(3*x) + sin(2*x) + sin(x))*sin(3*x) + 9*sin(2*x)^2 + 18*sin(
2*x)*sin(x) + 9*sin(x)^2 + 6*(6*(sin(2*x) + sin(x))*sin(3*x)^2 + sin(3*x)^3
 + (2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*
cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) +
9*sin(x)^2 + 6*cos(x) + 1)*sin(3*x) + 3*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(
3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 +
 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*s
in(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x)))
)*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(6*(sin(2*x) + sin(x))*sin(3
*x)^2 + sin(3*x)^3 + (2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 +
 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*
sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(3*x))*sin(2/3*arctan2(sin(
3/2*x), cos(3/2*x))) + 6*cos(x) + 1)*arctan2(cos(1/3*arctan2(sin(3/2*x), co
s(3/2*x))), sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))) + 1) - (2*(3*cos(2*x)
+ 3*cos(x) + 2)*cos(3*x)^3 + cos(3*x)^4 + 6*(sin(2*x) + sin(x))*sin(3*x)^3
+ sin(3*x)^4 + 3*(6*(cos(x) + 1)*cos(2*x) + 3*cos(2*x)^2 + 3*cos(x)^2 + 3*s
in(2*x)^2 + 6*sin(2*x)*sin(x) + 3*sin(x)^2 + 6*cos(x) + 2)*cos(3*x)^2 + 9*(
2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(
2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x
)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(4/
3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos
(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2
+ 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*
sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))
)^2 + (2*(3*cos(2*x) + 3*cos(x) + 2)*cos(3*x) + 2*cos(3*x)^2 + 6*(3*cos(x)
+ 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(
x) + 9*sin(x)^2 + 6*cos(x) + 2)*sin(3*x)^2 + 9*(2*(3*cos(2*x) + 3*cos(x) +
1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos
(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin
(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(4/3*arctan2(sin(3/2*x), cos(3
```

```
/2*x)))^2 + 9*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*c
os(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin
(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos
(x) + 1)*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 2*(9*(2*cos(x) + 1)*c
os(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9
*sin(x)^2 + 9*cos(x) + 2)*cos(3*x) + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)
^2 + 9*cos(x)^2 + 6*(3*(2*cos(2*x) + 2*cos(x) + 1)*cos(3*x)^2 + cos(3*x)^3
+ (cos(3*x) + 1)*sin(3*x)^2 + 3*(2*(3*cos(x) + 2)*cos(2*x) + 3*cos(2*x)^2 +
 3*cos(x)^2 + 3*sin(2*x)^2 + 6*sin(2*x)*sin(x) + 3*sin(x)^2 + 4*cos(x) + 1)
*cos(3*x) + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 3*(2*(3
*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x)
 + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2
+ 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(2/3*ar
ctan2(sin(3/2*x), cos(3/2*x))) + 6*((sin(2*x) + sin(x))*cos(3*x) + sin(2*x)
 + sin(x))*sin(3*x) + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*co
s(x) + 1)*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(3*(2*cos(2*x) + 2*c
os(x) + 1)*cos(3*x)^2 + cos(3*x)^3 + (cos(3*x) + 1)*sin(3*x)^2 + 3*(2*(3*co
s(x) + 2)*cos(2*x) + 3*cos(2*x)^2 + 3*cos(x)^2 + 3*sin(2*x)^2 + 6*sin(2*x)*
sin(x) + 3*sin(x)^2 + 4*cos(x) + 1)*cos(3*x) + 6*(3*cos(x) + 1)*cos(2*x) +
9*cos(2*x)^2 + 9*cos(x)^2 + 6*((sin(2*x) + sin(x))*cos(3*x) + sin(2*x) + si
n(x))*sin(3*x) + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x)
+ 1)*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*((sin(2*x) + sin(x))*cos(
3*x)^2 + 2*(sin(2*x) + sin(x))*cos(3*x) + sin(2*x) + sin(x))*sin(3*x) + 9*s
in(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*(6*(sin(2*x) + sin(x))*sin(
3*x)^2 + sin(3*x)^3 + (2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2
+ 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18
*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(3*x) + 3*(2*(3*cos(2*x) +
 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*
x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x
)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(2/3*arctan2(sin(3
/2*x), cos(3/2*x))))*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(6*(sin(2
*x) + sin(x))*sin(3*x)^2 + sin(3*x)^3 + (2*(3*cos(2*x) + 3*cos(x) + 1)*cos(
3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 +
 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(3*x))*s
in(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*cos(x) + 1)*arctan2(cos(1/3*arc
tan2(sin(3/2*x), cos(3/2*x))), -sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))) +
1) - 8*(6*cos(2*x)^2*cos(3/2*x) + 6*cos(3/2*x)*sin(2*x)^2 + 2*(2*(3*cos(x)
+ 1)*cos(3/2*x) + 3*x*sin(3/2*x))*cos(2*x) + 2*(3*cos(x)^2 - 3*x*sin(x) + 3
*sin(x)^2 + 2*cos(x))*cos(3/2*x) - 2*(3*(x - 2*sin(x))*cos(3/2*x) + 2*sin(3
/2*x))*sin(2*x) + (6*x*cos(x) + 3*x - 4*sin(x))*sin(3/2*x))*cos(3*x) - 8*(2
*(6*cos(x) + 1)*cos(3/2*x) + 3*x*sin(3/2*x))*cos(2*x) - 8*(6*cos(x)^2 - 3*x
*sin(x) + 6*sin(x)^2 + 2*cos(x))*cos(3/2*x) - 2*(3*(2*cos(2*x) + 2*cos(x) +
 1)*cos(3*x)^2 + cos(3*x)^3 + (cos(3*x) + 1)*sin(3*x)^2 + 3*(2*(3*cos(x) +
2)*cos(2*x) + 3*cos(2*x)^2 + 3*cos(x)^2 + 3*sin(2*x)^2 + 6*sin(2*x)*sin(x)
+ 3*sin(x)^2 + 4*cos(x) + 1)*cos(3*x) + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2
```

```
*x)^2 + 9*cos(x)^2 + 3*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2
 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + si
n(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^
2 + 6*cos(x) + 1)*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 3*(2*(3*cos(2*
x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*co
s(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin
(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(2/3*arctan2(s
in(3/2*x), cos(3/2*x))) + 6*((sin(2*x) + sin(x))*cos(3*x) + sin(2*x) + sin(
x))*sin(3*x) + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) +
1)*cos(5/3*arctan2(sin(3/2*x), cos(3/2*x))) - 2*(8*(3*x*sin(3/2*x) - cos(3/
2*x))*cos(3*x)^2 + 72*cos(2*x)^2*cos(3/2*x) + 8*cos(3/2*x)*sin(3*x)^2 + 72*
cos(3/2*x)*sin(2*x)^2 + 8*(9*x*cos(2*x)*sin(3/2*x) - (9*x*sin(x) + 2)*cos(3
/2*x) - 3*(3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + 2*x -
2*sin(x))*sin(3/2*x))*cos(3*x) + 72*(2*cos(3/2*x)*cos(x) + x*sin(3/2*x))*co
s(2*x) + 8*(9*cos(x)^2 - 9*x*sin(x) + 9*sin(x)^2 - 1)*cos(3/2*x) + 24*((3*x
*sin(3/2*x) - 2*cos(3/2*x))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*co
s(2*x) - (9*x*sin(x) + 6*cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^
2*cos(2*x)^2 + 9*a^2*cos(x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*
sin(2*x)*sin(x) + 9*a^2*sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) +
 3*a^2*cos(x) + a^2)*cos(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*si
n(2*x) + a^2*sin(x))*sin(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*c
os(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*
sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*
x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(
2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(
2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*
(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*
cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x)
+ 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*s
in(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - (3*x*cos(3/2*x) + 2*sin(3/2*x))*sin
(3*x) - 3*(3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + x - 2*
sin(x))*sin(3/2*x))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) - 3*(2*(3*cos(
2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*
cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*s
in(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(1/3*arctan2
(sin(3/2*x), cos(3/2*x))) - 216*(a^2*cos(3*x)^3 + 9*a^2*cos(2*x)^2 + 9*a^2*
cos(x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2*sin(x)^2 + 3*(
2*a^2*cos(2*x) + 2*a^2*cos(x) + a^2)*cos(3*x)^2 + 6*a^2*cos(x) + (a^2*cos(3
*x) + a^2)*sin(3*x)^2 + a^2 + 3*(3*a^2*cos(2*x)^2 + 3*a^2*cos(x)^2 + 3*a^2*
sin(2*x)^2 + 6*a^2*sin(2*x)*sin(x) + 3*a^2*sin(x)^2 + 4*a^2*cos(x) + a^2 +
2*(3*a^2*cos(x) + 2*a^2)*cos(2*x))*cos(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*
x) + 6*(a^2*sin(2*x) + a^2*sin(x) + (a^2*sin(2*x) + a^2*sin(x))*cos(3*x))*s
in(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6
*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) +
6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*
```

```
x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)
^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*
x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^
2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x
) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) +
3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x)
)*sin(3*x)), x) - 8*((3*x*cos(3/2*x) + 2*sin(3/2*x))*cos(3*x) + 3*(x - 2*si
n(x))*cos(3/2*x) - 6*cos(3/2*x)*sin(2*x) + 2*sin(3/2*x))*sin(3*x) - 24*(3*(
x - 2*sin(x))*cos(3/2*x) + 2*sin(3/2*x))*sin(2*x) + 24*(3*x*cos(x) + x - 2*
sin(x))*sin(3/2*x))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) - 2*(8*(3*x*si
n(3/2*x) - cos(3/2*x))*cos(3*x)^2 + 72*cos(2*x)^2*cos(3/2*x) + 8*cos(3/2*x)
*sin(3*x)^2 + 72*cos(3/2*x)*sin(2*x)^2 + 8*(9*x*cos(2*x)*sin(3/2*x) - (9*x*
sin(x) + 2)*cos(3/2*x) - 3*(3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(2*x) + 3*(3*
x*cos(x) + 2*x - 2*sin(x))*sin(3/2*x))*cos(3*x) + 72*(2*cos(3/2*x)*cos(x) +
 x*sin(3/2*x))*cos(2*x) + 8*(9*cos(x)^2 - 9*x*sin(x) + 9*sin(x)^2 - 1)*cos(
3/2*x) - 3*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(
x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*
x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x)
 + 1)*cos(1/3*arctan2(sin(3/2*x), cos(3/2*x))) - 216*(a^2*cos(3*x)^3 + 9*a^
2*cos(2*x)^2 + 9*a^2*cos(x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) +
 9*a^2*sin(x)^2 + 3*(2*a^2*cos(2*x) + 2*a^2*cos(x) + a^2)*cos(3*x)^2 + 6*a^
2*cos(x) + (a^2*cos(3*x) + a^2)*sin(3*x)^2 + a^2 + 3*(3*a^2*cos(2*x)^2 + 3*
a^2*cos(x)^2 + 3*a^2*sin(2*x)^2 + 6*a^2*sin(2*x)*sin(x) + 3*a^2*sin(x)^2 +
4*a^2*cos(x) + a^2 + 2*(3*a^2*cos(x) + 2*a^2)*cos(2*x))*cos(3*x) + 6*(3*a^2
*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x) + (a^2*sin(2*x) + a^
2*sin(x))*cos(3*x))*sin(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*co
s(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*s
in(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x
*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2
*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2
*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(
4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*c
os(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) +
 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*si
n(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - 8*((3*x*cos(3/2*x) + 2*sin(3/2*x))*c
os(3*x) + 3*(x - 2*sin(x))*cos(3/2*x) - 6*cos(3/2*x)*sin(2*x) + 2*sin(3/2*x
))*sin(3*x) - 24*(3*(x - 2*sin(x))*cos(3/2*x) + 2*sin(3/2*x))*sin(2*x) + 24
*(3*x*cos(x) + x - 2*sin(x))*sin(3/2*x))*cos(2/3*arctan2(sin(3/2*x), cos(3/
2*x))) + 2*(3*(2*cos(2*x) + 2*cos(x) + 1)*cos(3*x)^2 + cos(3*x)^3 + (cos(3*
x) + 1)*sin(3*x)^2 + 3*(2*(3*cos(x) + 2)*cos(2*x) + 3*cos(2*x)^2 + 3*cos(x)
^2 + 3*sin(2*x)^2 + 6*sin(2*x)*sin(x) + 3*sin(x)^2 + 4*cos(x) + 1)*cos(3*x)
 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*((sin(2*x) + s
in(x))*cos(3*x) + sin(2*x) + sin(x))*sin(3*x) + 9*sin(2*x)^2 + 18*sin(2*x)*
sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(1/3*arctan2(sin(3/2*x), cos(3/2*x))
) + 72*(a^2*cos(3*x)^4 + a^2*sin(3*x)^4 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x)
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+ 2*a^2)*cos(3*x)^3 + 9*a^2*cos(2*x)^2 + 9*a^2*cos(x)^2 + 6*(a^2*sin(2*x) +
 a^2*sin(x))*sin(3*x)^3 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2
*sin(x)^2 + 3*(3*a^2*cos(2*x)^2 + 3*a^2*cos(x)^2 + 3*a^2*sin(2*x)^2 + 6*a^2
*sin(2*x)*sin(x) + 3*a^2*sin(x)^2 + 6*a^2*cos(x) + 2*a^2 + 6*(a^2*cos(x) +
a^2)*cos(2*x))*cos(3*x)^2 + 6*a^2*cos(x) + (2*a^2*cos(3*x)^2 + 9*a^2*cos(2*
x)^2 + 9*a^2*cos(x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2*s
in(x)^2 + 6*a^2*cos(x) + 2*a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + 2*a^2)*
cos(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x))*sin(3*x)^2 + a^2 + 2*(9*a^2*cos
(2*x)^2 + 9*a^2*cos(x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^
2*sin(x)^2 + 9*a^2*cos(x) + 2*a^2 + 9*(2*a^2*cos(x) + a^2)*cos(2*x))*cos(3*
x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*((a^2*sin(2*x) + a^2*sin(x))*cos(3
*x)^2 + a^2*sin(2*x) + a^2*sin(x) + 2*(a^2*sin(2*x) + a^2*sin(x))*cos(3*x))
*sin(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) +
 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x)
+ 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/
2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(
x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(
2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*
a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos
(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x)
+ 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(
x))*sin(3*x)), x) + 8*(x*cos(3*x)^2*cos(3/2*x) - 4*(3*cos(x) + 1)*cos(2*x)*
sin(3/2*x) - 6*cos(2*x)^2*sin(3/2*x) - 6*sin(2*x)^2*sin(3/2*x) + 2*((x - 2*
sin(x))*cos(3/2*x) - 2*cos(3/2*x)*sin(2*x) - 2*cos(2*x)*sin(3/2*x) - 2*cos(
x)*sin(3/2*x))*cos(3*x) + (x - 4*sin(x))*cos(3/2*x) - 4*(3*sin(3/2*x)*sin(x
) + cos(3/2*x))*sin(2*x) - 2*(3*cos(x)^2 + 3*sin(x)^2 + 2*cos(x))*sin(3/2*x
))*sin(3*x) + 8*(3*(x - 4*sin(x))*cos(3/2*x) + 2*sin(3/2*x))*sin(2*x) - 8*(
3*x*cos(x) + x - 2*sin(x))*sin(3/2*x) - 2*(6*(sin(2*x) + sin(x))*sin(3*x)^2
 + sin(3*x)^3 + (2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3
*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2
*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(3*x) + 3*(2*(3*cos(2*x) + 3*cos
(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 +
 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 +
18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(4/3*arctan2(sin(3/2*x),
 cos(3/2*x))) + 3*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*
(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))
*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6
*cos(x) + 1)*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x))))*sin(5/3*arctan2(sin(
3/2*x), cos(3/2*x))) + 2*(8*(3*x*cos(3/2*x) + sin(3/2*x))*sin(3*x)^2 - 8*co
s(3*x)^2*sin(3/2*x) - 48*(3*cos(x) + 1)*cos(2*x)*sin(3/2*x) - 72*cos(2*x)^2
*sin(3/2*x) - 72*sin(2*x)^2*sin(3/2*x) - 144*sin(2*x)*sin(3/2*x)*sin(x) - 1
6*((3*cos(x) + 1)*sin(3/2*x) + 3*cos(2*x)*sin(3/2*x))*cos(3*x) + 216*(a^2*s
in(3*x)^3 + 6*(a^2*sin(2*x) + a^2*sin(x))*sin(3*x)^2 + (a^2*cos(3*x)^2 + 9*
a^2*cos(2*x)^2 + 9*a^2*cos(x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x)
 + 9*a^2*sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) +
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 a^2)*cos(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x))*sin(3*x))*integrate(1/3*(
x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*x) +
 x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2*x)
+ 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16
*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16
*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x
)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x
) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(
4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*s
in(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + 8*(9*x
*cos(3/2*x)*sin(2*x) - (3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(3*x) - 3*(3*x*si
n(3/2*x) - 2*cos(3/2*x))*cos(2*x) + (9*x*sin(x) + 6*cos(x) + 2)*cos(3/2*x)
- 3*(3*x*cos(x) + x)*sin(3/2*x))*sin(3*x) - 8*(9*cos(x)^2 + 9*sin(x)^2 + 6*
cos(x) + 1)*sin(3/2*x) - 24*((3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(3*x) + 3*(
3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(x) + 6*cos(x) + 2)*cos(3
/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*a^2*cos(x)^2 + a^2*sin(3*
x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x) + 9*a^2*sin(x)^2 + 6*a^2*c
os(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) + a^2)*cos(3*x) + 6*(3*a^2*c
os(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*sin(x))*sin(3*x))*integrate(1
/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2*
x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/2
*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2
+ 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2
+ 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*s
in(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*c
os(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) +
12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a
^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) - (3
*x*cos(3/2*x) + 2*sin(3/2*x))*sin(3*x) - 3*(3*x*cos(3/2*x) + 2*sin(3/2*x))*
sin(2*x) + 3*(3*x*cos(x) + x - 2*sin(x))*sin(3/2*x))*sin(2/3*arctan2(sin(3/
2*x), cos(3/2*x))) + 3*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2
 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + si
n(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^
2 + 6*cos(x) + 1)*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))))*sin(4/3*arctan2
(sin(3/2*x), cos(3/2*x))) + 2*(8*(3*x*cos(3/2*x) + sin(3/2*x))*sin(3*x)^2 -
 8*cos(3*x)^2*sin(3/2*x) - 48*(3*cos(x) + 1)*cos(2*x)*sin(3/2*x) - 72*cos(2
*x)^2*sin(3/2*x) - 72*sin(2*x)^2*sin(3/2*x) - 144*sin(2*x)*sin(3/2*x)*sin(x
) - 16*((3*cos(x) + 1)*sin(3/2*x) + 3*cos(2*x)*sin(3/2*x))*cos(3*x) + 216*(
a^2*sin(3*x)^3 + 6*(a^2*sin(2*x) + a^2*sin(x))*sin(3*x)^2 + (a^2*cos(3*x)^2
 + 9*a^2*cos(2*x)^2 + 9*a^2*cos(x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*s
in(x) + 9*a^2*sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos
(x) + a^2)*cos(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x))*sin(3*x))*integrate(
1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3/2*x) + 6*x*cos(2*x)*cos(3/2
*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin(3/2*x) + 6*x*sin(2*x)*sin(3/
2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) + x)*cos(3/2*x))/(a^2*cos(4*x)^2
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 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 16*a^2*cos(x)^2 + a^2*sin(4*x)^2
 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 + 48*a^2*sin(2*x)*sin(x) + 16*a^2*
sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*cos(3*x) + 6*a^2*cos(2*x) + 4*a^2*
cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(3*x) +
 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*
a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(3*x)), x) + 8
*(9*x*cos(3/2*x)*sin(2*x) - (3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(3*x) - 3*(3
*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) + (9*x*sin(x) + 6*cos(x) + 2)*cos(3/
2*x) - 3*(3*x*cos(x) + x)*sin(3/2*x))*sin(3*x) - 8*(9*cos(x)^2 + 9*sin(x)^2
 + 6*cos(x) + 1)*sin(3/2*x) + 3*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + c
os(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(
2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) +
9*sin(x)^2 + 6*cos(x) + 1)*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))))*sin(2/
3*arctan2(sin(3/2*x), cos(3/2*x))) + 2*(6*(sin(2*x) + sin(x))*sin(3*x)^2 +
sin(3*x)^3 + (2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*co
s(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)
*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(3*x))*sin(1/3*arctan2(sin(3/2*x),
cos(3/2*x))))/((sqrt(2)*a*cos(3*x)^4 + sqrt(2)*a*sin(3*x)^4 + 2*(3*sqrt(2)*
a*cos(2*x) + 3*sqrt(2)*a*cos(x) + 2*sqrt(2)*a)*cos(3*x)^3 + 6*(sqrt(2)*a*si
n(2*x) + sqrt(2)*a*sin(x))*sin(3*x)^3 + 9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*
a*cos(x)^2 + 9*sqrt(2)*a*sin(2*x)^2 + 18*sqrt(2)*a*sin(2*x)*sin(x) + 9*sqrt
(2)*a*sin(x)^2 + 3*(3*sqrt(2)*a*cos(2*x)^2 + 3*sqrt(2)*a*cos(x)^2 + 3*sqrt(
2)*a*sin(2*x)^2 + 6*sqrt(2)*a*sin(2*x)*sin(x) + 3*sqrt(2)*a*sin(x)^2 + 6*sq
rt(2)*a*cos(x) + 6*(sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(2*x) + 2*sqrt(2)*a)*c
os(3*x)^2 + 9*(sqrt(2)*a*cos(3*x)^2 + 9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*a*
cos(x)^2 + sqrt(2)*a*sin(3*x)^2 + 9*sqrt(2)*a*sin(2*x)^2 + 18*sqrt(2)*a*sin
(2*x)*sin(x) + 9*sqrt(2)*a*sin(x)^2 + 6*sqrt(2)*a*cos(x) + 2*(3*sqrt(2)*a*c
os(2*x) + 3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(3*x) + 6*(3*sqrt(2)*a*cos(x)
+ sqrt(2)*a)*cos(2*x) + 6*(sqrt(2)*a*sin(2*x) + sqrt(2)*a*sin(x))*sin(3*x)
+ sqrt(2)*a)*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9*(sqrt(2)*a*cos(
3*x)^2 + 9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*a*cos(x)^2 + sqrt(2)*a*sin(3*x)
^2 + 9*sqrt(2)*a*sin(2*x)^2 + 18*sqrt(2)*a*sin(2*x)*sin(x) + 9*sqrt(2)*a*si
n(x)^2 + 6*sqrt(2)*a*cos(x) + 2*(3*sqrt(2)*a*cos(2*x) + 3*sqrt(2)*a*cos(x)
+ sqrt(2)*a)*cos(3*x) + 6*(3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(2*x) + 6*(sq
rt(2)*a*sin(2*x) + sqrt(2)*a*sin(x))*sin(3*x) + sqrt(2)*a)*cos(2/3*arctan2(
sin(3/2*x), cos(3/2*x)))^2 + (2*sqrt(2)*a*cos(3*x)^2 + 9*sqrt(2)*a*cos(2*x)
^2 + 9*sqrt(2)*a*cos(x)^2 + 9*sqrt(2)*a*sin(2*x)^2 + 18*sqrt(2)*a*sin(2*x)*
sin(x) + 9*sqrt(2)*a*sin(x)^2 + 6*sqrt(2)*a*cos(x) + 2*(3*sqrt(2)*a*cos(2*x
) + 3*sqrt(2)*a*cos(x) + 2*sqrt(2)*a)*cos(3*x) + 6*(3*sqrt(2)*a*cos(x) + sq
rt(2)*a)*cos(2*x) + 2*sqrt(2)*a)*sin(3*x)^2 + 9*(sqrt(2)*a*cos(3*x)^2 + 9*s
qrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*a*cos(x)^2 + sqrt(2)*a*sin(3*x)^2 + 9*sqrt(
2)*a*sin(2*x)^2 + 18*sqrt(2)*a*sin(2*x)*sin(x) + 9*sqrt(2)*a*sin(x)^2 + 6*s
qrt(2)*a*cos(x) + 2*(3*sqrt(2)*a*cos(2*x) + 3*sqrt(2)*a*cos(x) + sqrt(2)*a)
*cos(3*x) + 6*(3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(2*x) + 6*(sqrt(2)*a*sin(
2*x) + sqrt(2)*a*sin(x))*sin(3*x) + sqrt(2)*a)*sin(4/3*arctan2(sin(3/2*x),
```

```
cos(3/2*x)))^2 + 9*(sqrt(2)*a*cos(3*x)^2 + 9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(
2)*a*cos(x)^2 + sqrt(2)*a*sin(3*x)^2 + 9*sqrt(2)*a*sin(2*x)^2 + 18*sqrt(2)*
a*sin(2*x)*sin(x) + 9*sqrt(2)*a*sin(x)^2 + 6*sqrt(2)*a*cos(x) + 2*(3*sqrt(2
)*a*cos(2*x) + 3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(3*x) + 6*(3*sqrt(2)*a*co
s(x) + sqrt(2)*a)*cos(2*x) + 6*(sqrt(2)*a*sin(2*x) + sqrt(2)*a*sin(x))*sin(
3*x) + sqrt(2)*a)*sin(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 6*sqrt(2)*a*
cos(x) + 2*(9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*a*cos(x)^2 + 9*sqrt(2)*a*sin
(2*x)^2 + 18*sqrt(2)*a*sin(2*x)*sin(x) + 9*sqrt(2)*a*sin(x)^2 + 9*sqrt(2)*a
*cos(x) + 9*(2*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(2*x) + 2*sqrt(2)*a)*cos(3*
x) + 6*(3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(2*x) + 6*(sqrt(2)*a*cos(3*x)^3
+ 9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*a*cos(x)^2 + 9*sqrt(2)*a*sin(2*x)^2 +
18*sqrt(2)*a*sin(2*x)*sin(x) + 9*sqrt(2)*a*sin(x)^2 + 3*(2*sqrt(2)*a*cos(2*
x) + 2*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(3*x)^2 + (sqrt(2)*a*cos(3*x) + sqr
t(2)*a)*sin(3*x)^2 + 6*sqrt(2)*a*cos(x) + 3*(3*sqrt(2)*a*cos(2*x)^2 + 3*sqr
t(2)*a*cos(x)^2 + 3*sqrt(2)*a*sin(2*x)^2 + 6*sqrt(2)*a*sin(2*x)*sin(x) + 3*
sqrt(2)*a*sin(x)^2 + 4*sqrt(2)*a*cos(x) + 2*(3*sqrt(2)*a*cos(x) + 2*sqrt(2)
*a)*cos(2*x) + sqrt(2)*a)*cos(3*x) + 6*(3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos
(2*x) + 3*(sqrt(2)*a*cos(3*x)^2 + 9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*a*cos(
x)^2 + sqrt(2)*a*sin(3*x)^2 + 9*sqrt(2)*a*sin(2*x)^2 + 18*sqrt(2)*a*sin(2*x
)*sin(x) + 9*sqrt(2)*a*sin(x)^2 + 6*sqrt(2)*a*cos(x) + 2*(3*sqrt(2)*a*cos(2
*x) + 3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(3*x) + 6*(3*sqrt(2)*a*cos(x) + sq
rt(2)*a)*cos(2*x) + 6*(sqrt(2)*a*sin(2*x) + sqrt(2)*a*sin(x))*sin(3*x) + sq
rt(2)*a)*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(sqrt(2)*a*sin(2*x) +
 sqrt(2)*a*sin(x) + (sqrt(2)*a*sin(2*x) + sqrt(2)*a*sin(x))*cos(3*x))*sin(3
*x) + sqrt(2)*a)*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(sqrt(2)*a*co
s(3*x)^3 + 9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*a*cos(x)^2 + 9*sqrt(2)*a*sin(
2*x)^2 + 18*sqrt(2)*a*sin(2*x)*sin(x) + 9*sqrt(2)*a*sin(x)^2 + 3*(2*sqrt(2)
*a*cos(2*x) + 2*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(3*x)^2 + (sqrt(2)*a*cos(3
*x) + sqrt(2)*a)*sin(3*x)^2 + 6*sqrt(2)*a*cos(x) + 3*(3*sqrt(2)*a*cos(2*x)^
2 + 3*sqrt(2)*a*cos(x)^2 + 3*sqrt(2)*a*sin(2*x)^2 + 6*sqrt(2)*a*sin(2*x)*si
n(x) + 3*sqrt(2)*a*sin(x)^2 + 4*sqrt(2)*a*cos(x) + 2*(3*sqrt(2)*a*cos(x) +
2*sqrt(2)*a)*cos(2*x) + sqrt(2)*a)*cos(3*x) + 6*(3*sqrt(2)*a*cos(x) + sqrt(
2)*a)*cos(2*x) + 6*(sqrt(2)*a*sin(2*x) + sqrt(2)*a*sin(x) + (sqrt(2)*a*sin(
2*x) + sqrt(2)*a*sin(x))*cos(3*x))*sin(3*x) + sqrt(2)*a)*cos(2/3*arctan2(si
n(3/2*x), cos(3/2*x))) + 6*((sqrt(2)*a*sin(2*x) + sqrt(2)*a*sin(x))*cos(3*x
)^2 + sqrt(2)*a*sin(2*x) + sqrt(2)*a*sin(x) + 2*(sqrt(2)*a*sin(2*x) + sqrt(
2)*a*sin(x))*cos(3*x))*sin(3*x) + 6*(sqrt(2)*a*sin(3*x)^3 + 6*(sqrt(2)*a*si
n(2*x) + sqrt(2)*a*sin(x))*sin(3*x)^2 + (sqrt(2)*a*cos(3*x)^2 + 9*sqrt(2)*a
*cos(2*x)^2 + 9*sqrt(2)*a*cos(x)^2 + 9*sqrt(2)*a*sin(2*x)^2 + 18*sqrt(2)*a*
sin(2*x)*sin(x) + 9*sqrt(2)*a*sin(x)^2 + 6*sqrt(2)*a*cos(x) + 2*(3*sqrt(2)*
a*cos(2*x) + 3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(3*x) + 6*(3*sqrt(2)*a*cos(
x) + sqrt(2)*a)*cos(2*x) + sqrt(2)*a)*sin(3*x) + 3*(sqrt(2)*a*cos(3*x)^2 +
9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*a*cos(x)^2 + sqrt(2)*a*sin(3*x)^2 + 9*sq
rt(2)*a*sin(2*x)^2 + 18*sqrt(2)*a*sin(2*x)*sin(x) + 9*sqrt(2)*a*sin(x)^2 +
6*sqrt(2)*a*cos(x) + 2*(3*sqrt(2)*a*cos(2*x) + 3*sqrt(2)*a*cos(x) + sqrt(2)
```

```
*a)*cos(3*x) + 6*(3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(2*x) + 6*(sqrt(2)*a*s
in(2*x) + sqrt(2)*a*sin(x))*sin(3*x) + sqrt(2)*a)*sin(2/3*arctan2(sin(3/2*x
), cos(3/2*x))))*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 6*(sqrt(2)*a*si
n(3*x)^3 + 6*(sqrt(2)*a*sin(2*x) + sqrt(2)*a*sin(x))*sin(3*x)^2 + (sqrt(2)*
a*cos(3*x)^2 + 9*sqrt(2)*a*cos(2*x)^2 + 9*sqrt(2)*a*cos(x)^2 + 9*sqrt(2)*a*
sin(2*x)^2 + 18*sqrt(2)*a*sin(2*x)*sin(x) + 9*sqrt(2)*a*sin(x)^2 + 6*sqrt(2
)*a*cos(x) + 2*(3*sqrt(2)*a*cos(2*x) + 3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(
3*x) + 6*(3*sqrt(2)*a*cos(x) + sqrt(2)*a)*cos(2*x) + sqrt(2)*a)*sin(3*x))*s
in(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + sqrt(2)*a)*sqrt(a))
```

Giac [F]

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+a*cos(x))^(3/2),x, algorithm="giac")
[Out] integrate(x/(a*cos(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x}{(a+a\,\cos(x))^{3/2}}\,dx$$

```
[In] int(x/(a + a*cos(x))^(3/2),x)
[Out] int(x/(a + a*cos(x))^(3/2), x)
```

3.183 $\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$

Optimal result . . . 1118
Rubi [N/A] . . . 1118
Mathematica [N/A] . . . 1119
Maple [N/A] (verified) . . . 1119
Fricas [N/A] . . . 1119
Sympy [N/A] . . . 1119
Maxima [N/A] . . . 1120
Giac [N/A] . . . 1120
Mupad [N/A] . . . 1120

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x(a+a\cos(x))^{3/2}}, x\right)$$

```
[Out] Unintegrable(1/x/(a+a*cos(x))^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

```
[In] Int[1/(x*(a + a*Cos[x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*(a + a*Cos[x])^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

```
[In] Integrate[1/(x*(a + a*Cos[x])^(3/2)),x]
[Out] Integrate[1/(x*(a + a*Cos[x])^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+\cos\left(x\right)a\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/x/(a+cos(x)*a)^(3/2),x)
[Out] int(1/x/(a+cos(x)*a)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{\left(a\cos\left(x\right)+a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cos(x) + a)/(a^2*x*cos(x)^2 + 2*a^2*x*cos(x) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 9.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{x\left(a\left(\cos\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(a+a*cos(x))**(3/2),x)
[Out] Integral(1/(x*(a*(cos(x) + 1))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{(a\cos(x)+a)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(x) + a)^(3/2)*x), x)
```

Giac [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{(a\cos(x)+a)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*cos(x) + a)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 14.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{x\left(a+a\cos(x)\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(a + a*cos(x))^(3/2)),x)
[Out] int(1/(x*(a + a*cos(x))^(3/2)), x)
```

3.184 $\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$

Optimal result 1121
Rubi [N/A] 1121
Mathematica [N/A] 1122
Maple [N/A] (verified) 1122
Fricas [F(-2)] 1122
Sympy [N/A] 1122
Maxima [N/A] 1123
Giac [N/A] 1123
Mupad [N/A] 1123

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \text{Int}\left(\frac{\sqrt[3]{a+a\cos(c+dx)}}{x}, x\right)$$

```
[Out] Unintegrable((a+a*cos(d*x+c))^(1/3)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

```
[In] Int[(a + a*Cos[c + d*x])^(1/3)/x,x]
```

```
[Out] Defer[Int][(a + a*Cos[c + d*x])^(1/3)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3)/x,x]
[Out] Integrate[(a + a*Cos[c + d*x])^(1/3)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a+\cos(dx+c)\,a)^{\frac{1}{3}}}{x}dx$$

```
[In] int((a+cos(d*x+c)*a)^(1/3)/x,x)
[Out] int((a+cos(d*x+c)*a)^(1/3)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a\left(\cos(c+dx)+1\right)}}{x}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/3)/x,x)
[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{(a\cos(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^(1/3)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{(a\cos(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

```
[In] integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + a)^(1/3)/x, x)
```

Mupad [N/A]

Not integrable

Time = 14.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{(a+a\,\cos(c+d\,x))^{1/3}}{x}\,dx$$

```
[In] int((a + a*cos(c + d*x))^(1/3)/x,x)
[Out] int((a + a*cos(c + d*x))^(1/3)/x, x)
```

3.185 $\int \frac{x^3}{a+b\cos(x)}\,dx$

Optimal result . . . 1124
Rubi [A] (verified) . . . 1125
Mathematica [A] (verified) . . . 1128
Maple [F] . . . 1128
Fricas [B] (verification not implemented) . . . 1129
Sympy [F] . . . 1130
Maxima [F(-2)] . . . 1130
Giac [F] . . . 1130
Mupad [F(-1)] . . . 1130

Optimal result

Integrand size = 12, antiderivative size = 383

$$\begin{aligned}\int \frac{x^3}{a+b\cos(x)}\,dx = &-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\\ &-\frac{3x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{3x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\\ &-\frac{6ix\,\mathrm{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{6ix\,\mathrm{PolyLog}\left(3,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\\ &+\frac{6\,\mathrm{PolyLog}\left(4,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{6\,\mathrm{PolyLog}\left(4,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

```
[Out] -I*x^3*ln(1+b*exp(I*x)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)+I*x^3*ln(1+b*ex
p(I*x)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)-3*x^2*polylog(2,-b*exp(I*x)/(a-
(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)+3*x^2*polylog(2,-b*exp(I*x)/(a+(a^2-b^2)^
(1/2)))/(a^2-b^2)^(1/2)-6*I*x*polylog(3,-b*exp(I*x)/(a-(a^2-b^2)^(1/2)))/(a
^2-b^2)^(1/2)+6*I*x*polylog(3,-b*exp(I*x)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1
/2)+6*polylog(4,-b*exp(I*x)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)-6*polylog(
4,-b*exp(I*x)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3402, 2296, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{x^3}{a+b\cos(x)}\,dx = &-\frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\\ &-\frac{6ix\,\text{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{6ix\,\text{PolyLog}\left(3,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\\ &+\frac{6\,\text{PolyLog}\left(4,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{6\,\text{PolyLog}\left(4,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\\ &-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{ix^3\log\left(1+\frac{be^{ix}}{\sqrt{a^2-b^2}+a}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

```
[In] Int[x^3/(a + b*Cos[x]),x]
```

```
[Out] ((-I)*x^3*Log[1 + (b*E^(I*x))/(a - Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] + (I*
x^3*Log[1 + (b*E^(I*x))/(a + Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] - (3*x^2*Po
lyLog[2, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + (3*x^2*Po
lyLog[2, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] - ((6*I)*x*
PolyLog[3, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + ((6*I)*
x*PolyLog[3, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] + (6*Po
lyLog[4, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2] - (6*PolyLo
g[4, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{e^{ix}x^3}{b+2ae^{ix}+be^{2ix}}\,dx \\ &= \frac{(2b)\int \frac{e^{ix}x^3}{2a-2\sqrt{a^2-b^2}+2be^{ix}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{ix}x^3}{2a+2\sqrt{a^2-b^2}+2be^{ix}}\,dx}{\sqrt{a^2-b^2}}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{ix^3 \log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3 \log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&\quad + \frac{(3i)\int x^2 \log\left(1+\frac{2be^{ix}}{2a-2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}} - \frac{(3i)\int x^2 \log\left(1+\frac{2be^{ix}}{2a+2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}} \\
&= -\frac{ix^3 \log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3 \log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&\quad - \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&\quad + \frac{6\int x \operatorname{PolyLog}\left(2,-\frac{2be^{ix}}{2a-2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}} - \frac{6\int x \operatorname{PolyLog}\left(2,-\frac{2be^{ix}}{2a+2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}} \\
&= -\frac{ix^3 \log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3 \log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&\quad - \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&\quad - \frac{6ix \operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{6ix \operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&\quad + \frac{(6i)\int \operatorname{PolyLog}\left(3,-\frac{2be^{ix}}{2a-2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}} - \frac{(6i)\int \operatorname{PolyLog}\left(3,-\frac{2be^{ix}}{2a+2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}} \\
&= -\frac{ix^3 \log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3 \log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&\quad + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{6ix \operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \\
&\quad + \frac{6ix \operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{6\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,\frac{bx}{-a+\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{ix}\right)}{\sqrt{a^2-b^2}} \\
&\quad - \frac{6\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,-\frac{bx}{a+\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{ix}\right)}{\sqrt{a^2-b^2}}
\end{aligned}
$$

$$= -\frac{ix^3 \log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{ix^3 \log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$
$$- \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$
$$- \frac{6ix \operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{6ix \operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$
$$+ \frac{6 \operatorname{PolyLog}\left(4,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{6 \operatorname{PolyLog}\left(4,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{a+b\cos(x)}\,dx$$
$$= \frac{-ix^3 \log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right) + ix^3 \log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right) - 3x^2 \operatorname{PolyLog}\left(2,\frac{be^{ix}}{-a+\sqrt{a^2-b^2}}\right) + 3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a+\sqrt{}}\right.}{}$$

```
[In] Integrate[x^3/(a + b*Cos[x]),x]
```

```
[Out] ((-I)*x^3*Log[1 + (b*E^(I*x))/(a - Sqrt[a^2 - b^2])] + I*x^3*Log[1 + (b*E^(
I*x))/(a + Sqrt[a^2 - b^2])] - 3*x^2*PolyLog[2, (b*E^(I*x))/(-a + Sqrt[a^2
 - b^2])] + 3*x^2*PolyLog[2, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))] - (6*I)*x
*PolyLog[3, (b*E^(I*x))/(-a + Sqrt[a^2 - b^2])] + (6*I)*x*PolyLog[3, -((b*E
^(I*x))/(a + Sqrt[a^2 - b^2]))] + 6*PolyLog[4, (b*E^(I*x))/(-a + Sqrt[a^2 -
 b^2])] - 6*PolyLog[4, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^2 - b^
2]
```

Maple [F]

$$\int \frac{x^3}{a+\cos(x)\,b}dx$$

```
[In] int(x^3/(a+cos(x)*b),x)
```

```
[Out] int(x^3/(a+cos(x)*b),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(317) = 634$.

Time $= 0.36$ (sec) , antiderivative size $= 1030$, normalized size of antiderivative $= 2.69$

$$\int \frac{x^3}{a + b\cos(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b*x^3*sqrt((a^2 - b^2)/b^2)*log((a*cos(x) + I*a*sin(x) + (b*cos(x)
+ I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b) + I*b*x^3*sqrt((a^2 - b^2)/b^2)
*log((a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2)
 + b)/b) + I*b*x^3*sqrt((a^2 - b^2)/b^2)*log((a*cos(x) - I*a*sin(x) + (b*co
s(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b) - I*b*x^3*sqrt((a^2 - b^2)
/b^2)*log((a*cos(x) - I*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)
/b^2) + b)/b) - 3*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(x) + I*a*sin(x)
 + (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 3*b*x^2*sqrt
((a^2 - b^2)/b^2)*dilog(-(a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*sin(x))*s
qrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-(a*
cos(x) - I*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b
+ 1) + 3*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(x) - I*a*sin(x) - (b*cos
(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 6*I*b*x*sqrt((a^2 - b
^2)/b^2)*polylog(3, -(a*cos(x) + I*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(
(a^2 - b^2)/b^2))/b) + 6*I*b*x*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(x)
+ I*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) + 6*I*b*x*
sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(x) - I*a*sin(x) + (b*cos(x) - I*b*
sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 6*I*b*x*sqrt((a^2 - b^2)/b^2)*polylog(3
, -(a*cos(x) - I*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/
b) + 6*b*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cos(x) + I*a*sin(x) + (b*cos(
x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 6*b*sqrt((a^2 - b^2)/b^2)*poly
log(4, -(a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b
^2))/b) + 6*b*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cos(x) - I*a*sin(x) + (b
*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 6*b*sqrt((a^2 - b^2)/b^2)
*polylog(4, -(a*cos(x) - I*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b
^2)/b^2))/b))/(a^2 - b^2)
```

Sympy [F]

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \int \frac{x^3}{a+b\cos\left(x\right)}\,dx$$

```
[In] integrate(x**3/(a+b*cos(x)),x)
[Out] Integral(x**3/(a + b*cos(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*cos(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \int \frac{x^3}{b\cos\left(x\right)+a}\,dx$$

```
[In] integrate(x^3/(a+b*cos(x)),x, algorithm="giac")
[Out] integrate(x^3/(b*cos(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \int \frac{x^3}{a+b\,\cos\left(x\right)}\,dx$$

```
[In] int(x^3/(a + b*cos(x)),x)
[Out] int(x^3/(a + b*cos(x)), x)
```

3.186 $\int \frac{x^2}{a+b\cos(c+dx)}\,dx$

Optimal result . . . 1131
Rubi [A] (verified) . . . 1131
Mathematica [A] (verified) . . . 1134
Maple [F] . . . 1135
Fricas [B] (verification not implemented) . . . 1135
Sympy [F] . . . 1136
Maxima [F(-2)] . . . 1136
Giac [F] . . . 1136
Mupad [F(-1)] . . . 1137

Optimal result

Integrand size = 16, antiderivative size = 329

$$\begin{aligned}\int \frac{x^2}{a+b\cos(c+dx)}\,dx = &-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}+\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}\\ &-\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}+\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}\\ &-\frac{2i\,\mathrm{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3}+\frac{2i\,\mathrm{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3}\end{aligned}$$

```
[Out] -I*x^2*ln(1+b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)+I*x^2*l
n(1+b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)-2*x*polylog(2,-
b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)+2*x*polylog(2,-b*
exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)-2*I*polylog(3,-b*ex
p(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2)+2*I*polylog(3,-b*exp(
I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{3402, 2296, 2221, 2611, 2320, 6724\}$

$$\int \frac{x^2}{a+b\cos(c+dx)}\,dx = -\frac{2i\,\text{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}} + \frac{2i\,\text{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}} - \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}} + \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}} - \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d\sqrt{a^2-b^2}}$$

```
[In] Int[x^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-I)*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b
^2]*d) + (I*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) - (2*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))
])/(Sqrt[a^2 - b^2]*d^2) + (2*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[
a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^2) - ((2*I)*PolyLog[3, -((b*E^(I*(c + d*x
)))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^3) + ((2*I)*PolyLog[3, -((b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = 2\int \frac{e^{i(c+dx)}x^2}{b+2ae^{i(c+dx)}+be^{2i(c+dx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{i(c+dx)}x^2}{2a-2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{i(c+dx)}x^2}{2a+2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$
$$+ \frac{(2i)\int x\log\left(1+\frac{2be^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}d} - \frac{(2i)\int x\log\left(1+\frac{2be^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}d}$$

$$= -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$
$$- \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}$$
$$+ \frac{2\int \text{PolyLog}\left(2,-\frac{2be^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}d^2} - \frac{2\int \text{PolyLog}\left(2,-\frac{2be^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)\,dx}{\sqrt{a^2-b^2}d^2}$$

$$= -\frac{ix^2 \log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix^2 \log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} - \frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}$$
$$+ \frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} - \frac{(2i)\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}\left(2,\frac{bx}{-a+\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx)}\right)}{\sqrt{a^2-b^2}d^3}$$
$$+ \frac{(2i)\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}\left(2,-\frac{bx}{a+\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx)}\right)}{\sqrt{a^2-b^2}d^3}$$
$$= -\frac{ix^2 \log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix^2 \log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} - \frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}$$
$$+ \frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} - \frac{2i\,\mathrm{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3} + \frac{2i\,\mathrm{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{a+b\cos(c+dx)}\,dx$$
$$= \frac{-2dx\,\mathrm{PolyLog}\left(2,\frac{be^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right) - i\left(d^2x^2\log\left(1-\frac{be^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right) - d^2x^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right) + 2idx\,\mathrm{PolyLog}\left(2\right.\right.}{\sqrt{a^2-b^2}d^3}$$

```
[In] Integrate[x^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*d*x*PolyLog[2, (b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - I*(d^2*x^2
*Log[1 - (b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - d^2*x^2*Log[1 + (b*E
^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + (2*I)*d*x*PolyLog[2, -((b*E^(I*(c
+ d*x)))/(a + Sqrt[a^2 - b^2]))] + 2*PolyLog[3, (b*E^(I*(c + d*x)))/(-a + S
qrt[a^2 - b^2])] - 2*PolyLog[3, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])
)]))/(Sqrt[a^2 - b^2]*d^3)
```

Maple [F]

$$\int \frac{x^2}{a+\cos\left(dx+c\right)b}dx$$

```
[In] int(x^2/(a+cos(d*x+c)*b),x)
```

```
[Out] int(x^2/(a+cos(d*x+c)*b),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(283) = 566$.

Time $= 0.43$ (sec) , antiderivative size $= 1263$, normalized size of antiderivative $= 3.84$

$$\int \frac{x^2}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1)
- 2*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) -
 (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 2*
b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c) + (b*
cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 2*b*d*
x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c) - (b*cos(
d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - I*b*c^2*sq
rt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a
^2 - b^2)/b^2) + 2*a) + I*b*c^2*sqrt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c)
- 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - I*b*c^2*sqrt((a^2
 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b
^2)/b^2) - 2*a) + I*b*c^2*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) - 2*I
*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) + (I*b*d^2*x^2 - I*b*c^2
)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x + c) + (b*cos(d*x
 + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (-I*b*d^2*x^2 + I
*b*c^2)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x + c) - (b*c
os(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (-I*b*d^2*x
^2 + I*b*c^2)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) - I*a*sin(d*x + c)
+ (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (I*b*
d^2*x^2 - I*b*c^2)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) - I*a*sin(d*x
+ c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) +
2*I*b*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(d*x + c) + I*a*sin(d*x + c)
+ (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 2*I*b*sqr
t((a^2 - b^2)/b^2)*polylog(3, -(a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(
```

```
d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 2*I*b*sqrt((a^2 -
b^2)/b^2)*polylog(3, -(a*cos(d*x + c) - I*a*sin(d*x + c) + (b*cos(d*x + c)
- I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 2*I*b*sqrt((a^2 - b^2)/b^2)
*polylog(3, -(a*cos(d*x + c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin
(d*x + c))*sqrt((a^2 - b^2)/b^2))/b))/((a^2 - b^2)*d^3)
```

Sympy [F]

$$\int \frac{x^2}{a + b\cos(c + dx)}\,dx = \int \frac{x^2}{a + b\cos\left(c + dx\right)}\,dx$$

```
[In] integrate(x**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(x**2/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a + b\cos(c + dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^2}{a + b\cos(c + dx)}\,dx = \int \frac{x^2}{b\cos\left(dx + c\right) + a}\,dx$$

```
[In] integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\cos(c+dx)}\,dx = \int \frac{x^2}{a+b\,\cos(c+d\,x)}\,dx$$

```
[In] int(x^2/(a + b*cos(c + d*x)),x)
[Out] int(x^2/(a + b*cos(c + d*x)), x)
```

3.187 $\int \frac{x}{a+b\cos(c+dx)}\,dx$

Optimal result . . . 1138
Rubi [A] (verified) . . . 1138
Mathematica [B] (verified) . . . 1140
Maple [B] (verified) . . . 1141
Fricas [B] (verification not implemented) . . . 1141
Sympy [F] . . . 1142
Maxima [F(-2)] . . . 1142
Giac [F] . . . 1143
Mupad [F(-1)] . . . 1143

Optimal result

Integrand size = 14, antiderivative size = 214

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} - \frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}$$

```
[Out] -I*x*ln(1+b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)+I*x*ln(1+
b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)-polylog(2,-b*exp(I*
(d*x+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)+polylog(2,-b*exp(I*(d*x+c
))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3402, 2296, 2221, 2317, 2438}

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = -\frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}} + \frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}} - \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d\sqrt{a^2-b^2}}$$

```
[In] Int[x/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-I)*x*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2
]*d) + (I*x*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 -
```

```
 b^2]*d) - PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))]/(Sqrt[a
^2 - b^2]*d^2) + PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))]/(
Sqrt[a^2 - b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\int \frac{e^{i(c+dx)}x}{b+2ae^{i(c+dx)}+be^{2i(c+dx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{\sqrt{a^2-b^2}}$$

$$
\begin{aligned}
&= -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} \\
&\quad + \frac{i\int \log\left(1+\frac{2be^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)dx}{\sqrt{a^2-b^2}d} - \frac{i\int \log\left(1+\frac{2be^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)dx}{\sqrt{a^2-b^2}d} \\
&= -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{2a-2\sqrt{a^2-b^2}}\right)}{x}dx, x, e^{i(c+dx)}\right)}{\sqrt{a^2-b^2}d^2} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{2a+2\sqrt{a^2-b^2}}\right)}{x}dx, x, e^{i(c+dx)}\right)}{\sqrt{a^2-b^2}d^2} \\
&= -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} + \frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} \\
&\quad - \frac{\operatorname{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{\operatorname{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 756 vs. $2(214) = 428$.

Time $= 0.94$ (sec) , antiderivative size $= 756$, normalized size of antiderivative $= 3.53$

$$
\begin{aligned}
&\int \frac{x}{a+b\cos(c+dx)}\,dx \\
&= \frac{2(c+dx)\operatorname{arctanh}\left(\frac{(a+b)\cot\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right) - 2\left(c+\arccos\left(-\frac{a}{b}\right)\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right) + \left(\arccos\left(-\frac{a}{b}\right) - 2\right.}{}
\end{aligned}
$$

```
[In] Integrate[x/(a + b*Cos[c + d*x]),x]

[Out] (2*(c + d*x)*ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]] - 2*(c +
ArcCos[-(a/b)])*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]] + (Ar
cCos[-(a/b)] - (2*I)*ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]] +
 (2*I)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])*Log[Sqrt[-a^2
 + b^2]/(Sqrt[2]*Sqrt[b]*E^((I/2)*(c + d*x))*Sqrt[a + b*Cos[c + d*x]])] + (
ArcCos[-(a/b)] + (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]
] - ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[(Sqrt[-a^2
```

```
+ b^2]*E^((I/2)*(c + d*x)))/(Sqrt[2]*Sqrt[b]*Sqrt[a + b*Cos[c + d*x]])] - (
ArcCos[-(a/b)] - (2*I)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])*Log[((a + b)*(-a + b - I*Sqrt[-a^2 + b^2])*(1 + I*Tan[(c + d*x)/2]))/(b*
(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2]))] - (ArcCos[-(a/b)] + (2*I)*Arc
Tanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])*Log[((a + b)*(I*a - I*b
 + Sqrt[-a^2 + b^2])*(I + Tan[(c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*T
an[(c + d*x)/2]))] + I*(PolyLog[2, ((a - I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[
-a^2 + b^2]*Tan[(c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2
]))] - PolyLog[2, ((a + I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-a^2 + b^2]*Tan[(
c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2]))]))/(Sqrt[-a^2
 + b^2]*d^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(188) = 376$.

Time = 0.84 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.93

method	result
risch	$-\frac{i\ln\left(\frac{-b\,e^{i(dx+c)}+\sqrt{a^2-b^2}-a}{\sqrt{a^2-b^2}-a}\right)x}{d\sqrt{a^2-b^2}}+\frac{i\ln\left(\frac{b\,e^{i(dx+c)}+\sqrt{a^2-b^2}+a}{\sqrt{a^2-b^2}+a}\right)x}{d\sqrt{a^2-b^2}}-\frac{i\ln\left(\frac{-b\,e^{i(dx+c)}+\sqrt{a^2-b^2}-a}{\sqrt{a^2-b^2}-a}\right)c}{d^2\sqrt{a^2-b^2}}+\frac{i\ln\left(\frac{b\,e^{i(dx+c)}+\sqrt{a^2-b^2}+a}{\sqrt{a^2-b^2}+a}\right.}{d^2\sqrt{a^2-b^2}}$

```
[In] int(x/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d/(a^2-b^2)^(1/2)*ln((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/((a^2-b^2)^(1
/2)-a))*x+I/d/(a^2-b^2)^(1/2)*ln((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/((a^2
-b^2)^(1/2)+a))*x-I/d^2/(a^2-b^2)^(1/2)*ln((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/
2)-a)/((a^2-b^2)^(1/2)-a))*c+I/d^2/(a^2-b^2)^(1/2)*ln((b*exp(I*(d*x+c))+(a^
2-b^2)^(1/2)+a)/((a^2-b^2)^(1/2)+a))*c-1/d^2/(a^2-b^2)^(1/2)*dilog((-b*exp(
I*(d*x+c))+(a^2-b^2)^(1/2)-a)/((a^2-b^2)^(1/2)-a))+1/d^2/(a^2-b^2)^(1/2)*di
log((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/((a^2-b^2)^(1/2)+a))+2*I/d^2*c/(-a
^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(I*(d*x+c))+2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(184) = 368$.

Time = 0.42 (sec) , antiderivative size = 915, normalized size of antiderivative = 4.28

$$\int \frac{x}{a+b\cos(c+dx)}\,dx$$

$$= \frac{-i\,bc\sqrt{\frac{a^2-b^2}{b^2}}\log\left(2\,b\cos(dx+c)+2i\,b\sin(dx+c)+2\,b\sqrt{\frac{a^2-b^2}{b^2}}+2\,a\right)+i\,bc\sqrt{\frac{a^2-b^2}{b^2}}\log\left(2\,b\cos(dx+c\right.}{}$$

```
[In] integrate(x/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b*c*sqrt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c)
 + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + I*b*c*sqrt((a^2 - b^2)/b^2)*log(2*b*c
os(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - I*b*c
*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqr
t((a^2 - b^2)/b^2) - 2*a) + I*b*c*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x +
c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) - b*sqrt((a^2 -
b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) + (b*cos(d*x + c) + I*b
*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + b*sqrt((a^2 - b^2)/b^2)*
dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - b*sqrt((a^2 - b^2)/b^2)*dilog(-(a*
cos(d*x + c) - I*a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt(
(a^2 - b^2)/b^2) + b)/b + 1) + b*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x +
c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2
)/b^2) + b)/b + 1) - (I*b*d*x + I*b*c)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x
 + c) + I*a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 -
b^2)/b^2) + b)/b) - (-I*b*d*x - I*b*c)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x
 + c) + I*a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 -
b^2)/b^2) + b)/b) - (-I*b*d*x - I*b*c)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x
 + c) - I*a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 -
b^2)/b^2) + b)/b) - (I*b*d*x + I*b*c)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x
+ c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b
^2)/b^2) + b)/b))/((a^2 - b^2)*d^2)
```

Sympy [F]

$$\int \frac{x}{a + b\cos(c + dx)}\,dx = \int \frac{x}{a + b\cos\left(c + dx\right)}\,dx$$

```
[In] integrate(x/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(x/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a + b\cos(c + dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = \int \frac{x}{b\cos(dx+c)+a}\,dx$$

```
[In] integrate(x/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate(x/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = \int \frac{x}{a+b\,\cos(c+d\,x)}\,dx$$

```
[In] int(x/(a + b*cos(c + d*x)),x)
[Out] int(x/(a + b*cos(c + d*x)), x)
```

3.188 $\int \frac{1}{x(a+b\cos(x))}\,dx$

Optimal result 1144
Rubi [N/A] 1144
Mathematica [N/A] 1145
Maple [N/A] (verified) 1145
Fricas [N/A] 1145
Sympy [N/A] 1145
Maxima [N/A] 1146
Giac [N/A] 1146
Mupad [N/A] 1146

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \text{Int}\left(\frac{1}{x(a+b\cos(x))}, x\right)$$

```
[Out] Unintegrable(1/x/(a+b*cos(x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{x(a+b\cos(x))}\,dx$$

```
[In] Int[1/(x*(a + b*Cos[x])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Cos[x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(a+b\cos(x))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{x(a+b\cos(x))}\,dx$$

```
[In] Integrate[1/(x*(a + b*Cos[x])),x]
[Out] Integrate[1/(x*(a + b*Cos[x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+\cos\left(x\right)b\right)}dx$$

```
[In] int(1/x/(a+cos(x)*b),x)
[Out] int(1/x/(a+cos(x)*b),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{(b\cos\left(x\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*cos(x)),x, algorithm="fricas")
[Out] integral(1/(b*x*cos(x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{x\left(a+b\cos\left(x\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*cos(x)),x)
[Out] Integral(1/(x*(a + b*cos(x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{(b\cos(x)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*cos(x)),x, algorithm="maxima")
[Out] integrate(1/((b*cos(x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{(b\cos(x)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*cos(x)),x, algorithm="giac")
[Out] integrate(1/((b*cos(x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 14.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{x\ (a+b\ \cos(x))}\,dx$$

```
[In] int(1/(x*(a + b*cos(x))),x)
[Out] int(1/(x*(a + b*cos(x))), x)
```

3.189 $\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx$

Optimal result . . . 1147
Rubi [A] (verified) . . . 1148
Mathematica [B] (warning: unable to verify) . . . 1151
Maple [B] (verified) . . . 1152
Fricas [B] (verification not implemented) . . . 1152
Sympy [F(-1)] . . . 1153
Maxima [F(-2)] . . . 1154
Giac [F] . . . 1154
Mupad [F(-1)] . . . 1154

Optimal result

Integrand size = 18, antiderivative size = 296

$$\begin{aligned}\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = &-\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d}\\ &+\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d}\\ &-\frac{f\log(a+b\cos(c+dx))}{\left(a^2-b^2\right)d^2}-\frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d^2}\\ &+\frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d^2}-\frac{b(e+fx)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

```
[Out] -f*ln(a+b*cos(d*x+c))/(a^2-b^2)/d^2-I*a*(f*x+e)*ln(1+b*exp(I*(d*x+c))/(a-(a
^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+I*a*(f*x+e)*ln(1+b*exp(I*(d*x+c))/(a+(a^2
-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-a*f*polylog(2,-b*exp(I*(d*x+c))/(a-(a^2-b^2
)^(1/2)))/(a^2-b^2)^(3/2)/d^2+a*f*polylog(2,-b*exp(I*(d*x+c))/(a+(a^2-b^2)^
(1/2)))/(a^2-b^2)^(3/2)/d^2-b*(f*x+e)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c
))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3405, 3402, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = -\frac{af\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}} + \frac{af\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}} - \frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)} - \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{d\left(a^2-b^2\right)^{3/2}} - \frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[In] Int[(e + f*x)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-I)*a*(e + f*x)*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*d) + (I*a*(e + f*x)*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2
 - b^2])])/((a^2 - b^2)^(3/2)*d) - (f*Log[a + b*Cos[c + d*x]])/((a^2 - b^2)
*d^2) - (a*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))])/((a^
2 - b^2)^(3/2)*d^2) + (a*f*PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
 b^2]))])/((a^2 - b^2)^(3/2)*d^2) - (b*(e + f*x)*Sin[c + d*x])/((a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{a\int \frac{e+fx}{a+b\cos(c+dx)}\,dx}{a^2-b^2} + \frac{(bf)\int \frac{\sin(c+dx)}{a+b\cos(c+dx)}\,dx}{(a^2-b^2)\,d} \\ &= -\frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{(2a)\int \frac{e^{i(c+dx)}(e+fx)}{b+2ae^{i(c+dx)}+be^{2i(c+dx)}}\,dx}{a^2-b^2} \\ &\quad - \frac{f\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(c+dx)\right)}{(a^2-b^2)\,d^2}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{f\log(a+b\cos(c+dx))}{(a^2-b^2)\,d^2} - \frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} \\
&\quad + \frac{(2ab)\int \frac{e^{i(c+dx)}(e+fx)}{2a-2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{(a^2-b^2)^{3/2}} - \frac{(2ab)\int \frac{e^{i(c+dx)}(e+fx)}{2a+2\sqrt{a^2-b^2}+2be^{i(c+dx)}}\,dx}{(a^2-b^2)^{3/2}} \\
&= -\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} \\
&\quad - \frac{f\log(a+b\cos(c+dx))}{(a^2-b^2)\,d^2} - \frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} \\
&\quad + \frac{(iaf)\int \log\left(1+\frac{2be^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)\,dx}{(a^2-b^2)^{3/2}\,d} - \frac{(iaf)\int \log\left(1+\frac{2be^{i(c+dx)}}{2a+2\sqrt{a^2-b^2}}\right)\,dx}{(a^2-b^2)^{3/2}\,d} \\
&= -\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} \\
&\quad - \frac{f\log(a+b\cos(c+dx))}{(a^2-b^2)\,d^2} - \frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} \\
&\quad + \frac{(af)\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx)}\right)}{(a^2-b^2)^{3/2}\,d^2} \\
&\quad - \frac{(af)\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{2a+2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx)}\right)}{(a^2-b^2)^{3/2}\,d^2} \\
&= -\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} + \frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} \\
&\quad - \frac{f\log(a+b\cos(c+dx))}{(a^2-b^2)\,d^2} - \frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2} \\
&\quad + \frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2} - \frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 933 vs. $2(296) = 592$.

Time = 10.62 (sec) , antiderivative size = 933, normalized size of antiderivative = 3.15

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \frac{-bde\sin(c+dx)+bcf\sin(c+dx)-bf(c+dx)\sin(c+dx)}{(a-b)(a+b)d^2(a+b\cos(c+dx))}$$
$$+\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2a(de-cf)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+f\log\left(\sec^2\left(\frac{1}{2}(c+dx)\right)\right)-f\log\left((a+b\cos(c+dx)\right.\right.}{}$$

```
[In] Integrate[(e + f*x)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-(b*d*e*Sin[c + d*x]) + b*c*f*Sin[c + d*x] - b*f*(c + d*x)*Sin[c + d*x])/(
(a - b)*(a + b)*d^2*(a + b*Cos[c + d*x])) + (Cos[(c + d*x)/2]^2*((2*a*(d*e
- c*f)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]) + f*Log[Sec[(c + d*x)/2]^2] - f*Log[(a + b*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2] - (I*a*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(Sqrt[a + b] - Sqrt[-
a + b]*Tan[(c + d*x)/2])/(I*Sqrt[-a + b] + Sqrt[a + b])] + PolyLog[2, (Sqrt
[-a + b]*(1 - I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] - I*Sqrt[a + b])]))/(Sqrt[
-a + b]*Sqrt[a + b]) + (I*a*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(I*(Sqrt[a +
 b] + Sqrt[-a + b]*Tan[(c + d*x)/2]))/(Sqrt[-a + b] + I*Sqrt[a + b])] + Pol
yLog[2, (Sqrt[-a + b]*(1 - I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] + I*Sqrt[a +
b])]))/(Sqrt[-a + b]*Sqrt[a + b]) - (I*a*f*(Log[1 + I*Tan[(c + d*x)/2]]*Log
[(Sqrt[a + b] + Sqrt[-a + b]*Tan[(c + d*x)/2])/(I*Sqrt[-a + b] + Sqrt[a + b
])] + PolyLog[2, (Sqrt[-a + b]*(1 + I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] - I*
Sqrt[a + b])]))/(Sqrt[-a + b]*Sqrt[a + b]) + (I*a*f*(Log[1 + I*Tan[(c + d*x
)/2]]*Log[(I*(Sqrt[a + b] - Sqrt[-a + b]*Tan[(c + d*x)/2]))/(Sqrt[-a + b] +
 I*Sqrt[a + b])] + PolyLog[2, (Sqrt[-a + b]*(1 + I*Tan[(c + d*x)/2]))/(Sqrt
[-a + b] + I*Sqrt[a + b])]))/(Sqrt[-a + b]*Sqrt[a + b]))*(a*d*e + a*d*f*x +
 b*f*Sin[c + d*x])*(Sqrt[a + b] - Sqrt[-a + b]*Tan[(c + d*x)/2])*(Sqrt[a +
b] + Sqrt[-a + b]*Tan[(c + d*x)/2]))/((a^2 - b^2)*d^2*(a + b*Cos[c + d*x])*
(a*(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*f*Log[1 + I*Tan[(c + d*
x)/2]]) + b*f*Sin[c + d*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(270) = 540$.

Time = 3.34 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.28

method	result
risch	$\frac{2i(fx+e)(a\,e^{i(dx+c)}+b)}{d(-a^2+b^2)(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b)} - \frac{2f\ln(e^{i(dx+c)})}{(-a^2+b^2)d^2} + \frac{f\ln(b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+b)}{(-a^2+b^2)d^2} + \frac{2iae\arctan\left(\frac{2b\,e^{i(dx+c)}+2a}{2\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{\frac{3}{2}}d} +$

```
[In] int((f*x+e)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(f*x+e)*(a*exp(I*(d*x+c))+b)/d/(-a^2+b^2)/(b*exp(2*I*(d*x+c))+2*a*exp(I
*(d*x+c))+b)-2/(-a^2+b^2)/d^2*f*ln(exp(I*(d*x+c)))+1/(-a^2+b^2)/d^2*f*ln(b*
exp(2*I*(d*x+c))+2*a*exp(I*(d*x+c))+b)+2*I/(-a^2+b^2)^(3/2)/d*a*e*arctan(1/
2*(2*b*exp(I*(d*x+c))+2*a)/(-a^2+b^2)^(1/2))+I/(-a^2+b^2)/d*f*a/(a^2-b^2)^(
1/2)*ln((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/((a^2-b^2)^(1/2)-a))*x-I/(-a^
2+b^2)/d*f*a/(a^2-b^2)^(1/2)*ln((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/((a^2-
b^2)^(1/2)+a))*x+I/(-a^2+b^2)/d^2*f*a/(a^2-b^2)^(1/2)*ln((-b*exp(I*(d*x+c))
+(a^2-b^2)^(1/2)-a)/((a^2-b^2)^(1/2)-a))*c-I/(-a^2+b^2)/d^2*f*a/(a^2-b^2)^(
1/2)*ln((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/((a^2-b^2)^(1/2)+a))*c+1/(-a^2
+b^2)/d^2*f*a/(a^2-b^2)^(1/2)*dilog((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/(
(a^2-b^2)^(1/2)-a))-1/(-a^2+b^2)/d^2*f*a/(a^2-b^2)^(1/2)*dilog((b*exp(I*(d*
x+c))+(a^2-b^2)^(1/2)+a)/((a^2-b^2)^(1/2)+a))-2*I/(-a^2+b^2)^(3/2)/d^2*a*f*
c*arctan(1/2*(2*b*exp(I*(d*x+c))+2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1482 vs. $2(266) = 532$.

Time = 0.46 (sec) , antiderivative size = 1482, normalized size of antiderivative = 5.01

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(
d*x + c) + I*a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2
 - b^2)/b^2) + b)/b + 1) - (a*b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2
)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(d*x + c) + I*b*si
n(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (a*b^2*f*cos(d*x + c) + a^2
*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c) + (b*
cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - (a*b^2
```

```
*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I
*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2)
 + b)/b + 1) - (-I*a^2*b*d*f*x - I*a^2*b*c*f + (-I*a*b^2*d*f*x - I*a*b^2*c*
f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - (I
*a^2*b*d*f*x + I*a^2*b*c*f + (I*a*b^2*d*f*x + I*a*b^2*c*f)*cos(d*x + c))*sq
rt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - (I*a^2*b*d*f*x + I*a^
2*b*c*f + (I*a*b^2*d*f*x + I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^2)
*log((a*cos(d*x + c) - I*a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c
))*sqrt((a^2 - b^2)/b^2) + b)/b) - (-I*a^2*b*d*f*x - I*a^2*b*c*f + (-I*a*b^
2*d*f*x - I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x +
 c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^
2)/b^2) + b)/b) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 - a*b^2)*f - (I*a^2*
b*d*e - I*a^2*b*c*f + (I*a*b^2*d*e - I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 -
 b^2)/b^2))*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2
)/b^2) + 2*a) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 - a*b^2)*f - (-I*a^2*b
*d*e + I*a^2*b*c*f + (-I*a*b^2*d*e + I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 -
 b^2)/b^2))*log(2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2
)/b^2) + 2*a) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 - a*b^2)*f - (I*a^2*b*
d*e - I*a^2*b*c*f + (I*a*b^2*d*e - I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b
^2)/b^2))*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)
/b^2) - 2*a) + ((a^2*b - b^3)*f*cos(d*x + c) + (a^3 - a*b^2)*f - (-I*a^2*b*
d*e + I*a^2*b*c*f + (-I*a*b^2*d*e + I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 -
b^2)/b^2))*log(-2*b*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2
)/b^2) - 2*a) + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*d*e)*sin(d*x + c))/(
(a^4*b - 2*a^2*b^3 + b^5)*d^2*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \int \frac{fx+e}{(b\cos(dx+c)+a)^2}\,dx$$

```
[In] integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*cos(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \text{Hanged}$$

```
[In] int((e + f*x)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] \text{Hanged}
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 1155

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if  not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
         #is checked before calling the grading function that is passed.
         #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```